Teleportation with insurance of an entangled atomic state via cavity decay

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We propose a scheme to teleport an entangled state of two Λ-type three-level atoms via photons. The teleportation protocol involves the local redundant encoding protecting the initial entangled state and allowing for repeating the detection until quantum information transfer is successful. We also show how to manipulate a state of many Λ-type atoms trapped in a cavity.

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I. INTRODUCTION

Practical quantum computation requires considering systems containing a scalable number of qubits. Recently, schemes have been proposed that employ more than two qubits to perform various quantum information tasks [1–3]. There is also an interest in performing quantum teleportation of state of more than one qubit. Lee [4] has presented a setup for teleportation of an entangled state of two photons. The scheme, as some other schemes [2,5–12], uses photons because they propagate fast and can carry quantum information over long distances. On the other hand, photonic states are much worse for the storage of the quantum information than atomic states. The scheme does not provide a way to store the quantum information and therefore it will be difficult to use in quantum computing. Another problem is that the scheme works only with a 50% success rate. Bose *et al.* [7] have proposed a scheme to teleport the state of one atom using photonic states as carriers of quantum information. In the scheme, the quantum information is stored in atomic states but the probability of successful teleportation is about 50% . The protocol of the teleportation can be repeated to teleport an entangled state of two atoms. This method, however, has at the most only a 25% success rate. Recently, Browne *et al.* [8] have shown that, under weak driving conditions, generation of entanglement between distant atoms can be performed with arbitrarily high probability. It seems that weak driving can also increase the success rate in Ref. [7].

In the present work, we propose a scheme that allows the teleportation of an entangled state of two atoms with insurance. Our device employs atomic states for storage and photonic states to transfer quantum information. There are two distinguishing features of our protocol. The first of them is that the probability of successful teleportation of the initial entangled state is about 49%. The second of them is that the initial state is not lost when the detection stage is unsuccessful because of using *local redundant encoding* [13]. Hence the teleportation procedure can be repeated until the quantum transfer is successful.

The paper is organized as follows: In Sec. II, we describe the teleportation device in detail. In Sec. III, we show the operations manipulating a state of many atoms trapped in a cavity. In Secs. IV and V, we present the protocol of the teleportation. Section VI gives the numerical results.

II. MODEL

We consider the device composed of one cavity with three atoms inside, one cavity with two atoms inside, a 50-50 beam splitter, two lasers L_A and L_B with right- and two lasers L'_A and L'_B with left-circular polarized radiation and two detectors D_+ and D_-. The system is shown in Fig. 1. The atoms are assumed to be located in fixed positions along a line in a linear trap or an optical lattice inside an optical cavity. We also assume that the atoms are separated by at least one optical wavelength so they can be addressed individually by two different laser fields. The propagation directions of the two laser beams are very close to each other so as to allow for effective transfer of photons from one beam to the other mediated by the atom. Introducing two laser beams allows for resetting the atomic states.

The cavity with two atoms inside and two lasers (L_B, L'_B) with different polarizations belong to Bob. The sender, Alice, has the other parts of the device. All the trapped atoms are modeled by three-level Λ systems with an excited state $|2\rangle$ and two ground states $|0\rangle$ and $|1\rangle$ as shown in Fig. 2. The

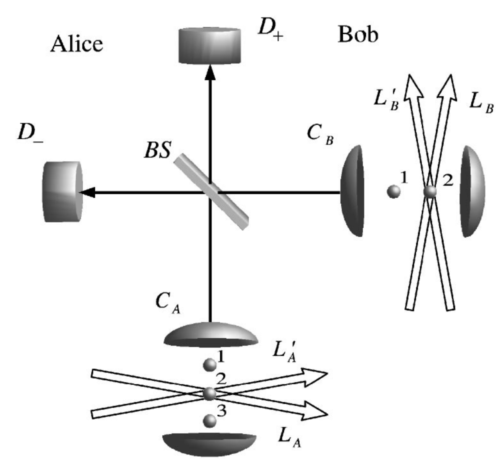

FIG. 1. Schematic representation of the entangled-state teleportation device. The state of Alice's atoms 1 and 2 is teleported to Bob's atoms 1 and 2.

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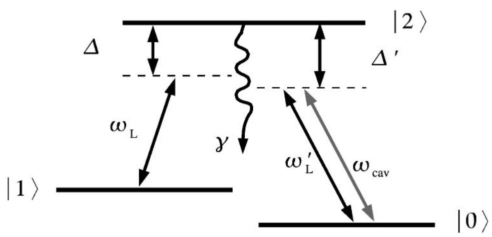

FIG. 2. Level scheme of one of the identical Λ atoms interacting with two classical laser fields and the quantized cavity mode.

excited state spontaneously decays with a rate γ. The transition $|0\rangle \leftrightarrow |2\rangle$ is coupled to the cavity mode with frequency ω_{cav} and coupling strength g. The transition is also driven by a classical laser field with frequency ω'_{L} which is the same as the cavity mode frequency. The coupling strength for this transition is denoted by Ω'. Another classical laser field with different polarization couples to the $|1\rangle \leftrightarrow |2\rangle$ transition with the coupling strength Ω. The frequency of the laser field is ω_{L}. We define two detunings $\Delta = (E_2 - E_1)/\hbar - \omega_{\text{L}}$ and $\Delta' = (E_2 - E_0)/\hbar - \omega_{\text{cav}}$.

The evolution of the system is conditional. Photon detection corresponds to the action of the operator,

$$C = \sqrt{\kappa}(a_A + i\epsilon a_B) , \tag{1}$$

where a_A and a_B denote the annihilation operators for Alice's and Bob's cavity modes, respectively, κ denotes the cavity decay rate, and ϵ is equal to unity when there is a click in the detector D_+ or minus unity for a click in D_-. Between the emissions evolution of the system is governed by the effective non-Hermitian Hamiltonian ($\hbar = 1$),

$$H = \sum_k (\Delta - i\gamma)\sigma_{22}^{(k)} - \sum_k \Delta_r \sigma_{00}^{(k)} - i\kappa a^\dagger a + \sum_k (\Omega \sigma_{21}^{(k)} + g a \sigma_{20}^{(k)} + \Omega' \sigma_{20}^{(k)} + \text{H.c.}) , \tag{2}$$

where $\Delta_r = \Delta' - \Delta$. In Eq. (2) we define $\sigma_{ij}^{(k)} \equiv (|i\rangle\langle j|)_k$, where $i,j = 0,1,2$ for the kth atom. In the far off resonance limit when $\Delta \gg \Omega$ and $\Delta' \gg \Omega', g$, we can eliminate adiabatically the level $|2\rangle$ [14–16]. The conditions have to be even more restrictive in our teleportation protocol because only then can we properly estimate phase-shift factors for long evolution times. Therefore we assume $10^{-1}\Delta \gg \Omega$ and $10^{-1}\Delta' \gg \Omega', g$. In order to simplify the Hamiltonian (2) we also assume that $\gamma \ll \Delta, \Delta'$ and the product of the excited level saturation parameter and the spontaneous decay rate is much smaller than the decay rate of the cavity mode ($\gamma g^2/\Delta'^2, \gamma\Omega'^2/\Delta'^2, \gamma\Omega^2/\Delta^2 \ll \kappa$). Under these conditions we can neglect the influence of the spontaneous decay rate on teleportation. Otherwise, the probability of success will be much lower as it was proved in Ref. [17]. These assumptions were also used in another quantum information process of entangled state preparation [18]. With these assumptions, after adiabatic elimination of the excited state $|2\rangle$ of the atoms, the Hamiltonian takes the form

$$H = -\sum_k \Delta_r \sigma_{00}^{(k)} - i\kappa a^\dagger a - \sum_k (\delta_1 \sigma_{11}^{(k)} + \delta_2 \sigma_{00}^{(k)} + \delta_3 a^\dagger a \sigma_{00}^{(k)}) - \sum_k (\delta_4 \sigma_{10}^{(k)} + \delta_5 a \sigma_{10}^{(k)} + \delta_6 a \sigma_{00}^{(k)} + \text{H.c.}) , \tag{3}$$

where $\delta_1 = \Omega^2/\Delta$, $\delta_2 = \Omega'^2/\Delta'$, $\delta_3 = g^2/\Delta'$, $\delta_4 = \Omega\Omega'(\Delta^{-1} + \Delta'^{-1})/2$, $\delta_5 = g\Omega(\Delta^{-1} + \Delta'^{-1})/2$, and $\delta_6 = g\Omega'/\Delta'$. The parameters $\delta_1 - \delta_6$ account for various contributions to the effective Hamiltonian, for example, δ_4 describes the transfer of photon from one laser beam to the other via coupling to the atom, δ_6 describes the transfer of photon from a cavity into the laser L' beam, etc. In this approximation all the atomic dynamics are restricted to the ground states $|0\rangle$ and $|1\rangle$ which can be treated as atomic qubits.

III. QUANTUM OPERATIONS

In our teleportation protocol we need certain transformations or quantum operations, which applied to a given state of the system (quantum register) transform it into another state. Such operations are performed with the unitary evolution operator e^{-iHt} applied to the state of the system. It is assumed that only one atom is illuminated at a time and that the laser fields are such that $\Omega \gg \Omega' \gg g$. It is useful to distinguish between the results of the action of the evolution operator onto particular states of the system and write down explicitly the results for some special cases. We list a number of local operations that can be performed by Alice and Bob on their states.

To fix and simplify the notation we label the Alice atoms with numbers (1,2,3) and Bob's atoms with numbers (1,2). Let us denote a state of the system of $n = (2 \text{ or } 3)$ atoms trapped in the cavity with y photons by $|x_1 \cdots x_n y\rangle$ where x_k is the kth atom state ($k = 1, \ldots, n$). This means that the state of the Alice part has the form $|x_1 x_2 x_3 y\rangle$ and the Bob part has the form $|x_1 x_2 y\rangle$, for example, $|1110\rangle = |1\rangle_1 |1\rangle_2 |1\rangle_3 |0\rangle$ means that at the Alice site atoms (1,2,3) are all in their state $|1\rangle$ and the cavity field is in state $|0\rangle$. In our notation, the first two or three numbers in the ket denote atomic states with labels increasing from left to right, and the last number is reserved for the field state. The joint state of the entire system can be described in the basis that is formed by the product states of the Alice part and the Bob part. Later on, we use the simplified notation $|x_1 x_2 x_3 y\rangle_A \otimes |x_1 x_2 y\rangle_B = |x_1 x_2 x_3 y; x_1 x_2 y\rangle$.

The simplest operation is just waiting for an arbitrary time t while all of the lasers are turned off. In this case both Ω and Ω' are set to zero, and we can use the simplified Hamiltonian given by

$$H = -\sum_{k=1}^{n} (\Delta_r \sigma_{00}^{(k)} + \delta_3 a^\dagger a \sigma_{00}^{(k)}) - i\kappa a^\dagger a . \tag{4}$$

During this operation the evolution of the system is given by

$$e^{-iHt}|x_1 \cdots y\rangle = e^{iN_0(\Delta_r + y\delta_3)t} e^{-y\kappa t}|x_1 \cdots y\rangle , \tag{5}$$

where N_0 is the number of atoms being in state $|0\rangle$ that are not illuminated by the laser field. In order to simplify the following transformations we assume $\delta_1 = \Delta_r$. Moreover, we want the probability of no collapse during the encoding stage to be close to unity. This can be done provided that $\delta_5 \gg \kappa$. Those assumptions imply that $\Delta_r \gg \kappa, \delta_3$ and therefore the Hamiltonian can be written as $H = -\Delta_r \sigma_{00}^{(k)}$ for the operation time $t \approx \Delta_r^{-1}$. Thus for short times the evolution simplifies to

$$e^{-iHt}|x_1 \cdots y\rangle = \alpha^{N_0}(t)|x_1 \cdots y\rangle , \tag{6}$$

where $\alpha(t) = e^{i\Delta_r t}$.

As the next local operation we consider the illumination of the kth atom by the laser field driving $|1\rangle \leftrightarrow |2\rangle$ transition $(\Omega \neq 0)$ while the second laser field coupled to $|2\rangle \leftrightarrow |0\rangle$ is turned off $(\Omega' = 0)$. We can use this laser field to get a number of useful transformations. The transformations times are of the order of δ_5^{-1} and therefore under conditions $\delta_1 = \Delta_r$ and $x_k + y > 0$ the system evolution can be well approximated by the relation

$$e^{-iHt}|x_1 \cdots x_k \cdots y\rangle = f_{N_0,\beta_k}(t)[\cos(\xi_k t)|x_1 \cdots x_k \cdots y\rangle + i\sin(\xi_k t) \times |x_1 \cdots x_k' \cdots y'\rangle] , \tag{7}$$

where $\xi_k = \sqrt{\beta_k}\delta_5$, $\beta_k = x_k + y$, $f_{N_0,\beta_k}(t) = \alpha^{N_0+1}(t)\exp\{i\ \delta_3[\beta_k(2N_0+1) - N_0]t/2\}$, $x_k' = x_k - (-1)^{x_k+1}$ and $y' = y + (-1)^{x_k+1}$. One can see that we are able to perform different transformations by illuminating the kth atom for different times.

We map the atomic state onto the cavity mode by choosing the interaction time $t^{(1)} = (\pi/2 + 2n\pi)/\delta_5$ where n is an integer, according to

$$|x_1 \cdots 1 \cdots 0\rangle \to i f_{N_0,1}(t^{(1)})|x_1 \cdots 0 \cdots 1\rangle , \tag{8}$$

$$|x_1 \cdots 0 \cdots 1\rangle \to i f_{N_0,1}(t^{(1)})|x_1 \cdots 1 \cdots 0\rangle . \tag{9}$$

Of course the interaction time $t^{(2)} = (3\pi/2 + 2n\pi)/\delta_5$ also leads to mapping of the atomic state,

$$|x_1 \cdots 1 \cdots 0\rangle \to -i f_{N_0,1}(t^{(2)})|x_1 \cdots 0 \cdots 1\rangle , \tag{10}$$

$$|x_1 \cdots 0 \cdots 1\rangle \to -i f_{N_0,1}(t^{(2)})|x_1 \cdots 1 \cdots 0\rangle . \tag{11}$$

If we turn the laser on for time $t^{(3)} = (\pi/4 + 2n\pi)/\delta_5$ then we create a maximally entangled state of the illuminated atom and the cavity system according to

$$|x_1 \cdots 1 \cdots 0\rangle \to \frac{f_{N_0,1}(t^{(3)})}{\sqrt{2}}(i|x_1 \cdots 0 \cdots 1\rangle + |x_1 \cdots 1 \cdots 0\rangle) , \tag{12}$$

$$|x_1 \cdots 0 \cdots 1\rangle \to \frac{f_{N_0,1}(t^{(3)})}{\sqrt{2}}(i|x_1 \cdots 1 \cdots 0\rangle + |x_1 \cdots 0 \cdots 1\rangle) . \tag{13}$$

Interaction for time $t^{(4)} = (2n\pi)/\delta_5$ or $t^{(5)} = ([2n+1]\pi)/\delta_5$ gives only the phase factor,

$$|x_1 \cdots x_k \cdots y\rangle \to f_{N_0,1}(t^{(4)})|x_1 \cdots x_k \cdots y\rangle , \tag{14}$$

$$|x_1 \cdots x_k \cdots y\rangle \to -f_{N_0,1}(t^{(5)})|x_1 \cdots x_k \cdots y\rangle . \tag{15}$$

The transformations can also be performed for the states with $\beta_k = 2$. The states mapping can be done for the illumination time $t^{(6)} = (\pi/2 + 2m\pi)/(\sqrt{2}\delta_5)$ as

$$|x_1 \cdots 1 \cdots 1\rangle \to i f_{N_0,2}(t^{(6)})|x_1 \cdots 0 \cdots 2\rangle , \tag{16}$$

$$|x_1 \cdots 0 \cdots 2\rangle \to i f_{N_0,2}(t^{(6)})|x_1 \cdots 1 \cdots 1\rangle \tag{17}$$

or for the time $t^{(7)} = (3\pi/2 + 2m\pi)/(\sqrt{2}\delta_5)$ as

$$|x_1 \cdots 1 \cdots 1\rangle \to -i f_{N_0,2}(t^{(7)})|x_1 \cdots 0 \cdots 2\rangle , \tag{18}$$

$$|x_1 \cdots 0 \cdots 2\rangle \to -i f_{N_0,2}(t^{(7)})|x_1 \cdots 1 \cdots 1\rangle , \tag{19}$$

where m is an integer. Interaction for the time $t^{(8)} = (\pi/4 + 2m\pi)/(\sqrt{2}\delta_5)$ leads to the generation of a maximally entangled state,

$$|x_1 \cdots 1 \cdots 1\rangle \to \frac{1}{\sqrt{2}} f_{N_0,2}(t^{(8)})(i|x_1 \cdots 0 \cdots 2\rangle + |x_1 \cdots 1 \cdots 1\rangle) , \tag{20}$$

$$|x_1 \cdots 0 \cdots 2\rangle \to \frac{1}{\sqrt{2}} f_{N_0,2}(t^{(8)})(i|x_1 \cdots 1 \cdots 1\rangle + |x_1 \cdots 0 \cdots 2\rangle),$$

and illumination for the time $t^{(9)} = (2m\pi)/(\sqrt{2}\delta_5)$ or $t^{(10)} = ([2m+1]\pi)/(\sqrt{2}\delta_5)$ generates only a phase factor,

$$|x_1 \cdots x_k \cdots y\rangle \to f_{N_0,2}(t^{(9)})|x_1 \cdots x_k \cdots y\rangle , \tag{21}$$

$$|x_1 \cdots x_k \cdots y\rangle \to -f_{N_0,2}(t^{(10)})|x_1 \cdots x_k \cdots y\rangle . \tag{22}$$

There is a special state with $\beta_k = 0$. If a laser is turned on, the state accumulates a phase shift but the population of the state remains unchanged as described by

$$e^{-iHt}|x_1 \cdots 0 \cdots 0\rangle = \alpha^{N_0+1}(t)|x_1 \cdots 0 \cdots 0\rangle . \tag{23}$$

When we want to map an arbitrary superposition of the two atomic ground states onto the cavity mode then this feature of the state is very desired. However, when the detection stage is unsuccessful in our protocol, the population transfer is necessary to repeat the teleportation process. Therefore we have to use another local operation consisting in simultaneous applying of two laser fields with different polarizations (for instance, L_A and L_A'). For evolution times of the order of $t \approx \delta_4^{-1}$ we can neglect in the Hamiltonian all terms much smaller than δ_4, then the approximate state dynamics are given by

$$e^{-iHt}|x_1 \cdots 0 \cdots 0\rangle = \alpha^{N_0+1}(t)[i\sin(\delta_4 t)|x_1 \cdots 1 \cdots 0\rangle + \cos(\delta_4 t) \times |x_1 \cdots 0 \cdots 0\rangle] , \tag{24}$$

$$e^{-iHt}|x_1 \cdots 1 \cdots 0\rangle = \alpha^{N_0+1}(t)[i\sin(\delta_4 t)|x_1 \cdots 0 \cdots 0\rangle + \cos(\delta_4 t) \times |x_1 \cdots 1 \cdots 0\rangle] . \tag{25}$$

It is evident that by using $\pi/2$ pulse we can change the atom state even if the cavity field mode is empty. This case can be described by

$$|x_1\cdots 0\cdots 0\rangle \to i\alpha^{N_0+1}(t^{(11)})|x_1\cdots 1\cdots 0\rangle\ , \tag{26}$$

$$|x_1\cdots 1\cdots 0\rangle \to i\alpha^{N_0+1}(t^{(11)})|x_1\cdots 0\cdots 0\rangle\ , \tag{27}$$

where $t^{(11)}=\pi/(2\delta_4)$. It is also possible to create superposition of two ground states by using a $\pi/4$ pulse,

$$|x_1\cdots 0\cdots 0\rangle \to \frac{\alpha^{N_0+1}(t^{(12)})}{\sqrt{2}}(|x_1\cdots 0\cdots 0\rangle + i|x_1\cdots 1\cdots 0\rangle)\ , \tag{28}$$

$$|x_1\cdots 1\cdots 0\rangle \to \frac{\alpha^{N_0+1}(t^{(12)})}{\sqrt{2}}(|x_1\cdots 1\cdots 0\rangle + i|x_1\cdots 0\cdots 0\rangle)\ , \tag{29}$$

where $t^{(12)}=\pi/(4\delta_4)$.

The set of transformations listed above forms necessary ingredients for the teleportation protocols we present in the next two sections.

IV. TELEPORTATION OF A STATE OF A SINGLE ATOM

Let us first demonstrate the idea of teleportation with insurance on teleportation of a single quantum state. The unknown state which Alice wants to teleport is stored in her atom $a|0\rangle+b|1\rangle$. In order to be sure that any detection event does not destroy the original state Alice has to employ local redundant encoding [13]. This technique entangles the atom with another atom which plays a role of backup qubit. Instead of another atom we use the cavity mode as a backup qubit to simplify the following considerations. The entangled state of the atom and the cavity mode should be given by

$$a(|00\rangle_A + |11\rangle_A) + b(|01\rangle_A + |10\rangle_A)\ . \tag{30}$$

Let the initial state of the Alice cavity mode be in a superposition of a vacuum state and one photon state $(|1\rangle+|0\rangle)$. This state can be easily generated using another atom placed in the cavity. The initial Alice's state is given by

$$a(|01\rangle_A + |00\rangle_A) + b(|11\rangle_A + |10\rangle_A)\ . \tag{31}$$

One can see that Alice needs to swap one pair of the state amplitudes without exchanging the second pair of amplitudes to code the initial state of her system to the form given by Eq. (30). The objective has to be achieved using only the quantum operations presented in Sec. III. This can be done and Alice performs this in the encoding stage. The whole teleportation protocol consists of four stages: (A) The encoding stage, (B) The detection stage I, (C) The detection stage II, and (D) The recovery stage.

A. Encoding stage

Alice needs four steps to code her initial state.

(i) First, Alice illuminates her atom using both her lasers L_A and L'_A for time $t_1=\pi/(4\delta_4)$. This corresponds to operations given by Eqs. (28) and (29). Then the unnormalized state of Alice system becomes

$$|\tilde{\Psi}\rangle_A = a(|01\rangle_A + i|11\rangle_A + |00\rangle_A + i|10\rangle_A) + b(|11\rangle_A + i|01\rangle_A + |10\rangle_A + i|00\rangle_A)\ . \tag{32}$$

(ii) Next, Alice turns off her laser L'_A while the laser L_A still illuminates the atom. Alice needs to perform transformations (14) and (22) to differentiate the phase factor of the state $|11\rangle$ from phase factors of other states. This operation is similar to the quantum phase gate. Alice turns off the laser L_A after time t_2 which has to fulfill the conditions $t_2\delta_5=2n\pi$ and $t_2\sqrt{2}\delta_5=(2m+1)\pi$. This can be done only approximately for $n=6$ and $m=8$. After this step the state of Alice's system can be well approximated by

$$|\tilde{\Psi}\rangle_A = a(e^{i\delta_3(t_2/2)}|01\rangle_A - ie^{i\delta_3 t_2}|11\rangle_A + |00\rangle_A + ie^{i\delta_3(t_2/2)}|10\rangle_A) + b(-e^{i\delta_3 t_2}|11\rangle_A + ie^{i\delta_3(t_2/2)}|01\rangle_A + e^{i\delta_3(t_2/2)}|10\rangle_A + i|00\rangle_A)\ . \tag{33}$$

(iii) In the third step Alice just waits for time t_3 while her lasers are turned off. The evolution of the Alice system is given by Eq. (6). If condition $\alpha(t_3)=-e^{i\delta_3(t_2/2)}$ is satisfied then the state becomes

$$|\tilde{\Psi}\rangle_A = a(e^{i\delta_3(t_2/2)}(|01\rangle_A + i|11\rangle_A) + |00\rangle_A - i|10\rangle_A) + b(e^{i\delta_3(t_2/2)}\times(|11\rangle_A + i|01\rangle_A) - |10\rangle_A + i|00\rangle_A)\ . \tag{34}$$

(iv) In the fourth step of the encoding stage Alice again turns on both her lasers L_A and L'_A for time $t_1=\pi/(4\delta_4)$ performing operations (28) and (29). After this step she turns the lasers off and her system state is given by

$$|\tilde{\Psi}\rangle_A = e^{i\delta_3(t_2/2)}i(b|01\rangle_A + a|11\rangle_A) + a|00\rangle_A - b|10\rangle_A\ . \tag{35}$$

Although Alice's system state (35) differs from desired state (30) by phase factors, the initial state of Alice's atom is protected and cannot be destroyed by any detection event. If Alice observes a photon then her system is projected onto state $b|00\rangle_A+a|10\rangle_A$, otherwise when she does not register any click she is left with the state $a|00\rangle_A-b|10\rangle_A$. One can see that Alice needs only a one-qubit flip gate or a one-qubit phase gate to recover her original state.

In the encoding stage Bob performs only operation (12). Initially his atom is prepared in state $|1\rangle$ and the field mode in his cavity is empty. He turns on his laser L_B for time $t_4=\pi/(4\delta_5)$ creating a maximally entangled state of his atom and field mode in his cavity,

$$|\Psi\rangle_B = \frac{1}{\sqrt{2}}(|10\rangle_B + i|01\rangle_B)\ . \tag{36}$$

We assume that Bob's operation terminates at the same instant of time as the fourth Alice's step and therefore after the encoding stage the joint state of their systems is given by

$$|\tilde{\Psi}\rangle = [e^{i\delta_3(t_2/2)}i(b|01\rangle_A + a|11\rangle_A) + a|00\rangle_A - b|10\rangle_A] \otimes (|10\rangle_B + i|01\rangle_B)\ . \tag{37}$$

B. Detection stage I

The second stage of the protocol performs the joint detection of the Alice cavity and the Bob cavity modes. Alice just waits for a finite time $t_d \gg \kappa^{-1}$ registering clicks in her detectors. The stage will be successful only if a single click occurs. Nevertheless, we expect that even if Alice does not register any click or registers two clicks the original state can be recovered because of using local redundant encoding. All lasers are turned off and therefore the evolution of the joint state of Alice's and Bob's systems is given by

$$|\tilde{\Psi}\rangle = [e^{i\delta_3(t_2/2)} i e^{-\kappa t}(b e^{i\Delta_r t} e^{i\delta_3 t}|01\rangle_A + a|11\rangle_A) + a e^{i\Delta_r t}|00\rangle_A - b|10\rangle_A] \otimes (|10\rangle_B + i e^{i\Delta_r t} e^{i\delta_3 t} e^{-\kappa t}|01\rangle_B) . \qquad (38)$$

If Alice does not detect any photon during the stage then $\exp(-\kappa t_d) \approx 0$ and thus the joint state becomes

$$|\tilde{\Psi}\rangle = (b|10\rangle_A - a e^{i\Delta_r t_d}|00\rangle_A) \otimes |10\rangle_B . \qquad (39)$$

We chose such time t_d that $\exp(i\Delta_r t_d) = -1$ and therefore Alice's system state is $a|00\rangle_A + b|10\rangle_A$. This is one of the two unsuccessful cases and one can see that the original state is not lost.

If evolution is interrupted by the click of one of the detectors then the jump operator (1) acts on the joint state (38). Next, in the absence of any laser field the evolution is given by

$$\begin{aligned}|\tilde{\Psi}\rangle = {} & e^{i\delta_3(t_2/2)}(b e^{i\Delta_r t} e^{i\delta_3 t_j}|00\rangle_A|10\rangle_B + a|10\rangle_A|10\rangle_B) \\ & + i\epsilon e^{i\Delta_r t} e^{i\delta_3 t_j}(a e^{i\Delta_r t}|00\rangle_A|00\rangle_B - b|10\rangle_A|00\rangle_B) \\ & + i e^{-\kappa t} e^{i\delta_3(t_2/2)} e^{i\Delta_r t} e^{i\delta_3 t_j}[e^{i\Delta_r t} e^{i\delta_3 t_j} e^{i\delta_3(t-t_j)} b(i\epsilon|01\rangle_A|00\rangle_B \\ & + |00\rangle_A|01\rangle_B) + a(i\epsilon|11\rangle_A|00\rangle_B + e^{i\delta_3(t-t_j)}|10\rangle_A|01\rangle_B)] .\end{aligned} \qquad (40)$$

If Alice registers the second click at time t_c then the global system state becomes

$$|\tilde{\Psi}\rangle = b e^{i\Delta_r t_c} e^{i\delta_3 t_c}(\epsilon + \epsilon_1)|00\rangle_A|00\rangle_B + a(\epsilon + \epsilon_1 e^{i\delta_3(t_c - t_j)}) \times |10\rangle_A|00\rangle_B . \qquad (41)$$

Although Alice has performed local redundant encoding her initial state is destroyed in this case. This is due to the presence of the factors $(\epsilon + \epsilon_1)$ and $(\epsilon + \epsilon_1 e^{i\delta_3(t_c - t_j)})$ changing the population of the two states in a random way. When the cavity mode is not empty then the second term of Hamiltonian (4) leads to accumulation of phase shifts. These phase shifts depend on the number of atoms N_0 which are in state $|0\rangle$. Therefore only the states $|01\rangle_A$ and $|01\rangle_B$ collect the phase $\delta_3(t - t_j)$ in superposition (40). If all the states had the same N_0 in this stage then recovery of the initial state would be possible. Therefore an entangled state of two atoms after using local redundant encoding can be recovered in both unsuccessful detection cases. Recovery of a state of a single atom is possible for every detection event in another scheme as it has been shown very recently in Ref. [19].

If Alice registers only one click during the detection time t_d then quantum information transfer will be successful and under the assumption $\exp(i\Delta_r t_d) = -1$ the joint state will be given by

$$\begin{aligned}|\tilde{\Psi}\rangle = {} & e^{i\delta_3(t_2/2)}(a|10\rangle_A|10\rangle_B - b e^{i\delta_3 t_j}|00\rangle_A|10\rangle_B) \\ & + i\epsilon e^{i\delta_3 t_j}(a|00\rangle_A|00\rangle_B + b|10\rangle_A|00\rangle_B) .\end{aligned} \qquad (42)$$

One can see that Alice's and Bob's systems are still in the entangled state. In order to complete the quantum transfer Alice has to remove this entanglement by making a measurement of her atom state.

C. Detection stage II

In the third stage of the protocol Alice measures the state of her atom while Bob waits with his laser turned off. She needs two steps to perform the detection.

(i) First, Alice maps her atom state onto her cavity mode. She turns on her laser for time $t_4 = \pi/(2\delta_5)$ performing transformations (8) and (23).

(ii) Next, she waits for time t_d making a measurement of the fields leaking from the cavities. We again assume that $\exp(i\Delta_r t_d) = -1$. All lasers are turned off thus the evolution of the joint system is given by

$$\begin{aligned}|\tilde{\Psi}\rangle = {} & i e^{-\kappa t} e^{i\delta_3 t} e^{i\delta_3(t_4/2)}|01\rangle_A \otimes (a e^{i\delta_3(t_2/2)}|10\rangle_B \\ & + b i\epsilon e^{i\delta_3 t_j} e^{i\Delta_r t_4} e^{i\Delta_r t}|00\rangle_B) - e^{i\delta_3 t_j}|00\rangle_A \otimes (b e^{i\delta_3(t_2/2)}|10\rangle_B \\ & - a i\epsilon e^{i\Delta_r t_4} e^{i\Delta_r t}|00\rangle_B) .\end{aligned} \qquad (43)$$

If Alice detects one photon during time t_d then Bob's system state after this step is given by

$$|\tilde{\Psi}\rangle = a|10\rangle_B + b\theta|00\rangle_B , \qquad (44)$$

where $\theta = -i\epsilon \exp(i\Delta_r t_4)\exp[i\delta_3(t_j - t_2/2)]$. Otherwise, when Alice does not detect any photon in this stage then Bob's system state becomes

$$|\tilde{\Psi}\rangle = b|10\rangle_B + a\phi|00\rangle_B , \qquad (45)$$

where $\phi = i\epsilon \exp(i\Delta_r t_4)\exp(-i\delta_3 t_2/2)$. After the measurement Alice's system remains in state $|00\rangle_A$.

D. Recovery stage

After the detection stage II Alice informs Bob about her measurements results. Now, Bob has to perform local operations to recover the initial state of Alice's atom. His actions depend on the results.

If a click occurs in the third stage then he needs two steps. First, he waits for such time t_θ that $\theta e^{i\Delta_r t_\theta} = 1$. Second, he illuminates his atom, using both his lasers, for time $t_5 = \pi/(2\delta_4)$. In this way he realizes a one-qubit flip gate using transformations (26) and (27).

If no photon is detected during the third stage then Bob simply waits for time t_ϕ that the condition $\phi e^{i\Delta_r t_\phi} = 1$ is satisfied.

After the protocol is over Bob's atom state is given by $a|0\rangle + b|1\rangle$.

V. TELEPORTATION OF AN ENTANGLED STATE

The teleportation protocol we propose here makes it possible to teleport an entangled state of two atoms with insurance. The state to be teleported is an entangled state of atoms 1 and 2 at the Alice site, which is

$$|\psi_0\rangle = a\,|1\rangle_1|0\rangle_2 + b\,|0\rangle_1|1\rangle_2. \tag{46}$$

The third atom in the Alice cavity and both Bob's atoms are prepared in their states $|1\rangle$. The field modes in both cavities are initially empty. Thus the joint state of the entire system is initially given by

$$|\Psi(0)\rangle = (a\,|1010\rangle_A + b\,|0110\rangle_A) \otimes |110\rangle_B = a\,|1010;110\rangle + b\,|0110;110\rangle\,. \tag{47}$$

We assume that this state is given or prepared before the protocol starts.

The teleportation protocol consists of five stages: (A) the preparation stage, (B) the encoding stage, (C) the detection stage I, (D) the detection stage II, and (E) the recovery stage. In each stage there are a number of steps to be performed in order to get, finally, the required result.

A. Preparation stage

The aim of the preparation stage of the protocol is to create a maximally entangled state of the third Alice atom and the first Bob atom. This can be done by following the distant atom entangling technique of Ref. [20]. This stage consists of three steps.

(i) First, Alice and Bob perform transformation given by Eq. (8). They simply illuminate, using lasers L_A and L_B their atoms, i.e., Alice's atom 3 and Bob's atom 1, for the time $t_1=\pi/(2\delta_5)$. After this operation each cavity is in one photon state.

(ii) Next, they wait until either of Alice's detectors clicks. All lasers are turned off and therefore, before the detection event, evolution of their systems is described by Eq. (6). One photon registered by Alice corresponds to an action of the collapse operator (1) and leads to the creation of a maximally entangled state of both cavity fields.

(iii) After the detection event, Alice and Bob have to turn on the lasers L_A and L_B, immediately. They illuminate the two atoms for time t_1 performing the transformation given by Eq. (9). This operation leads to mapping and storage of the entangled state of both cavity fields in the state of Alice's atom 3 and Bob's atom 1. This concludes creating a maximally entangled state of the two atoms and then the global system state is given by

$$|\Psi\rangle = a|1000;110\rangle + ai\epsilon e^{(i/2)\delta_3 t_1}|1010;010\rangle + b|0100;110\rangle + bi\epsilon e^{(i/2)\delta_3 t_1}|0110;010\rangle\,. \tag{48}$$

B. Encoding stage

The encoding stage is introduced to apply the local redundant encoding [13] in which Alice codes the entangled state of her first two atoms (atoms 1 and 2), that is to be teleported, to the entangled state of four atoms (atoms 1, 2, 3 of Alice and atom 1 of Bob). The third Alice atom and the first Bob atom are the backup atoms which allow protection of the teleported state in the case of the protocol failure in the detection stage. The encoding consists of a sequence of four steps.

(i) First of them is mapping the state of the first Alice atom onto the cavity mode by illuminating (using laser L_A) the atom for time t_1. This corresponds to the transformations given by Eqs. (8) and (23). During the operation Bob's lasers are turned off and therefore he uses transformation (6). After the operation the unnormalized joint state becomes

$$|\widetilde{\Psi}\rangle = ia\alpha_1 e^{i(3/2)\delta_3 t_1}|0001;110\rangle - a\epsilon\alpha_1 e^{i(3/2)\delta_3 t_1}|0011;010\rangle + b|0100;110\rangle + ib\epsilon e^{(i/2)\delta_3 t_1}|0110;010\rangle\,, \tag{49}$$

where $\alpha_1=\alpha(t_1)$.

(ii) The second step of the encoding stage is illuminating the third Alice atom. One can see that $\beta_2=1$ in all terms of the superposition (49). The purpose of the second operation is to make β_2's different. The Rabi frequency scales with β and therefore we can perform independently different transformations for different values of β. Alice switches the laser L_A on for the appropriate interaction time leading to the transformations (8), (9), (16), and (23). It is clear that the time has to satisfy conditions $t_2\delta_5=\pi/2+2n\pi$ and $t_2\sqrt{2}\delta_5=\pi/2+2m\pi$. This can be done only approximately for $n=7$ and $m=10$. During this step Bob waits with lasers turned off thus the evolution of state of his system is given by Eq. (6). After this operation we achieve the state close to

$$|\widetilde{\Psi}\rangle = b|0100;110\rangle - b\epsilon\alpha_2 e^{i\delta_3[(1/2)t_1+t_2]}|0101;010\rangle - a\alpha_1\alpha_2 e^{i(3/2)\delta_3(t_1+t_2)}|0010;110\rangle - ia\epsilon\alpha_1\alpha_2^2 e^{i\delta_3[(3/2)t_1+4t_2]} \times|0002;010\rangle\,, \tag{50}$$

where $\alpha_2=\alpha(t_2)$. We neglect the low populated states in the superposition (50) but we include them as all other imperfections of the operation in our numerical calculations.

(iii) The third step is the most important at the encoding stage. The previous two steps are intended to prepare the third one, which creates the entangled state of three atoms and the cavity field. In order to make the entangled state Alice has to swap one pair of the state amplitudes without exchanging the second pair of amplitudes. Alice can do that by turning on the L_A laser and illuminating her second atom for the time which leads to completing the transformations (14), (16), (17), and (23). Here, we meet the same problem as in the second step because the illuminating time has to satisfy two conditions: $t_3\delta_5=2n\pi$ and $t_3\sqrt{2}\delta_5=\pi/2+2m\pi$. We can find an approximate solution for $n=3$ and $m=4$. In this step Bob's lasers are turned off. Just as in the previous step, we neglect the states for which the population is close to zero and obtain

$$|\tilde{\Psi}\rangle = a\alpha_1\alpha_2 e^{i(3/2)\delta_3(t_1+t_2)}|0010;110\rangle - a\epsilon\alpha_1\alpha_2^2\alpha_3^2 e^{i\delta_3[(3/2)t_1+4t_2+4t_3]}|0101;010\rangle - b\alpha_3 e^{i(3/2)\delta_3 t_3}|0100;110\rangle + ib\epsilon\alpha_2\alpha_3^2 e^{i\delta_3[(1/2)t_1+t_2+4t_3]} \times|0002;010\rangle , \quad (51)$$

where $\alpha_3=\alpha(t_3)$. Although the entangling is already done, one can see that the state (51) is not protected yet. For instance, if the two-photon state is detected then the initial state of the first two Alice atoms will be lost.

(iv) In order to change the state (51) into a protected state Alice illuminates her third atom using the L_A laser. This is the fourth step of the encoding stage. Alice needs to perform transformations (14), (17), and (23), therefore the illumination time has to fulfill the conditions $t_4\delta_5=2n\pi$ and $t_4\sqrt{2}\delta_5 =\pi/2+2m\pi$. It is obvious that the time is the same as for the previous operation time and thus $\alpha_3=\alpha_4=\alpha(t_4)$. We again neglect low populated states. During this step Bob performs two operations. First, Bob waits for time $t_4-t_1/2$ with lasers turned off. Next he creates a maximally entangled state of his second atom and his cavity. For this purpose he turns the laser L_B on for time $t_1/2$ performing the transformation given by Eq. (12). Alice and Bob perform their actions in such a way that they end the fourth step at the same time. Then the global system is given by

$$|\tilde{\Psi}\rangle = (a\alpha_1\alpha_2 e^{i(3/2)\delta_3(t_1+t_2)}|0010\rangle_A - b|0100\rangle_A) \otimes (i|101\rangle_B + |110\rangle_B) - (a\epsilon\alpha_1\alpha_2^2\alpha_3^2 e^{i\delta_3[(7/4)t_1+4t_2+(7/2)t_3]}|0101\rangle_A + b\epsilon\alpha_2\alpha_3^3 e^{i\delta_3[(3/4)t_1+t_2+(13/2)t_3]}|0011\rangle_A) \otimes (i|001\rangle_B + |010\rangle_B) . \quad (52)$$

This is the end of the encoding stage. If we wanted to store the protected state we would map the cavity state to the first Alice atom state. Then we would have the entangled state of four atoms. However, we want the photonic state to be the state (52) because we use the cavity field for quantum information transfer.

C. Detection stage I

The third stage of the protocol is the first detection stage, in which Alice just waits for time $t_D \gg \kappa^{-1}$ making a measurement of the fields leaking from the cavities. The detection of one photon only leads to the quantum information transfer. If Alice does not detect any photon or detects two photons the teleportation process will be unsuccessful. However, even then, quantum information will be safe owing to the local redundant encoding. In the absence of any laser field the evolution is given by Eq. (5). If Alice does not detect any photon in this stage the state evolves into

$$|\Psi\rangle = - a\alpha_1\alpha_2 e^{i(3/2)\delta_3(t_1+t_2)}|0010;110\rangle + b|0100;110\rangle . \quad (53)$$

This is one of two unsuccessful cases. The initial state which Alice wanted to teleport is modified by phase shift factors but it is not lost. The modified initial state is stored in the second and third Alice atoms. In order to repeat the whole protocol Alice has to reset her first atom. She turns on both her lasers (L_A and L_A') for the time $t_5=\pi/(2\delta_4)$ performing transformation (26). During the resetting operation both Bob's lasers are turned off.

If the evolution given by Eq. (5) is interrupted by a collapse at time $t_j < t_D$ then the jump operator C acts on the global system state. After that the transformation (5) continues changing the state. If Alice registers the second click of either of her detectors the joint state becomes

$$|\Psi\rangle = a\alpha_1\alpha_2\alpha_3^{-1} e^{i\delta_3(t_1+3t_2-3t_3)}|0100;000\rangle + b|0010;000\rangle . \quad (54)$$

It is evident that the Alice initial state is not destroyed also in the second case when the step is unsuccessful. Before the protocol can be repeated Alice has to prepare her first atom in the state $|1\rangle$ and Bob has to prepare both of his atoms in the state $|1\rangle$ using transformation (26). They reset the atoms in two steps. First, Alice and Bob turn on all their lasers (L_A, L_A', L_B, and L_B') for the time $t_5=\pi/(2\delta_4)$. Alice and Bob illuminate their first atoms. Next, Bob illuminates for the time t_5, using both his lasers, his second atom while Alice waits with lasers turned off.

If there is no second photon detection then the quantum information transfer is done and the global system state is given by

$$|\tilde{\Psi}\rangle = a\epsilon\alpha_1\alpha_2^2\alpha_3^2 e^{i\delta_3[(7/4)t_1+4t_2+(7/2)t_3+t_j]}|0100;010\rangle - b\epsilon_1|0100;100\rangle + a\epsilon_1\alpha_1\alpha_2 e^{i(3/2)\delta_3(t_1+t_2)}|0010;100\rangle + b\epsilon\alpha_2\alpha_3^3 e^{i\delta_3[(3/4)t_1+t_2+(13/2)t_3+t_j]}|0010;010\rangle . \quad (55)$$

After this stage Alice shares the information about her initial state with Bob. Now Alice's initial state can be send to Bob, but it is also possible for Bob to send it back to Alice. We will not consider the case when Bob sends back the state.

D. Detection stage II

In the fourth stage of the protocol Alice measures the state of her third atom. During the stage Bob waits with lasers turned off. This stage consists of two steps.

(i) First, Alice turns the L_A laser on and illuminates the third atom performing transformations (8) and (23).

(ii) After this she turns the laser off and the evolution is given by Eq. (5) which leads to the joint state

$$|\tilde{\Psi}\rangle = a\epsilon\alpha_1\alpha_2^2\alpha_3^2 e^{i\delta_3[(7/4)t_1+4t_2+(7/2)t_3+t_j]}|0100;010\rangle - b\epsilon_1|0100;100\rangle + e^{i(\Delta_r+3\delta_3)t_D}e^{-\kappa t_D} \times[ib\epsilon\alpha_1\alpha_2\alpha_3^3 e^{i\delta_3[(9/4)t_1+t_2+(13/2)t_3+t_j]}|0001;010\rangle + ia\epsilon_1\alpha_1^2\alpha_2 e^{i(3/2)\delta_3(2t_1+t_2)}|0001;100\rangle] . \quad (56)$$

Alice again waits for time t_D making a measurement of the fields leaking from the cavities. The detection of one photon corresponds to an action of the jump operator C on the global state (56). In this case the state becomes

$$|\Psi\rangle = a\epsilon\epsilon_1\alpha_1\alpha_3^{-3}e^{i\delta_3[(3/4)t_1+(1/2)t_2-(13/2)t_3-t_j]}|0000;100\rangle + b|0000;010\rangle . \quad (57)$$

Otherwise, when Alice has not detected any photon, the joint state is given by

$$|\Psi\rangle = -a\epsilon\epsilon_1\alpha_1\alpha_2^2\alpha_3^2e^{i\delta_3[(7/4)t_1+4t_2+(7/2)t_3+t_j]}|0100;010\rangle + b|0100;100\rangle . \quad (58)$$

E. Recovery stage

Generally, Alice and Bob may need the protocol to be repeated several times until Alice registers only one click in the third stage. It is easy to prove that after $\mathcal{N}$ repetitions of the protocol Bob's system state is given by $a\theta|100\rangle_B+b|010\rangle_B$ if Alice has detected one photon in the fourth stage and $a\phi|010\rangle_B+b|100\rangle_B$ if Alice has not registered any detection in the fourth stage, where

$$\theta = \epsilon\epsilon_1\alpha_1^{\mathcal{N}+1}\alpha_2^{\mathcal{N}}\alpha_3^{-3} \times\exp\left[(i/2)\delta_3\left(\frac{3}{2}t_1+t_2-13t_3-2t_j\right)\right]\mu_0^{\mathcal{N}_0}\mu_2^{\mathcal{N}_2},$$

$$\phi = -\epsilon\epsilon_1\alpha_1^{\mathcal{N}+1}\alpha_2^{\mathcal{N}+2}\alpha_3^{2} \times\exp\left[(i/2)\delta_3\left(\frac{7}{2}t_1+8t_2+7t_3+2t_j\right)\right]\mu_0^{\mathcal{N}_0}\mu_2^{\mathcal{N}_2},$$

$\mu_0=-\exp[i3/2\,\delta_3(t_1+t_2)]$, and $\mu_2=\alpha_3^{-1}\exp[i\delta_3(t_1+3t_2-3t_3)]$. We denote by $\mathcal{N}_0$ and $\mathcal{N}_2$ numbers of repetitions caused by zero and two-photon detections in the third stage.

In order to obtain the original state which Alice wanted to teleport, the phase shift factor θ or ϕ has to be removed by Bob. In case of no photon detection in the fourth stage Bob also has to swap the amplitudes of his system states. This is the objective of the fifth stage of the protocol. During the stage both Alice's lasers are turned off.

In the case of detection of one photon in the fourth stage Bob needs three steps to remove the phase shift factor θ.

(i) First, Bob illuminates, using the L_B laser, his first atom for the time t_1 in order to perform transformations (8) and (23).

(ii) Next, he turns off the laser and waits for such time t_θ that $-\theta\alpha_1^2e^{i\Delta_r t_\theta}e^{i2\delta_3 t_1}=1$.

(iii) Finally, he again turns the L_B laser on illuminating his first atom for the time t_1. In this way Bob performs transformations (9) and (23).

If no photon has been detected in the fourth stage Bob performs four operations to remove the phase shift factor ϕ and to exchange the amplitudes.

(i) First, he turns the L_B laser on and illuminates his first atom for the time t_1. He performs transformations (8) and (23) in this step.

(ii) Next, he illuminates, using the L_B laser, his second atom. He turns the laser off after the time t_1 when transformations (8) and (9) are done.

(iii) Next, he waits with lasers turned off for such time t_ϕ that $\phi e^{i\Delta_r t_\phi}=1$.

(iv) Finally, he illuminates his first atom using the L_B laser. He turns the laser off after the time t_1. In this way he performs transformations (9) and (23).

After the last stage of the protocol Bob's system state is given by $a|100\rangle_B+b|010\rangle_B$.

VI. NUMERICAL RESULTS

In order to simplify the above considerations we have used some approximations and therefore the fidelity of the teleported state and the probability of the successful teleportation process have to be calculated numerically. Both quantities depend on the moduli of the amplitudes a and b of the initial state [7,17]. Therefore we need to calculate average values of the fidelity and probability taken over all input states. We compute the averages using the method of quantum trajectories [21,22]. In order to take also into account such imperfections as spontaneous emission from the excited states, we have performed the numerical calculations with the full Hamiltonian (2). We can get individual trajectory by generating a random initial state and performing the whole teleportation protocol. The initial state will be successfully teleported when photon detections are only registered during the second step of the preparation stage, the detection stage I or the second step of the detection stage II. The state will be lost when either of the detectors click during other steps of the protocol. The trajectories in which the initial state is destroyed are counted as the unsuccessful cases. The average probability of a successful teleportation process is then given by the ratio of the number of successful trajectories to the number of all trajectories. The initial state can be also destroyed by spontaneous atomic emission. However, spontaneous atomic emission cannot be detected by D_- or D_+ and therefore such cases will be erroneously counted as successful. Since trajectories in which spontaneous emission occurs cannot be rejected, a nonzero spontaneous decay rate leads to lowering the average fidelity. Therefore the average fidelity has to be taken over all trajectories in which measurement indicates success. Since the average probability of a successful teleportation is not experimentally accessible because of inability to reject trajectories in which spontaneous emission occurs, we define the average probability of a successful measurement by the ratio of the number of trajectories in which measurement indicates success to the number of all trajectories. The average probability of a successful measurement is experimentally accessible and thus we calculate it instead of the probability of a successful teleportation. However, as is mentioned in Sec. II, we want to suppress the influence of the spontaneous decay rate on teleportation. We can achieve this by choosing a sufficiently small value of the spontaneous decay rate.

Before choosing numerical values for all parameters let us collect all the aforementioned assumptions and rewrite them in a compact form $(10^{-1}\Delta \gg \Omega \gg \Omega' \gg g; \Delta' \gg \gamma; \delta_5 \gg \kappa \gg \gamma\Omega^2/\Delta^2; \Delta_r=\delta_1)$. Now, it is easy to check that the parameter values $(\Delta;\Omega;\Omega';g;\gamma;\kappa)/2\pi=(2\times10^3;10;0.84;0.07;10^{-4};10^{-7})$MHz satisfy the conditions. In order to make the average values reliable, we generate 30 000 trajectories. We have got the average fidelity about

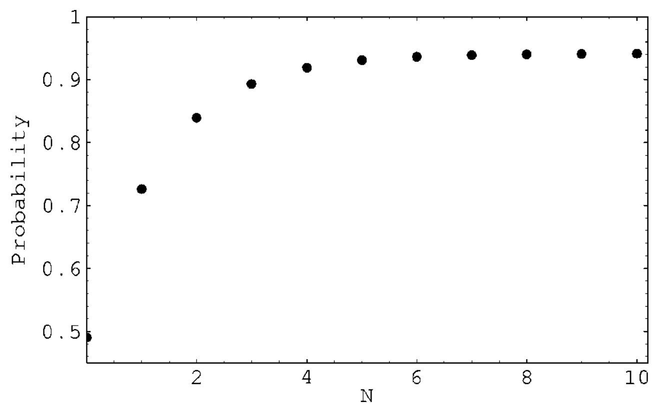

FIG. 3. The average probability of successful measurement as a function of the number $\mathcal{N}$ of the repetitions of the first detection stage.

$F=0.98$ and the average probability of successful measurement $P=0.94$. We have also found that for these parameters the probability that run in which spontaneous emission occurs will be erroneously counted as successful is only 0.1% . These results show that the probability of success is much higher than the successful teleportation probability in other schemes [4,7,17]. This is due to the fact that the initial state is not lost in our scheme when Alice's measurement is unsuccessful contrary to the other schemes, when the initial state is lost and the probability of success is equal or less than 0.5. Owing to the local redundant encoding technique used in our scheme the initial state is protected and therefore the protocol can be repeated until only one photon is detected in the detection stage. Figure 3 shows the probability to transmit the quantum state in the first try and in the subsequent repetitions. As it is seen the probability that a single try will lead to the successful transfer of the initial state is about 0.49. Moreover, one can see that the probability to achieve the successful teleportation process after $\mathcal{N}$ repetitions saturates very quickly. Therefore the protocol does not require a great number of repetitions.

As mentioned above, there are some imperfections in the encoding stage. This is obvious that the imperfections decrease the average fidelity of the teleported state. Also transformations recovering the original state can be done only approximately. In order to show the influence of the imperfections on the average fidelity we plot the average fidelity as a function of the number of the repetitions in Fig. 4.

One can see that the average fidelity decreases with increasing $\mathcal{N}$. Thus if higher fidelity is required, this can be achieved by rejecting the cases with too high a number of repetitions. In order to show this improvement of average teleportation fidelity, let us plot the average fidelity as a function the average probability. As it is evident from Fig. 5 the increase of average fidelity can be achieved by accepting a lower success rate. Moreover, the increase of the average fidelity and the decrease of the average probability is higher for a small number of cases counted as successful. When the repetition number limiting the successful cases is high the points become indistinguishable. Therefore the teleportation scheme will work properly even when we set the maximal number of repetitions to 6 as is clearly illustrated in Fig. 5.

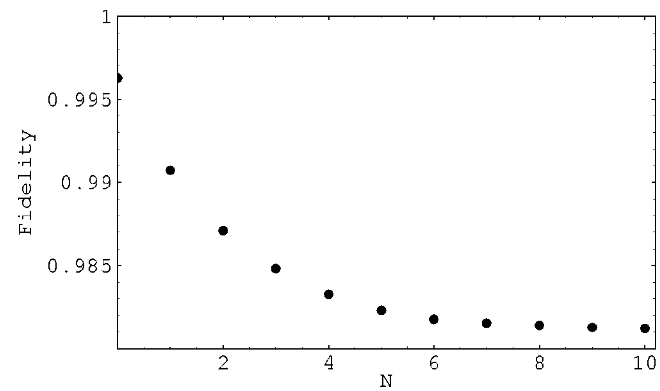

FIG. 4. The average fidelity as a function of the number $\mathcal{N}$ of the repetitions of the first detection stage.

In our calculations we use parameters for which approximations generated by adiabatical elimination of the excited state are good enough. Only for restrictive conditions $10^{-1}\Delta \gg \Omega \gg \Omega' \gg g$ can we calculate δ_1 sufficiently accurately and estimate proper phase shift factors for long evolution times. Otherwise the fidelity is drastically decreased. However, such assumptions lead to a very small value of δ_5 and an unrealistic value of the cavity decay rate. Moreover, very small κ makes the teleportation time orders of magnitude longer than any reported decoherence time for entanglement between atoms. Therefore we use conditions $(\Delta \gg \Omega \gg \Omega' \gg g; \Delta' \gg \gamma; \delta_5 \gg \kappa \gg \gamma\Omega^2/\Delta^2; \Delta_r=\delta_1)$ in our next simulations. We also use a much shorter version of the second step of the encoding stage. The same operation, up to phase shift factors, can be performed by the transformations (8), (9), (18), and (23). Therefore the illumination time of the third Alice atom has to fulfill the conditions $t_2\delta_5=\pi/2+2n\pi$ and $t_2\sqrt{2}\delta_5=3\pi/2+2m\pi$. An approximate solution can be found for $n=1$ and $m=1$. This approximation is slightly less accurate but makes the encoding stage almost two times shorter. These conditions and the reduced time t_2 allow us to choose the parameters values

$$(\Delta;\Omega;\Omega';g;\gamma;\kappa)/2\pi = (2\times 10^3;80;6.3;0.5;10^{-4};1.5\times 10^{-5})\ \text{MHz}.$$

One can see that the cavity decay rate is still orders of mag-

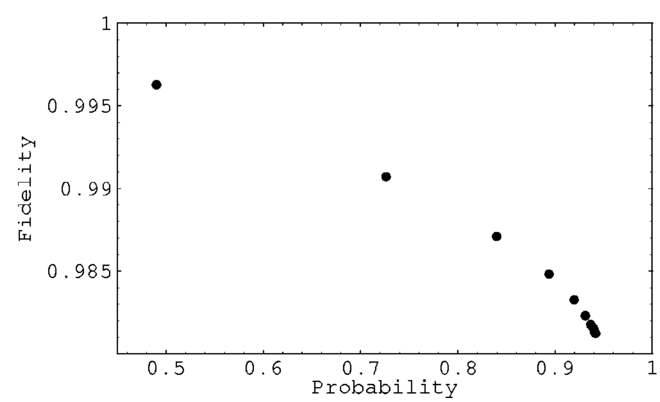

FIG. 5. The average fidelity vs the average probability of successful measurement. The points are for $\mathcal{N}=0,1,\ldots$.

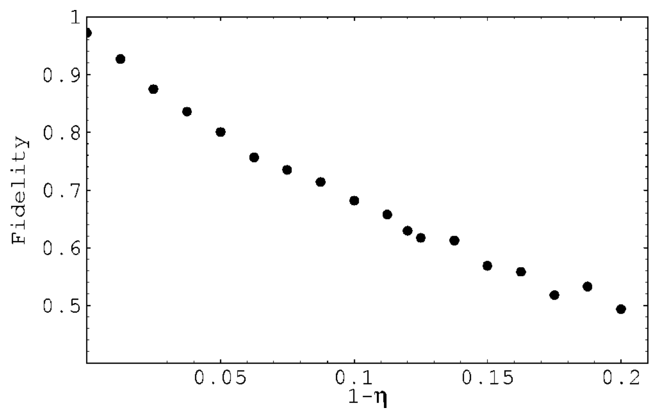

FIG. 6. The average fidelity vs the detector inefficiency. The averages are taken over 2000 trajectories. The parameter regime is $(\Delta;\Omega;\Omega';g;\gamma;\kappa)/2\pi = (2\times10^3;80;6.3;0.5;10^{-4};1.5\times10^{-5})$ MHz.

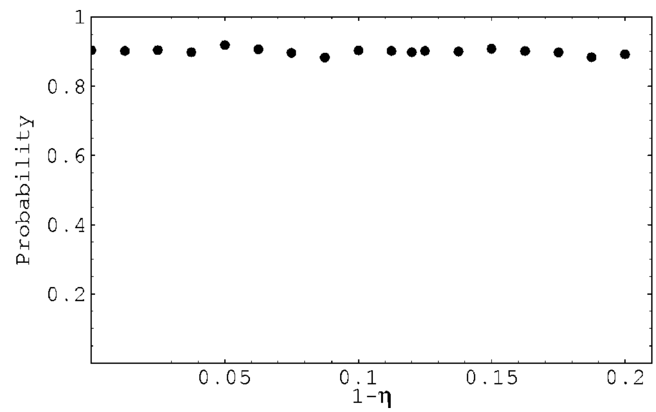

FIG. 7. The average probability of successful measurement vs the detector inefficiency. The averages are taken over 2000 trajectories. The parameter regime is $(\Delta;\Omega;\Omega';g;\gamma;\kappa)/2\pi=(2\times10^3;80;6.3;0.5;10^{-4};1.5\times10^{-5})$ MHz.

nitude below currently reported values [23–28] but further increasing Ω reduces the average fidelity. We cannot also increase κ without increasing of Ω because then the average probability of successful measurement is lowered. For these parameters it is necessary to compute δ_1 numerically. We also compute numerically t_2 and t_3 to improve the average fidelity. In order to further decrease the time needed to complete the teleportation we use the detection time $t_D=4\kappa^{-1}$ instead of $t_D=10\kappa^{-1}$. We have generated 10 000 trajectories and we obtain the average fidelity of 0.97 and the average probability of successful measurement of 0.90. For these parameters the probability that run in which spontaneous emission occurs will be erroneously counted as successful is about 1% . We have also found that the average teleportation time is about 0.1 s. Recently, Roos *et al.* have reported the lifetime of entanglement between atoms exceeding 0.1 s [29].

In the above discussion, we have assumed perfect detectors and perfect mirrors. In real experiments, detectors have efficiency η less than unity and mirrors are not perfect also because of absorption. Therefore a photon emitted from the cavities can never be recorded. On the other hand, there are "dark counts," which give a detector click although no photon leaks out of the cavities' mirrors. There is no way of knowing for sure that the detection results are correct. Therefore if one of these imperfections occurs then Alice and Bob will perform improper operations. Of course, this type of error reduces the average fidelity of the teleported state. Figure 6 shows the influence of the detector inefficiency on the average fidelity. One can see that the scheme is sensitive to the detector inefficiency. For the efficiency $\eta=88\%$ reported by Takeuchi *et al.* [30] and parameters $(\Delta;\Omega;\Omega';g;\gamma;\kappa)/2\pi=(2\times10^3;80;6.3;0.5;10^{-4};1.5\times10^{-5})$ MHz we have generated 10 000 trajectories and we have found that the average fidelity is only 0.63. On the other hand, the probability of successful measurement does not depend on the detector efficiency as is evident from Fig. 7.

In order to suppress the influence of these imperfections on the teleportation it is necessary to use photon detectors with high efficiencies [30–32] and low dark count rates.

VII. CONCLUSIONS

In this paper we have presented a scheme performing quantum teleportation of atomic entangled states via cavity decay. The distinguishing feature of our protocol is using the local redundant encoding technique. We have shown the feasibility of the technique in detail for atoms trapped in a cavity and manipulated by laser fields. Since the technique codes the initial state in the way that the state is secure during the detection stage, the encoding procedure and the detection stage can be repeated until only one photon is detected. However, our protocol is not immune against other sorts of decoherence such as spontaneous atomic emission, the detector inefficiency, and dark counts which destroy the teleported state and have to be suppressed. The numerical calculations show that the average probability of success of the protocol is about 0.94 while the average probability of successful teleportation without the insurance does not exceed 0.5. Moreover, we have shown that not more than six repetitions are enough to obtain high average values of the probability and the fidelity of the teleportation. We have also shown that although the average fidelity is as high as 0.984, one can still increase it by rejecting the cases with too many repetitions and accepting a lower success rate. In addition, we have shown how to manipulate states of many atoms trapped in a cavity using two lasers. We believe that the analytical results presented in Sec. III can be helpful for a description of various atomic systems in optical cavities.

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