Positron annihilation in the positron-helium ion $e^+[\mathrm{He}(2\,^3S)]$

Alexei M. Frolov
Department of Physics, Queen's University, Kingston, Ontario, Canada K7L 3N6
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The (e^-,e^+)-pair annihilation is considered in the positron-helium $e^+[\mathrm{He}(2\,^3S)]$ four-body ion, in which the positron forms a bound state with the helium atom in its lowest triplet state [i.e., in the $2\,^3S$ $(L=0)$ state]. A number of bound-state properties of this system have been determined for the $e^+[\mathrm{He}(2\,^3S)]$ ions, where the $^\infty$He, ^{4}He, and ^{3}He helium nuclei are considered. Some applications related to the positron annihilation in the $e^+[\mathrm{He}(2\,^3S)]$ ion are also discussed.

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I. INTRODUCTION

As is well known the positron e^+ does not form any bound state with the He atom, if this atom is in its ground singlet $1\,^1S$ $(L=0)$ state [1,2]. This follows from the very small dipole and quadrupole polarizabilities of the He atom in the ground $1\,^1S$ $(L=0)$ state. In general, the effective potential $V(r)$ of an atom in a nondegenerate S $(L=0)$ state has the following asymptotic form (at large r) [3]:

$$V(r) \sim -\frac{\alpha_1}{2r^4} - \left(\frac{\alpha_2}{2} + 3\beta_1\right)\frac{1}{r^6} + O\left(\frac{1}{r^7}\right), \qquad (1)$$

where α_1, α_2, and β_1 $(\beta_1 < 0)$ are the dipole polarizability, quadrupole polarizability, and nonadiabatic term (or distortion term), respectively. For the ground $1\,^1S$ $(L=0)$ state of the helium the atom the α_1, α_2, and β_1 values are small and the positron cannot form any bound state with such an atom. The situation, however, changes drastically for the excited states of the helium atom. In particular, the positron can form a bound state with the helium atom in its lowest triplet state [or $2\,^3S$ $(L=0)$ state] [4,5]. The arising positron-helium $e^+[\mathrm{He}(2\,^3S)]$ four-body ion is of considerable interest in numerous applications.

First, note that the $e^+[\mathrm{He}(2\,^3S)]$ ion is an extremely weakly bound Coulomb four-body system, i.e., the analysis of its geometry is very interesting. Second, the two-photon (e^-,e^+)-pair annihilation in some spin states of this positron-helium ion is strictly prohibited, and therefore only the three-photon annihilation is possible. In other words, a significant difference can be found in the lifetimes of different spin states in the $e^+[\mathrm{He}(2\,^3S)]$ ion. Furthermore, a relatively large lifetime of the quartet-spin state allows one to discuss a significant number of applications, including the positron conservation and annihilation from the accelerated (relativistic) positron-helium ions. In addition to this the hyperfine structure of the considered positron-helium ions $e^+[\mathrm{He}(2\,^3S)]$ is very interesting. To solve these and other similar problems one needs to know various bound-state properties of the positron-helium ions. The main goal of this study is to determine a large number of bound-state properties of the positron-helium $e^+[\mathrm{He}(2\,^3S)]$ ion and discuss some applications of this system related to the (e^-,e^+)-pair annihilation.

II. METHOD

The Hamiltonian of the considered positron-helium ion is written in the form (in atomic units $\hbar=1$, $m_e=1$, and $e=1$)

$$H = -\frac{1}{2M}\Delta_1 - \frac{1}{2}\Delta_2 - \frac{1}{2}\Delta_3 - \frac{1}{2}\Delta_4 + \frac{2}{r_{12}} - \frac{2}{r_{13}} - \frac{2}{r_{14}} - \frac{1}{r_{23}} - \frac{1}{r_{24}} + \frac{1}{r_{34}}, \qquad (2)$$

where the notation 1 designates the corresponding helium nucleus (below, the $^\infty$He, ^{3}He, ^{4}He nuclei are considered), the notation 2 means the positron, while 3 and 4 stand for electrons. The mass M means the nuclear mass of the considered helium isotope. In fact, all our present calculations have been performed in atomic units ($\hbar=1$, $m_e=1$, and $e=1$). In these units the following values for the nuclear masses [6,7] were used in our present calculations:

$$M_{^3He^{2+}} = 5495.8852, \quad M_{^4He^{2+}} = 7294.2996 \qquad (3)$$

The numerical values of other physical constants used in this study have been chosen from Refs. [6,7].

In this study the wave function of the $e^+[\mathrm{He}(2\,^3S)]$ ion is approximated by the variational expansion written in the basis of the six-dimensional (or four-body) Gaussian distribution originally proposed in Ref. [8]. The variational anzatz of fully correlated six-dimensional (or four-body) Gaussian distribution is written in the form [8]

$$\Psi_{L=0} = \mathcal{A}_{34}\sum_{k=1}^{N} C_k \exp(-\alpha_{12}^{(k)}r_{12}^2 - \alpha_{13}^{(k)}r_{13}^2 - \alpha_{23}^{(k)}r_{23}^2 - \alpha_{14}^{(k)}r_{14}^2 - \alpha_{24}^{(k)}r_{24}^2 - \alpha_{34}^{(k)}r_{34}^2), \qquad (4)$$

where C_k are the linear variational parameters, $\alpha_{ij}^{(k)}$ are the nonlinear parameters. The operator $\mathcal{A}_{34}$ designates the appropriate symmetrizer (or antisymmetrizer), i.e., a projection operator which produces the final wave function with the correct permutation symmetry. In fact, there is only one pair of identical particles (electrons) in all systems considered in this study. Therefore the operator $\mathcal{A}_{34}$ can be easily constructed in each of these cases.

TABLE I. The expectation values in atomic units ($m_e=1, \hbar=1, e=1$) of some properties for the ground states ($L=0$) of the positron-helium ions $e^+[^3\mathrm{He}(2\,^3S)]$, $e^+[^4\mathrm{He}(2\,^3S)]$, and $e^+[^\infty\mathrm{He}(2\,^3S)]$.

ion	$e^+[^3\mathrm{He}(2\,^3S)]$	$e^+[^4\mathrm{He}(2\,^3S)]$	$e^+[^\infty\mathrm{He}(2\,^3S)]$
E	-2.250227359	-2.250317665	-2.250593715
$\langle T\rangle$	2.250241630	2.250331901	2.250607820
$\langle V\rangle$	-4.500468604	-4.500649181	-4.501201112
χ[a]	0.32566×10^{-5}	0.32487×10^{-5}	0.32255×10^{-5}
$\langle r_{12}\rangle$	15.7719062	15.7664035	15.7495907
$\langle r_{13}\rangle$	8.04128962	8.03842264	8.02966246
$\langle r_{23}\rangle$	9.47212379	9.46941807	9.46115130
$\langle r_{34}\rangle$	15.3607970	15.3551045	15.3377112
$\langle r_{12}^2\rangle$	354.93970	354.67545	353.86846
$\langle r_{13}^2\rangle$	174.07621	173.94282	173.53542
$\langle r_{23}^2\rangle$	184.61159	184.47989	184.07772
$\langle r_{34}^2\rangle$	348.16400	347.89724	347.08256
$\langle r_{12}^3\rangle$	10860.14	10848.05	10811.14
$\langle r_{13}^3\rangle$	5366.951	5360.901	5342.417
$\langle r_{23}^3\rangle$	5479.220	5473.173	5454.716
$\langle r_{34}^3\rangle$	10755.99	10743.88	10706.88
$\langle r_{12}^4\rangle$	422650.1	422053.8	420233.9
$\langle r_{13}^4\rangle$	210012.8	209714.9	208805.5
$\langle r_{23}^4\rangle$	212021.6	211723.2	210812.3
$\langle r_{34}^4\rangle$	420889.4	420292.8	418471.9
$\langle r_{12}^{-1}\rangle$	0.09109743	0.09112044	0.09119079
$\langle r_{13}^{-1}\rangle$	1.05231354	1.05237398	1.05255887
$\langle r_{23}^{-1}\rangle$	0.28812206	0.28812935	0.28815161
$\langle r_{34}^{-1}\rangle$	0.10283353	0.10286343	0.10295488
$\langle r_{12}^{-2}\rangle$	0.01170523	0.01171006	0.01172662
$\langle r_{13}^{-2}\rangle$	4.01287782	4.01324438	4.01437300
$\langle r_{23}^{-2}\rangle$	0.24461941	0.24461551	0.24460184
$\langle r_{34}^{-2}\rangle$	0.01747345	0.01748194	0.01751216
$\langle -1/2\nabla_1^2\rangle$	2.01290539	2.01309380	2.01367511
$\langle -1/2\nabla_2^2\rangle$	0.12021867	0.12021590	0.12020425
$\langle -1/2\nabla_3^2\rangle$	1.06482930	1.06492088	1.06520187
$\langle \delta_{12}\rangle$	0.95927×10^{-6}	0.95980×10^{-6}	0.97023×10^{-6}
$\langle \delta_{13}\rangle$	1.272320	1.272491	1.273174
$\langle \delta_{23}\rangle$	0.0187995	0.0187989	0.0188440
$\langle \delta_{123}\rangle$	0.25073×10^{-5}	0.25091×10^{-5}	0.22981×10^{-5}

[a] $\chi=|1+\langle V\rangle/2\langle T\rangle|$ is the virial parameter which indicates the overall quality of the wave function used [19].

The appropriate energies and a number of other bound-state properties determined for the $e^+[^\infty\mathrm{He}(2\,^3S)]$, $e^+[^4\mathrm{He}(2\,^3S)]$, and $e^+[^3\mathrm{He}(2\,^3S)]$ ions can be found in Table I. To recalculate the obtained energies from a.u. to MHz and cm^{-1} the conversion factors $6.579\,683\,920\,61\times10^9$ and $2.194\,746\,306\,8\times10^7$ [6] must be used. As follows from Table I the $e^+[\mathrm{He}(2\,^3S)]$ ion is a very weakly bound four-body system. The total energy of this system almost coincides with the corresponding threshold energy $E_{tr}=-2.25$ a.u. which corresponds to the dissociation $e^+[\mathrm{He}(2\,^3S)]=\mathrm{Ps}+\mathrm{He}^+$ in the case of the helium ion with infinitely heavy nucleus. Here and below, the notation Ps designates the neutral two-body system e^+e^- (or positronium, for short).

The bound-state properties shown in Table I include the total E energy (in a.u.), the expectation values of the kinetic $\langle T\rangle$ and potential $\langle V\rangle$ energies for each of the considered $e^+[^3\mathrm{He}(2\,^3S)]$, $e^+[^4\mathrm{He}(2\,^3S)]$, and $e^+[^\infty\mathrm{He}(2\,^3S)]$ ions. Note that our variational energy for the $e^+[^\infty\mathrm{He}(2\,^3S)]$ ion from Table I ($E=-2.250\,593\,715$ a.u.) is slightly lower than the corresponding energy determined in Ref. [5] ($E=-2.250\,592\,4$ a.u.). The $e^+[^3\mathrm{He}(2\,^3S)]$ and $e^+[^4\mathrm{He}(2\,^3S)]$ ions were not considered in Ref. [5]. Also, a number of interparticle distances $\langle r_{ij}\rangle$ and their powers $\langle r_{ij}^n\rangle$ (where n

$=-2, -1, 1, 2, 3, 4$) are presented in Table I. These values can be used to understand the nontrivial geometry of the considered weakly bound $e^+[\mathrm{He}(2\,^3S)]$ ions. The expectation value of interparticle delta functions are also presented in Table I. Note that all expectation values $\langle\delta_{3,4,\ldots}\rangle$ which contain coordinates of both electrons simultaneously equal zero exactly, since these electrons form the triplet state. This means that we always have (in our notations the subscripts 3 and 4 designate the electrons)

$$\langle\delta_{34}\rangle = 0, \quad \langle\delta_{134}\rangle = 0, \quad \langle\delta_{234}\rangle = 0, \quad \langle\delta_{1234}\rangle = 0. \qquad (5)$$

These delta functions are not shown in Table I.

Table I also includes the single-particle kinetic energies $T_i = 1/2\langle\mathbf{p}_i^2\rangle = 1/2\langle -\nabla_i^2\rangle$ expectation values, where $i = 1,2,3(=4)$. It follows from Table I that the positron kinetic energy $1/2\langle\mathbf{p}_2^2\rangle$ is a really small value in comparison with the single-electron kinetic energy $1/2\langle\mathbf{p}_3^2\rangle \equiv 1/2\langle\mathbf{p}_4^2\rangle$. This is another indication that the $e^+[^\infty\mathrm{He}(2\,^3S)]$, $e^+[^4\mathrm{He}(2\,^3S)]$, and $e^+[^3\mathrm{He}(2\,^3S)]$ ions are extremely weakly bound systems in which the positron e^+ moves as an almost free particle. Note also that some of the properties presented in Table I can be used to determine a number of other bound-state properties, e.g., for the $\langle\mathbf{r}_{ik}\cdot\mathbf{r}_{jk}\rangle$ expectation value one finds

$$\langle\mathbf{r}_{ik}\cdot\mathbf{r}_{jk}\rangle = \frac{1}{2}(\langle\mathbf{r}_{ik}^2\rangle + \langle\mathbf{r}_{jk}^2\rangle - \langle\mathbf{r}_{ij}^2\rangle), \qquad (6)$$

where $i \neq j \neq k = (1,2,3,4)$. Analogous expressions for the $\langle\mathbf{p}_i\cdot\mathbf{p}_j\rangle$ expectation values are significantly more complicated than in the three-body case. The most interesting value, however, is the $\langle\mathbf{p}_2\cdot\mathbf{p}_3\rangle = \langle\mathbf{p}_+\cdot\mathbf{p}_-\rangle$ expectation value. This value determines the electron-positron momentum correlation in the $e^+[\mathrm{He}(2\,^3S)]$ ions. Furthermore, if the expectation value $\langle\mathbf{p}_2\cdot\mathbf{p}_3\rangle$ is known, then all other interparticle momentum correlations are uniformly determined from the following relations:

$$\langle\mathbf{p}_1\cdot\mathbf{p}_3\rangle = \frac{1}{2}\left(\left\langle -\frac{1}{2}\nabla_2^2\right\rangle - \left\langle -\frac{1}{2}\nabla_1^2\right\rangle\right) + \langle\mathbf{p}_2\cdot\mathbf{p}_3\rangle, \qquad (7)$$

$$\langle\mathbf{p}_3\cdot\mathbf{p}_4\rangle = \frac{1}{2}\left(\left\langle -\frac{1}{2}\nabla_1^2\right\rangle - \left\langle -\frac{1}{2}\nabla_2^2\right\rangle\right) - \left\langle -\frac{1}{2}\nabla_3^2\right\rangle - 2\langle\mathbf{p}_2\cdot\mathbf{p}_3\rangle, \qquad (8)$$

$$\langle\mathbf{p}_1\cdot\mathbf{p}_2\rangle = \left\langle -\frac{1}{2}\nabla_1^2\right\rangle + \frac{1}{2}\left\langle -\frac{1}{2}\nabla_3^2\right\rangle - 2\langle\mathbf{p}_2\cdot\mathbf{p}_3\rangle, \qquad (9)$$

where $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_4$, and $\mathbf{p}_1$ are the positron, electron(s), and nuclear momenta, respectively.

Note that the $\langle\mathbf{p}_i\cdot\mathbf{p}_j\rangle$ expectation values are of interest in a number of applications. In particular, these values can be used to evaluate the isotopic shifts in the $e^+[^M\mathrm{He}(2\,^3S)]$ ions. In general, the isotopic shift between the ground-state energies of the $e^+[^3\mathrm{He}(2\,^3S)]$ and $e^+[^4\mathrm{He}(2\,^3S)]$ ions can be evaluated as the following expectation value (in atomic units):

$$\Delta E = \frac{1}{2}\left(\frac{1}{M_{^3\mathrm{He}}} - \frac{1}{M_{^4\mathrm{He}}}\right)\langle\Psi|(\mathbf{p}_2+\mathbf{p}_3+\mathbf{p}_4)^2|\Psi\rangle = \left(\frac{1}{M_{^3\mathrm{He}}} - \frac{1}{M_{^4\mathrm{He}}}\right)\left(\frac{1}{2}\left\langle -\frac{1}{2}\nabla_2^2\right\rangle + \left\langle -\frac{1}{2}\nabla_3^2\right\rangle + 2\langle\mathbf{p}_2\cdot\mathbf{p}_3\rangle + \langle\mathbf{p}_3\cdot\mathbf{p}_4\rangle\right), \qquad (10)$$

where the subscripts 3, 4, and 2 are used to designate the electrons and positron, respectively. This formula exactly coincides with the expression for the three-electron atom/ion. In other words, from this formula one cannot see the explicit difference between the positron and electron. By generalizing the Vinti hypervirial theorem [9] to the one-center, three-electron systems one can reduce this formula to a number of different forms. Some of these forms show explicitly the difference between the positron and electrons.

In our present calculations we have used the wave function, Eq. (4), with 600 basis functions. To optimize the 6×600 nonlinear parameters the conjugate direction method (or Powell's method) has been applied. It is also interesting to note that by using the classical James and Coolidge [10] variational expansion it is hard to show the boundness of the $e^+[\mathrm{He}(2\,^3S)]$ ion in those cases when only one exponential function is used in all basis functions (see, e.g., Refs. [11,12]). The total number of nonlinear parameters in such cases equals 3. However, with the use of two different exponential functions (six nonlinear parameters) in two different families of basis functions this state is certainly bound. These results will be published elsewhere.

III. HYPERFINE STRUCTURE OF THE POSITRON-HELIUM IONS

By using our results from Table I one can determine the hyperfine structure of the considered $e^+[^3\mathrm{He}(2\,^3S)]$ and $e^+[^4\mathrm{He}(2\,^3S)]$ ions. The hyperfine splitting of the ground state in the $e^+[\mathrm{He}(2\,^3S)]$ ion is given by the expectation value of the following operator:

$$H_{HF} = \frac{8\pi}{3}(\mathbf{m}_+\cdot\mathbf{m}_N)\delta(\mathbf{r}_{N+}) + \frac{8\pi}{3}(\mathbf{m}_+\cdot\mathbf{m}_-)\delta(\mathbf{r}_{+-}) + \frac{8\pi}{3}(\mathbf{m}_-\cdot\mathbf{m}_N)\delta(\mathbf{r}_{N-}), \qquad (11)$$

where $\mathbf{m}_+$, $\mathbf{m}_-$, and $\mathbf{m}_N$ are the magnetic moments of the positron, electron, and nucleus, respectively. The Dirac delta function $\delta(\mathbf{r}_{ij})$ is determined traditionally, i.e., $\delta(\mathbf{r}_{ij}) = \delta(\mathbf{r}_i - \mathbf{r}_j)$, where $i \neq j = (+,-,N)$. In our present notations the subscripts $(+,-,N)$ correspond to the indexes 2, 3 ($\equiv 4$), and 1, respectively. Following Ref. [13] it can be shown that the spin-space expectation value $\langle H_{HF}\rangle$ (i.e., spin operator) for the $e^+[\mathrm{He}(2\,^3S)]$ ion can be represented in the form

$$\delta H_s = \langle H_{HF}\rangle = a(\mathbf{s}_+\cdot\mathbf{I}_N) - b(\mathbf{S}_-\cdot\mathbf{s}_+) - c(\mathbf{S}_-\cdot\mathbf{I}_N), \qquad (12)$$

where $\mathbf{s}_+$, $\mathbf{S}_-$, and $\mathbf{I}_N$ are the spin vectors of the positron, two electrons (which are already in the triplet state), and nucleus

[13], respectively. The sum of the first and third (i.e., last) term in Eq. (12) can be considered as the Fermi-Serge term in the Hamiltonian of the $e^+[^3\mathrm{He}(2\,^3S)]$ ion. In Eq. (12) the constants a, b, and c have the following values (in atomic units):

$$a = A\langle \delta(\mathbf{r}_{N+})\rangle = \frac{2\pi}{3}\alpha^2\frac{g_e g_N}{m_p}\langle \delta(\mathbf{r}_{N+})\rangle, \qquad (13)$$

$$b = B\langle \delta(\mathbf{r}_{+-})\rangle = \frac{2\pi}{3}\alpha^2 g_e^2\langle \delta(\mathbf{r}_{+-})\rangle, \qquad (14)$$

$$c = A\langle \delta(\mathbf{r}_{N-})\rangle = \frac{2\pi}{3}\alpha^2\frac{g_e g_N}{m_p}\langle \delta(\mathbf{r}_{N-})\rangle, \qquad (15)$$

where α is the fine-structure constant, m_e and m_p are the electron and proton masses, respectively. Here, we used the fact that in atomic units $m_e=1$ and the Bohr magneton equals $1/2$ exactly. The fine-structure constant α, proton mass m_p, and g factors used in calculations were chosen from Refs. [6,7],

$$\alpha = 7.297\,353\,08 \times 10^{-3},$$

$$m_p = 1836.152\,701,$$

$$g_e = 2.002\,319\,304\,386,$$

$$g_N(^3\mathrm{He}) = 4.255\,249\,6,$$

$$g_N(^4\mathrm{He}) = 0.$$

In fact, the hyperfine splitting is traditionally expressed in MHz. To recalculate the energies from a.u. to MHz the following conversion factor $6.579\,683\,920\,61 \times 10^9$(MHz/ a.u.) [6] has been used. Now, one can easily calculate the A and B constants for the $e^+[^3\mathrm{He}(2\,^3S)]$ and $e^+[^3\mathrm{He}(2\,^3S)]$ ions,

$$A(^3\mathrm{He}) = 3405.210\,335\,3 \quad \mathrm{MHz},$$

$$A(^4\mathrm{He}) = 0,$$

$$B(^3\mathrm{He}) = B(^4\mathrm{He}) = 733\,828.145\,34 \quad \mathrm{MHz}.$$

By using the A and B coefficients and δ-function expectation values from Table I one easily finds the a, b, and c coefficients in Eq. (12)

$$a(^3\mathrm{He}) = 3.2665 \quad \mathrm{kHz},$$

$$a(^4\mathrm{He}) = 0,$$

$$b(^3\mathrm{He}) = 13\,795.60 \quad \mathrm{MHz},$$

$$b(^4\mathrm{He}) = 13\,795.16 \quad \mathrm{MHz},$$

$$c(^3\mathrm{He}) = 4332.52 \quad \mathrm{MHz},$$

$$c(^4\mathrm{He}) = 0.$$

In turn, these coefficients allow one to determine the actual hyperfine splitting for different spin states in the $e^+[^4\mathrm{He}(2\,^3S)]$ and $e^+[^3\mathrm{He}(2\,^3S)]$ ions. It is interesting to note that the electron-nuclear and electron-positron hyperfine splittings in the $e^+[^3\mathrm{He}(2\,^3S)]$ ion have the same order of magnitude, while analogous positron-nuclear splitting is a very small value which can be always considered as a perturbation.

IV. POSITRON ANNIHILATION

Note that the helium atom in the lowest triplet state [or $2\,^3S$ ($L=0$) state] is a remarkably stable atomic system. At relatively low densities of upper solar atmosphere the lifetime of the helium atom in its $2\,^3S$ ($L=0$) state can be incredibly large (from a few weeks up to four–six months). This follows from the fact that optical transition from this state to the ground $1\,^1S$ ($L=0$) state is strictly forbidden. In fact, the presence of two groups of series in the optical spectrum of helium was the source of long-time confusion, since spectroscopists believed that they were dealing with the two different elements, which were named orthohelium and parahelium, respectively. The parahelium was also often called asterium (or nebulium). Later, it was shown that helium is definitely a simple chemical element. The double-helium mystery was finally solved only around the middle of 1920's by Pauli. For our present purposes it is important to note that the extremely long lifetime of the helium atom in its lowest $2\,^3S$ ($L=0$) state is sufficient to create and observe the $e^+[\mathrm{He}(2\,^3S)]$ four-body ion. In general, the stability of this $e^+[\mathrm{He}(2\,^3S)]$ ion sharply depends upon its total spin multiplicity.

The total multiplicity of the bound $e^+[\mathrm{He}(2\,^3S)]$ ion can be determined by using the fact that the two electrons are already in the triplet state. This means that only doublet and quartet spin configurations are possible for this ion. The corresponding terms occur in the ratio 1:1. The doublet state of the $e^+[\mathrm{He}(2\,^3S)]$ ion is unstable and rapidly decays by the Auger transition to the ground $1\,^1S$ ($L=0$) state of the helium atom. The kinetic energy of the emitted positron is ≈ 0.7285 eV. The preliminary evaluations indicate that the Auger transition rate in this case is quite comparable and even larger than the two-photon annihilation rate. The spin-averaged two-photon annihilation rate $\overline{\Gamma}_{2\gamma}$ in the $e^+[\mathrm{He}(2\,^3S)]$ ion which also includes the first-order radiative correction [14] can be evaluated as follows (for more details, see the Appendix):

$$\overline{\Gamma}_{2\gamma} = \left[1 - \frac{\alpha}{\pi}\left(5 - \frac{\pi^2}{4}\right)\right] n\frac{2}{6}\left(4\pi\alpha^4 c a_0^{-1}\right)\langle \delta(\mathbf{r}_{+-})\rangle \approx 1.333\,674 \times 10^{12}\langle \delta(\mathbf{r}_{+-})\rangle\ \mathrm{sec}^{-1}, \qquad (16)$$

where α is the fine-structure constant ($\alpha \approx 7.297\,353\,08 \times 10^{-3}$), c is the speed of light ($c \approx 2.997\,924\,58 \times 10^8$ m sec^{-1}), and a_0 is the Bohr radius ($a_0 \approx 0.529\,177\,249 \times 10^{-10}$ m) [6]. Also, in this equation n is the total number of (e^-,e^+) pairs and $\langle\delta(\mathbf{r}_{+-})\rangle$ is the expectation value of the electron-positron Dirac delta function.

Note that the $\bar{\Gamma}_{2\gamma}$ rate in Eq. (16) is the value which is averaged over the initial spin states (i.e., over all multiplicities of the doublet and quartet states in the $e^+[\mathrm{He}(2\,^3S)]$ ion). In particular, by using the expectation value of the $\delta(\mathbf{r}_{+-})$ function from Table I $[\langle\delta(\mathbf{r}_{+-})\rangle\approx 1.879\,72\times 10^{-2}]$ it is easy to obtain that $\bar{\Gamma}_{2\gamma}\approx 2.5069\times 10^{9}\mathrm{sec}^{-1}$, while the lifetime $1/\bar{\Gamma}_{2\gamma}\approx 3.9890\times 10^{-10}$sec. In the case of a pure doublet state one finds $\Gamma_{2\gamma}=3\bar{\Gamma}_{2\gamma}\approx 7.5207\times 10^{9}\mathrm{sec}^{-1}$. As expected this $\Gamma_{2\gamma}$ value is very close to the annihilation rate of the para-positronium $\Gamma_{2\gamma}(e^+e^-)=7.9852\times 10^{9}\,\mathrm{sec}^{-1}$. Such a coincidence follows from the fact that the 1*s* electron of the helium atom does not contribute significantly to the (e^-,e^+)-pair annihilation in the $e^+[\mathrm{He}(2\,^3S)]$ ions. In other words, the annihilation of the (e^-,e^+) pair in the $e^+[\mathrm{He}(2\,^3S)]$ ions proceeds without any substantial contribution from the internal atomic 1*s* electron. However, as is mentioned above, the two-photon annihilation in the doublet $^2\mathrm{S}_{J=1/2}$ state of the $[e^+\mathrm{He}(2\,^3S)]$ ion always proceeds in competition with the helium-positron Auger transition.

For the quartet spin state of the $e^+[\mathrm{He}(2\,^3S)]$ ion the situation differs from the doublet state considered above. In this case the Auger transition is forbidden (we assume that there are no collisions, i.e., the overall atomic density is relatively low). In other words, the decay of the quartet $^4\mathrm{S}_{J=3/2}$ state of the $e^+[\mathrm{He}(2\,^3S)]$ ion is possible only due to the (e^-,e^+)-pair annihilation. However, it is easy to understand that only the three-photon annihilation of the (e^-,e^+) pair can proceed in this state. The two-photon annihilation for the triplet (e^-,e^+) pair is strictly prohibited. For the quartet $^4\mathrm{S}_{J=3/2}$ state the formula for the three-photon annihilation rate takes the form

$$\begin{aligned}\Gamma_{3\gamma} &= \frac{16(\pi^2-9)}{9}n\alpha^5 c a_0^{-1}\langle\delta(\mathbf{r}_{+-})\rangle\\ &\approx 1.812\,365\times 10^{8} n\langle\delta(\mathbf{r}_{+-})\rangle\\ &\approx 6.813\,48\times 10^{6}\quad \mathrm{sec}^{-1},\end{aligned}\tag{17}$$

where the total number of the triplet (e^-,e^+) pairs in the considered case (i.e., $n=2$). Also, in this formula α is the fine-structure constant and c is the speed of light and a_0 is the Bohr radius. In Eq. (17) the expectation value $\langle\delta(\mathbf{r}_{+-})\rangle\approx 1.879\,72\times 10^{-2}$ has been used. Again the computed $\Gamma_{3\gamma}$ value is very close to the orthopositronium annihilation rate $\Gamma_{3\gamma}(e^+e^-)=7.2112\times 10^{6}\,\mathrm{sec}^{-1}$. The spin-averaged three-photon annihilation rate is $\bar{\Gamma}_{3\gamma}=2/3\Gamma_{3\gamma}\approx 4.542\,32\times 10^{6}\,\mathrm{sec}^{-1}$. The lifetime of the quartet $^4\mathrm{S}_{J=3/2}$ state in the $e^+[\mathrm{He}(2\,^3S)]$ ion is $\approx 1/\Gamma_{3\gamma}\approx 1.4677\times 10^{-7}$ sec, i.e., a relatively large value in comparison to the lifetime of any other positron containing a few-body system. This allows one to consider the $^4\mathrm{S}_{J=3/2}$ state of the $e^+[\mathrm{He}(2\,^3S)]$ ion as an ideal atomic system to keep positrons before the (e^-,e^+)-pair annihilation. A few related applications are considered below.

V. APPLICATIONS. DISCUSSION AND CONCLUSION

In general, the $^4\mathrm{S}_{J=3/2}$ state of the $e^+[\mathrm{He}(2\,^3S)]$ ion can be stabilized only in some combinations of the strong electromagnetic fields (below, EM fields, for short). If such a stabilizing EM field is rapidly transformed to some destabilizing field combination, then the annihilation rate in the $e^+[\mathrm{He}(2\,^3S)]$ ion suddenly increases 300–1000 times. This means that, in principle, we can observe and control (at least partially) the γ flash at the laboratory conditions. Another interesting application is related with a possibility to accelerate the $e^+[\mathrm{He}(2\,^3S)]$ ions to very large (i.e., relativistic) energies. In this case, due to the Doppler effect the emitted annihilation γ quanta can be observed as much harder γ rays. Indeed, the observed frequency (ω') of the emitted γ quantum is related to the incident frequency (ω) of the same γ quantum by the relation

$$\omega' = \gamma_L\omega(1-\beta\cos\theta),\tag{18}$$

where γ_L is the corresponding Lorentz γ factor, $\beta=v/c$ and θ is the angle relative between the emitted γ quantum and direction of $\mathbf{v}$ ($\mathbf{v}$ is the velocity of the accelerated $e^+[\mathrm{He}(2\,^3S)]$ ion**)**. By using modern accelerators one can reach $\gamma_L=100$–1000, i.e., the observed frequency ω' can be significantly larger than the incident annihilation frequency ω_a ($\omega_a\approx 1.235\,589\,72\times 10^{14}$ MHz).

Note also that the classical [15,16] one-photon annihilation of the (e^-,e^+) pair cannot proceed in the $e^+[\mathrm{He}(2\,^3S)]$ ion. This follows from the fact that the expectation value of the triple delta function $\delta(\mathbf{r}_{+--})\equiv\delta(\mathbf{r}_{234})$ equals zero exactly in the considered $e^+[\mathrm{He}(2\,^3S)]$ ion [17]. However, the one-photon annihilation can proceed "at the nuclear surface," since the expectation value of the corresponding delta function $\delta(\mathbf{r}_{123})$ is not zero (see Table I). In general, for the one-photon annihilation rate in the $e^+[\mathrm{He}(2\,^3S)]$ ion one can write

$$\Gamma_\gamma = \Gamma_\gamma^{(e)} + \Gamma_\gamma^{(n)} = y\frac{32\pi^2}{27}\alpha^8 c a_0^{-1}\langle\delta(\mathbf{r}_{+--})\rangle + \Gamma_\gamma^{(n)} = \Gamma_\gamma^{(n)},\tag{19}$$

where $\Gamma_\gamma^{(n)}\sim\langle\delta(\mathbf{r}_{123})\rangle$ is the one-photon annihilation at the nuclear surface and factor $y\approx 1$ [12]. In contrast with this in the positronium hydride PsH both channels of one-photon annihilation are open. In other words, by studying the one-photon annihilation in the $e^+[\mathrm{He}(2\,^3S)]$ ion one can determine the internal (nuclear) conversion rate for annihilation γ quanta. In the analogous five-body $e^+[(\mathrm{Li}^{3+}\mu^-)e_2^-(2\,^3S)]$ system such an internal conversion of the annihilation γ quanta can produce the photodetachment of the complex two-body nucleus $\mathrm{Li}^{3+}\mu^-$. In this case, the annihilation of the (e^-,e^+) pair is followed by the emission of negatively charged muon μ^-. The five-body ion $e^+[(\mathrm{Li}^{3+}\mu^-)e_2^-(2\,^3S)]$ with two electrons in the triplet states has the same electronic structure as the $e^+[\mathrm{He}(2\,^3S)]$ ion considered above. In particular, the positron can form the bound state with the four-body $(\mathrm{Li}^{3+}\mu^-)e_2^-$ quasiatom, if this quasiatom is in its $2\,^3S$ ($L=0$) state (electron state). Note that this five-body ion $e^+[(\mathrm{Li}^{3+}\mu^-)e_2^-(2\,^3S)]$ has a very complicated hyperfine structure and (e^-,e^+)-pair annihilation in this ion can be affected by the presence of negatively charged muon μ^-. The analysis of the five-body $e^+[(6\mathrm{Li}^{3+}\mu^-)e_2^-(2\,^3S)]$ and $e^+[(7\mathrm{Li}^{3+}\mu^-)e_2^-(2\,^3S)]$ ions is the goal of our future studies.

In conclusion, it is important to note that currently the $e^{+}[\mathrm{He}(2\,^{3}S)]$ ion is of significant experimental and theoretical interest. In particular, the study of the (e^{-},e^{+})-pair annihilation in this system can improve our understanding of many atomic and QED processes. Furthermore, the analysis of bound-state properties of this weakly bound four-body system is a quite complex problem which requires an extensive development of new numerical methods and algorithms.

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APPENDIX

Let us present the formulas which describe the two- and three-photon annihilation rates of the nonrelativistic (e^{-},e^{+}) pairs. In general, the two- $(\Gamma_{2\gamma})$ and three-photon $(\Gamma_{3\gamma})$ annihilation rates for the singlet and triplet (e^{-},e^{+}) pairs are written in the forms [18]

$$\Gamma_{2\gamma} = 4\pi\alpha^{4}ca_{0}^{-1}\langle\delta(\mathbf{r}_{+-})\rangle \approx 2.012\,350\times 10^{11}\langle\delta(\mathbf{r}_{+-})\rangle\ \mathrm{sec}^{-1} \tag{A1}$$

and

$$\Gamma_{3\gamma} = \frac{16(\pi^{2}-9)}{9}\alpha^{5}ca_{0}^{-1}\langle\delta(\mathbf{r}_{+-})\rangle \approx 5.675\,550 \times 10^{8}\langle\delta(\mathbf{r}_{+-})\rangle\ \mathrm{sec}^{-1}, \tag{A2}$$

respectively. Note that these formulas explicitly contain the expectation value of the electron-positron delta function $\delta(\mathbf{r}_{+-})$. For the considered positron-containing, many-electron compounds these expressions must be multiplied by the total number of the singlet/triplet electron-positron pairs (n), corresponding statistical weights of the considered singlet/triplet spin states, and by the factor which includes the lowest-order radiative correction to the two-photon annihilation rate [14].

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