# Generating a four-photon polarization-entangled cluster state

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We propose a scheme to generate a four-photon polarization-entangled cluster state by using only linear optical elements and four-photon coincidence detection. The scheme requires two single-photon states and one two-photon polarization entangled state as input resources. Then, we consider the realistic implementation of the scheme by using the spontaneous parametric down-conversion as photon resources. It is shown that under certain conditions, the four-photon polarization cluster state can be generated with a high fidelity, and the scheme is feasible by current experimental technology.

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Entangled state of two or more particles is not only a key ingredient for the tests of quantum nonlocality [1–3], but also a basic resource in achieving tasks of quantum information processing, such as quantum cryptography [4], quantum dense coding [5] and quantum teleportation [6]. Most of the research in quantum information processing is based on quantum entanglement of two qubits. Recently, there is much interest in quantum entanglement of many qubits. It has become clear that for the system shared by three and more parties there are several inequivalent classes of entangled states [7]. In Ref. [8], Briegel *et al.* introduced a class of entangled states, i.e., the cluster states. It has been shown that cluster states can be regarded as a resource for GHZ states [8] and are more immune to decoherence than GHZ states [9]. In Ref. [10], the proof of Bell's theorem without the inequalities was given for cluster states, and a new Bell inequality is considered, which is maximally violated by four-qubit cluster state and is not violated by the four-qubit GHZ state. On the other hand, cluster states have been shown to constitute a universal resource for quantum computation assisted by local measurement only [11].

Recently, experiments with entangled photons open a broad field of research. Entangled photon states can be used to test Bell's inequality [12] or to implement quantum information protocols, such as quantum teleportation [13], quantum dense coding [14] and quantum cryptography [15]. Several schemes have been proposed for generation of multiphoton polarization entangled GHZ state and W state [16]. The experimental realization of GHZ states by means of three or four photons was experimentally observed and used to verify quantum nonlocality [17–19]. More recently, the W state of three photons has also been observed in an experiment [20]. However, no scheme is proposed for generation of polarization entangled cluster state. The aim of this paper is to propose an experimentally feasible scheme for preparing a four-photon polarization entangled cluster state of the form [8]

$$|\text{Cluster}\rangle = \frac{1}{2}(|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4 + |H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4 + |V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4 - |V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4), \quad (1)$$

where $H$ and $V$ denotes, respectively, horizontal and vertical linear polarizations, and subscripts $i$ $(i=1,2,3,4)$ denote the spatial modes of photons.

The paper is organized as follows. At first, we explain the scheme for generation of four-photon polarization entangled cluster state in an ideal situation, in which a two-photon polarization entangled state and two single-photon resources are used as input states. Then, we consider the realistic implementation of the scheme by using the spontaneous parametric down-conversion as photon resources. It is shown that under certain conditions, the four-photon cluster state can be generated with a high fidelity, and the scheme is feasible by current experimental technology. The experimental scheme is based on postselection strategy, which has been used to demonstrate quantum information processing and generate multiphoton GHZ state and W state [13–20].

In the following we present a detailed analysis of the proposed scheme. The experimental setup is shown in Fig. 1, which requires two single-photon states and one two-photon polarization entangled state as input resources, and consists of two polarization beam splitters $\text{PBS}_i$ $(i=1,2)$, three half wave plates $\text{HWP}_i$ $(i=1,2,3)$, and four photon detectors $D_i$ $(i=1,2,3,4)$. The EPR source generates the input state

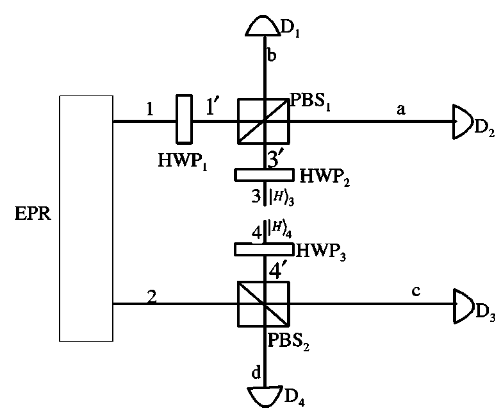


FIG. 1. The schematic diagram of the proposed scheme for generation of the four-photon cluster state. EPR denote entangled two photon state $\frac{1}{2}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)$. $\text{PBS}_i$ denotes polarization beam splitter, which transmits the horizontal polarization and reflect vertical polarization. $\text{HWP}_i$ denote the half wave plate, which implements transformation $|H\rangle\to(|H\rangle+|V\rangle)/\sqrt{2}$, $|V\rangle\to(|H\rangle-|V\rangle)/\sqrt{2}$. $D_i$ are photon number detectors.

 

$\Psi_{EPR}=(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)/\sqrt{2}$. In order to inject two further photons into the arrangement of linear optical devices, we use two single-photon sources: $\Psi_3=|H\rangle_3$ and $\Psi_4=|H\rangle_4$. These single-photon input modes 3 and 4 are horizontally polarized. Thus the input state of the system is

$$\frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)|H\rangle_3|H\rangle_4. \tag{2}$$

Now let the modes 1, 2, and 3 pass through the half wave plates $HWP_1$, $HWP_2$, and $HWP_3$, respectively. The transformation of the $HWP_i$ is given by

$$|H\rangle_i \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{i'}+|V\rangle_{i'}),$$

$$|V\rangle_i \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{i'}-|V\rangle_{i'}). \tag{3}$$

After passing these half wave plates, the state of the system becomes

$$\frac{1}{4}(|H\rangle_{1'}|H\rangle_2+|H\rangle_{1'}|V\rangle_2+|V\rangle_{1'}|H\rangle_2-|V\rangle_{1'}|V\rangle_2)(|H\rangle_{3'}+|V\rangle_{3'})(|H\rangle_{4'}+|V\rangle_{4'}). \tag{4}$$

Next the modes $1'$ and $3'$ are mixed at the polarization beam splitter $PBS_1$, and the modes 2 and $4'$ are mixed at the polarization beam splitter $PBS_2$. Since polarization beam splitter transmit the horizontal polarization and reflect vertical polarization, the two polarization beam splitters $PBS_1$ and $PBS_2$ transform state (4) into

$$\frac{1}{4}(|H\rangle_a|H\rangle_c+|H\rangle_a|V\rangle_d+|V\rangle_b|H\rangle_c-|V\rangle_b|V\rangle_d)(|H\rangle_b+|V\rangle_a)(|H\rangle_d+|V\rangle_c), \tag{5}$$

which can be rewritten as follows:

$$\begin{aligned}\frac{1}{4}(&|H\rangle_a|H\rangle_b|H\rangle_c|H\rangle_d+|H\rangle_a|H\rangle_b|V\rangle_c|V\rangle_d\\&+|V\rangle_a|V\rangle_b|H\rangle_c|H\rangle_d-|V\rangle_a|V\rangle_b|V\rangle_c|V\rangle_d+|H\rangle_a|H\rangle_b|H\rangle_c|V\rangle_c\\&+|H\rangle_a|V\rangle_a|H\rangle_c|H\rangle_d+|H\rangle_a|V\rangle_a|H\rangle_c|V\rangle_c+|H\rangle_a|H\rangle_b|H\rangle_d|V\rangle_d\\&+|H\rangle_a|V\rangle_a|H\rangle_d|V\rangle_d+|H\rangle_a|V\rangle_a|V\rangle_c|V\rangle_d+|H\rangle_b|V\rangle_b|H\rangle_c|H\rangle_d\\&+|H\rangle_b|V\rangle_b|H\rangle_c|V\rangle_c+|V\rangle_a|V\rangle_b|H\rangle_c|V\rangle_c-|H\rangle_b|V\rangle_b|H\rangle_d|V\rangle_d\\&-|H\rangle_b|V\rangle_b|V\rangle_c|V\rangle_d-|V\rangle_a|V\rangle_b|H\rangle_d|V\rangle_d).\end{aligned} \tag{6}$$

Since we consider only those terms, which correspond to the four-photon coincidence detection (one photon in each of the four beams), the state (6) of the system is projected into

$$|\Psi_c\rangle=\frac{1}{2}(|H\rangle_a|H\rangle_b|H\rangle_c|H\rangle_d+|H\rangle_a|H\rangle_b|V\rangle_c|V\rangle_d+|V\rangle_a|V\rangle_b|H\rangle_c|H\rangle_d-|V\rangle_a|V\rangle_b|V\rangle_c|V\rangle_d), \tag{7}$$

which is expected cluster state (1). The success probability of obtaining photons in state (7) is 0.25.

Now we consider the experimental realization of the present scheme. The main difficulty of the scheme in respect to an experimental demonstration consists in the requirement of single-photon sources and two-photon polarization entangled state as input states. The complete technology of these resources is yet to be established [21]. Moreover, the scheme needs a synchronized arrival of many photons into input ports of experimental setup, which will be experimentally challenging. Currently, the best accessible optical source in laboratory is spontaneous parametric down-conversion. In Fig. 2, we consider realistic implementation of the presented scheme by using the spontaneous parametric down-conversion as input photon resources. A laser pulse with angular frequency $\omega_0$ is divided into two by a beam splitter. Transmitted pulses of the frequency $\omega_0$ is used to pump a type-II phase-matched parametric down conversion crystal [17–19], which is arranged to emit polarization entangled photon state into the two input modes 1 and 2 with the same photon polarization. The generated state $|\Psi\rangle_{12}$ can be written as follows:

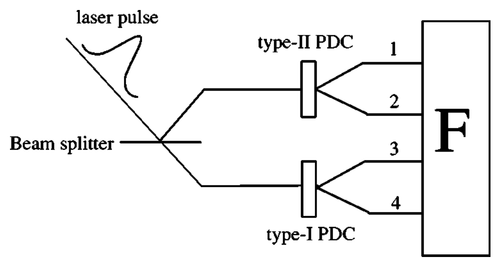


FIG. 2. This scheme input state for the four-photon polarization entangled cluster state. Type-I PDC denotes the type-I phase-matched parametric down conversion, and type-II PDC denotes the type-II phase-matched parametric down conversion. $F$ denotes the setup shown in Fig. 1.

$$\begin{aligned}|\Psi\rangle_{12}=(1-\gamma_1^2)\{&|0\rangle_1|0\rangle_2+\gamma_1(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\\&+\gamma_1^2[|2H\rangle_1|2H\rangle_2+|H\rangle_1|V\rangle_1|H\rangle_2|H\rangle_2+|2V\rangle_1|2V\rangle_2\\&+O(\gamma_1^3)]\},\end{aligned} \tag{8}$$

where $\gamma_1^2$ corresponds to the rate of one-photon-pair generation per pulse of the pump field. We assume that $\gamma_1$ is small, so that we neglect the terms with more than four photons. The reflected pulses of the frequency $\omega_0$ is used to pump a type-I phase-matched parametric down conversion crystal [22], which is arranged to produce down converted photon pairs into the two modes 3 and 4. The generated state $|\Psi\rangle_{34}$ can be written as follows:

$$|\Psi\rangle_{34}=\sqrt{1-\gamma_1^2}\{|0\rangle_3|0\rangle_4+\gamma_2|H\rangle_3|H\rangle_4+\gamma_2^2[|2H\rangle_3|2H\rangle_4+O(\gamma_2^3)]\}, \tag{9}$$

where we again assume that $\gamma_2$ is small and neglect the terms with more than four photons. Thus the input state of the realistic scheme is $|\Psi\rangle_{12}|\Psi\rangle_{34}$. Using such input state and four-photon coincidence detection, the output state of the setup shown in Fig. 1 is projected into

$$|\Psi_o\rangle = \frac{1}{\sqrt{\gamma_2^2 + 2\gamma_1^2}}\left[\frac{\gamma_1}{\sqrt{2}}(|H\rangle_a|H\rangle_b|H\rangle_c|H\rangle_d + |H\rangle_a|H\rangle_b|V\rangle_c|V\rangle_d + |V\rangle_a|V\rangle_b|H\rangle_c|H\rangle_d - |V\rangle_a|V\rangle_b|V\rangle_c|V\rangle_d) + \gamma_2|H\rangle_a|V\rangle_b|H\rangle_c|V\rangle_d\right], \tag{10}$$

which is superposition of a cluster state and a product state. In order to quantify how close the state (10) comes to the cluster state (7), we need calculate the fidelity

$$F = |\langle\Psi_c|\Psi_o\rangle|^2 = \frac{2\gamma_1^2}{\gamma_2^2 + 2\gamma_1^2} = \frac{2}{\frac{\gamma_2^2}{\gamma_1^2} + 2}. \tag{11}$$

It is obvious that the fidelity increases with decreasing ratio $\gamma_2/\gamma_1$. If we set $\gamma_2/\gamma_1 \leqslant 0.1$, once the four coincidence event is observed, we can prepare the cluster state with a fidelity larger than 0.995.

In addition, in order to obtain the efficiency of realistic scheme, we calculate the success probability of producing a four-photon coincidence event as follows:

$$P = (1-\gamma_1^2)^2(1-\gamma_2^2)\gamma_2^2(\gamma_2^2 + 2\gamma_1^2)/4. \tag{12}$$

If we choose parameter $\gamma_2/\gamma_1 = 0.1$ (the fidelity of scheme is $F \approx 0.995$), the success probability is $p = 0.503 \times 10^{-2}(1-\gamma_1^2)^2(1-0.01\gamma_1^2)\gamma_1^4$. In the current experiments for generating multiphoton entanglement by using the spontaneous parametric down-conversion [17–19], the photon pair generation rate per pulse $\gamma_1^2$ is of the order of $3\times10^{-4}$. Using stimulated parametric down-conversion, the photon pair generation rate per pulse can be four times higher than by spontaneous parametric down-conversion. If we choose $\gamma_1^2 = 3\times10^{-4}$, the success probability is $p = 4.5\times10^{-10}$. The generation rate for the four photon polarization entangled cluster state should be $p = 4.5\times10^{-2}$ per second for a laser pulse with a 100 MHz repetition rate. In practice, to implement the presented scheme experimentally, we need to pay attention to errors caused by the generation of three-photon pairs and dark counts of photon detectors. Since the photon pair generation rate per pulse $\gamma_1^2$ is of the order of $3\times10^{-4}$, the three-photon pair generation rate $O(\gamma_1^6)$ is $3\times10^{-4}$ lower than two-photon pair generation rate $O(\gamma_1^4)$. Moreover, the scheme is based on the four-photon coincidence detection, the dark counts of current photon detectors are quite low for current coincidence measurement [17–19], so that the errors caused by the generation of three-photon pairs and dark counts of photon detectors is negligible. The scheme is feasible by current experimental technology.

In conclusion, an ideal scheme is firstly proposed for the generation of the four-photon polarization entangled cluster states by using two single-photon resources, a two-photon polarization entangled state, linear optical devices and a four-photons coincidence detection. Then we consider the realistic implementation of the proposed scheme by using the spontaneous parametric down-conversion as photon resources of the proposed scheme. It is shown that under certain conditions, the four-photon cluster state can be generated with a high fidelity. The scheme is feasible by current experimental technology.

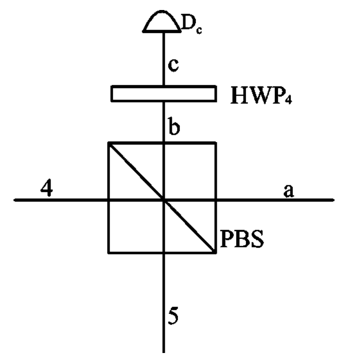


FIG. 3. The schematic diagram of the proposed scheme for generation of the seven-photon cluster state. PBS denotes polarization beam splitter. $HWP_4$ denote the half wave plate.

Finally, it should be pointed out that, if $N$ four-photon polarization cluster states (7) are prepared, we can generate the cluster state of $3N+1$ photons by using linear optical elements. The procedure is given in the Appendix.

## APPENDIX

In this appendix, we demonstrate how to generate cluster states of $3N+1$ photons. For simplicity, we only demonstrate how to generate a seven-photon polarization entangled cluster state. The first step of the scheme is to prepare a pair of four-photon polarization entangled cluster states (7), which can be rewritten in the form [8]

$$\frac{1}{4}(|H\rangle_1 + |V\rangle_1\sigma_{2,z})(|H\rangle_2 + |V\rangle_2\sigma_{3,z})(|H\rangle_3 + |V\rangle_3\sigma_{4,z}) \times (|H\rangle_4 + |V\rangle_4) \tag{A1}$$

and

$$\frac{1}{4}(|H\rangle_5 + |V\rangle_5\sigma_{6,z})(|H\rangle_6 + |V\rangle_6\sigma_{7,z})(|H\rangle_7 + |V\rangle_7\sigma_{8,z}) \times (|H\rangle_8 + |V\rangle_8), \tag{A2}$$

where $\sigma_{i,z} = |H\rangle_i\langle H| - |V\rangle_i\langle V|$. The modes 4 and 5 are injected into two input ports of the setup shown in Fig. 3. After passing through the polarization beam splitter, the state of the system becomes

$$\frac{1}{16}(|H\rangle_1 + |V\rangle_1\sigma_{2,z})(|H\rangle_2 + |V\rangle_2\sigma_{3,z})(|H\rangle_3 + |V\rangle_3\sigma_{4,z})(|H\rangle_a + |V\rangle_b)(|H\rangle_b + |V\rangle_a\sigma_{6,z})(|H\rangle_6 + |V\rangle_6\sigma_{7,z})(|H\rangle_7 + |V\rangle_7\sigma_{8,z}) \times (|H\rangle_8 + |V\rangle_8). \tag{A3}$$

Then mode $b$ pass through the half wave plate $HWP_4$

$$\frac{1}{16}(|H\rangle_1 + |V\rangle_1\sigma_{2,z})(|H\rangle_2 + |V\rangle_2\sigma_{3,z})(|H\rangle_3 + |V\rangle_3\sigma_{4,z}) \times\left(|H\rangle_a + \frac{1}{\sqrt{2}}(|H\rangle_c - |V\rangle_c)\right)\left(\frac{1}{\sqrt{2}}(|H\rangle_c + |V\rangle_c) + |V\rangle_a\sigma_{6,z}\right) \times(|H\rangle_6 + |V\rangle_6\sigma_{7,z})(|H\rangle_7 + |V\rangle_7\sigma_{8,z})(|H\rangle_8 + |V\rangle_8). \quad (A4)$$

If the detector $D_c$ detect one horizontal polarization photon, the state (A4) is projected into

$$\frac{1}{8\sqrt{2}}(|H\rangle_1 + |V\rangle_1\sigma_{2,z})(|H\rangle_2 + |V\rangle_2\sigma_{3,z})(|H\rangle_3 + |V\rangle_3\sigma_{4,z}) \times(|H\rangle_a + |V\rangle_a\sigma_{6,z})(|H\rangle_6 + |V\rangle_6\sigma_{7,z})(|H\rangle_7 + |V\rangle_7\sigma_{8,z}) \times(|H\rangle_8 + |V\rangle_8), \quad (A5)$$

which is seven-photon polarization cluster state [8]. If the detector $D_c$ detect one vertical polarization photon, the state (A4) is projected into

$$\frac{1}{8\sqrt{2}}(|H\rangle_1 + |V\rangle_1\sigma_{2,z})(|H\rangle_2 + |V\rangle_2\sigma_{3,z})(|H\rangle_3 + |V\rangle_3\sigma_{4,z}) \times(|H\rangle_a - |V\rangle_a\sigma_{6,z})(|H\rangle_6 + |V\rangle_6\sigma_{7,z})(|H\rangle_7 + |V\rangle_7\sigma_{8,z}) \times(|H\rangle_8 + |V\rangle_8), \quad (A6)$$

which can be transformed into Eq. (A5) by local operation. The success probability of the scheme is $\frac{1}{2}$. Once the $N$ four-photon polarization cluster states are prepared, it is obvious that the above procedure can be used to generate $(3N+1)$-photon polarization cluster state. The success probability of cluster state generation is $1/2^{N-1}$.


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