Quantum teleportation of an arbitrary two-qubit state and its relation to multipartite entanglement

Gustavo Rigolin*
Departamento de Raios Cósmicos e Cronologia, Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, Caixa Postal 6165, cep 13084-971, Campinas, São Paulo, Brazil
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We explicitly show a protocol in which an arbitrary two qubit state $|\phi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ is faithfully and deterministically teleported from Alice to Bob. We construct the 16 orthogonal generalized Bell states that can be used to teleport the two qubits. The local operations Bob must perform on his qubits in order to recover the teleported state are also constructed. They are restricted only to single-qubit gates. This means that a controlled-NOT gate is not necessary to complete the protocol. A generalization where N qubits are teleported is also shown. We define a generalized magic basis, which possesses interesting properties. These properties help us to suggest a generalized concurrence from which we construct a measure of entanglement that has a clear physical interpretation: A multipartite state has maximum entanglement if it is a genuine quantum teleportation channel.

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In 1993 Bennett *et al.* [1] discovered one of the most astonishing features of quantum mechanics: quantum teleportation. Using what they called an Einstein-Podolsky-Rosen (EPR) channel [$|\phi^+\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)$, for example] they showed that it is possible to transmit a one-qubit state $a|0\rangle + b|1\rangle$ from one location (Alice) to another (Bob) by sending two bits of classical information. Four years later, this protocol was experimentally demonstrated [2].

The next natural question was whether it was possible to teleport not just a single qubit, but rather N qubits. Lee *et al.* [3], in a very interesting work, showed that it was possible to teleport a two-qubit state $|\Phi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ from Alice to Bob using a four-entangled state and sending to him four bits of classical information. However, they did not explicitly construct the protocol and did not provide a generalization to N qubits.

Here we explicitly construct this protocol and present a generalization to N qubits. We create a set of 16 generalized Bell states to implement the teleportation. These states slightly differ from the generalized Bell states used in a probabilistic two-qubit teleportation protocol by Gao *et al.* [4]. We also show that the unitary operations Bob must apply on his qubits to faithfully obtain the teleported two qubits are restricted to single-qubit gates, i.e., gates such as controlled-NOT (CNOT) are not necessary to accomplish the protocol.

The motivation for defining this set of 16 generalized Bell states lies in the fact that by using them, we can easily construct a "magic basis." We show that this magic basis possesses the same interesting properties as the original one defined in Ref. [5]. With the aid of the magic basis we generalize Wootters concurrence [6] and define the entanglement of teleportation (E_T), which has a simple physical interpretation in terms of the efficiency an entangled $2N$-qubit state has to teleport an unknown N-qubit state.

In the original proposal [1] the teleportation of a single qubit $|\phi\rangle = a|0\rangle + b|1\rangle$ is executed as follows. Alice and Bob initially share a maximally two-qubit entangled state: $|\Psi^-\rangle = (1/\sqrt{2})(|01\rangle - |10\rangle)$. The joint system (qubit to be teleported plus EPR state) before Alice's measurement can be written as

$$|\Phi\rangle = |\phi\rangle \otimes |\Psi^-\rangle,$$

$$|\Phi\rangle = \frac{a}{\sqrt{2}}(|001\rangle - |010\rangle) + \frac{b}{\sqrt{2}}(|101\rangle - |110\rangle), \qquad (1)$$

where we use the convention that the first two qubits belong to Alice and the third one belongs to Bob ($|AAB\rangle$, $A \rightarrow$ Alice and $B \rightarrow$ Bob). Rewriting Eq. (1) in terms of the four Bell states $|\Phi^\pm\rangle = (1/\sqrt{2})(|00\rangle \pm |11\rangle)$ and $|\Psi^\pm\rangle = (1/\sqrt{2})(|01\rangle \pm |10\rangle)$ we get

$$|\Phi\rangle = \frac{1}{2}\{|\Psi^-\rangle(-a|0\rangle - b|1\rangle) + |\Psi^+\rangle(-a|0\rangle + b|1\rangle) + |\Phi^-\rangle(a|1\rangle + b|0\rangle) + |\Phi^+\rangle(a|1\rangle - b|0\rangle)\}. \qquad (2)$$

Alice now makes a Bell measurement and classically communicates the result to Bob. With this two-bit information at hand, Bob applies the appropriate unitary operation to obtain the state $|\phi\rangle$. See Table I.

Let us now explicitly present the protocol to teleport two arbitrary qubits. The state Alice wants to teleport is written as

$$|\phi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle, \qquad (3)$$

where a, b, c, and d are complex coefficients and we assume $|\phi\rangle$ to be normalized. We now define the 16 generalized Bell states [7], or G states for simplicity. We divide them into four groups.

Group 1:

$$|g_1\rangle = \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle), \qquad (4)$$

*Electronic address: rigolin@ifi.unicamp.br

TABLE I. The unitary transformations Bob must perform on his qubit, conditioned to Alice's measurement result, to complete the teleportation protocol. I is the identity operator and σ are the usual Pauli matrices.

Alice's result	Bob's operation	Bob's qubit
$\|\Psi^-\rangle$	I	$I(-a\|0\rangle - b\|1\rangle) \propto \|\phi\rangle$
$\|\Psi^+\rangle$	σ^z	$\sigma^z(-a\|0\rangle + b\|1\rangle) \propto \|\phi\rangle$
$\|\Phi^-\rangle$	σ^x	$\sigma^x(a\|1\rangle + b\|0\rangle) \propto \|\phi\rangle$
$\|\Phi^+\rangle$	$\sigma^z\sigma^x$	$\sigma^z\sigma^x(a\|1\rangle - b\|0\rangle) \propto \|\phi\rangle$

$$|g_2\rangle = \frac{1}{2}(|0000\rangle + |0101\rangle - |1010\rangle - |1111\rangle), \quad (5)$$

$$|g_3\rangle = \frac{1}{2}(|0000\rangle - |0101\rangle + |1010\rangle - |1111\rangle), \quad (6)$$

$$|g_4\rangle = \frac{1}{2}(|0000\rangle - |0101\rangle - |1010\rangle + |1111\rangle). \quad (7)$$

Group 2:

$$|g_5\rangle = \frac{1}{2}(|0001\rangle + |0100\rangle + |1011\rangle + |1110\rangle), \quad (8)$$

$$|g_6\rangle = \frac{1}{2}(|0001\rangle + |0100\rangle - |1011\rangle - |1110\rangle), \quad (9)$$

$$|g_7\rangle = \frac{1}{2}(|0001\rangle - |0100\rangle + |1011\rangle - |1110\rangle), \quad (10)$$

$$|g_8\rangle = \frac{1}{2}(|0001\rangle - |0100\rangle - |1011\rangle + |1110\rangle). \quad (11)$$

Group 3:

$$|g_9\rangle = \frac{1}{2}(|0010\rangle + |0111\rangle + |1000\rangle + |1101\rangle), \quad (12)$$

$$|g_{10}\rangle = \frac{1}{2}(|0010\rangle + |0111\rangle - |1000\rangle - |1101\rangle), \quad (13)$$

$$|g_{11}\rangle = \frac{1}{2}(|0010\rangle - |0111\rangle + |1000\rangle - |1101\rangle), \quad (14)$$

$$|g_{12}\rangle = \frac{1}{2}(|0010\rangle - |0111\rangle - |1000\rangle + |1101\rangle). \quad (15)$$

Group 4:

$$|g_{13}\rangle = \frac{1}{2}(|0011\rangle + |0110\rangle + |1001\rangle + |1100\rangle), \quad (16)$$

$$|g_{14}\rangle = \frac{1}{2}(|0011\rangle + |0110\rangle - |1001\rangle - |1100\rangle), \quad (17)$$

$$|g_{15}\rangle = \frac{1}{2}(|0011\rangle - |0110\rangle + |1001\rangle - |1100\rangle), \quad (18)$$

$$|g_{16}\rangle = \frac{1}{2}(|0011\rangle - |0110\rangle - |1001\rangle + |1100\rangle). \quad (19)$$

These states form an orthonormal basis $\Sigma_{j=1}^{16}|g_j\rangle\langle g_j| = I$ and $\langle g_j|g_k\rangle = \delta_{jk}$, which we call the generalized Bell basis [7], or G basis for brevity.

Alice and Bob must share one of the 16 G states, where we assume that the first two qubits are with Alice and the last two with Bob. These last two qubits with Bob will be used to "receive" the teleported state. For definiteness we assume Alice and Bob share the $|g_1\rangle$ state. Hence, the initial joint state is

$$\begin{aligned}|\Phi\rangle &= |\phi\rangle \otimes |g_1\rangle \\ &= \frac{a}{2}\{|000000\rangle + |000101\rangle + |001010\rangle + |001111\rangle\} \\ &\quad + \frac{b}{2}\{|010000\rangle + |010101\rangle + |011010\rangle + |011111\rangle\} \\ &\quad + \frac{c}{2}\{|000000\rangle + |100101\rangle + |101010\rangle + |101111\rangle\} \\ &\quad + \frac{d}{2}\{|110000\rangle + |110101\rangle + |111010\rangle + |111111\rangle\}.\end{aligned} \quad (20)$$

Here the first four qubits belong to Alice and the last two belong to Bob ($|AAAABB\rangle$, $A \rightarrow$ Alice and $B \rightarrow$ Bob). Using Eqs. (4)–(19) we can write Eq. (20) as

$$|\Phi\rangle = \frac{1}{4}\sum_{j=1}^{16} |g_j\rangle_A |\phi_j\rangle_B, \quad (21)$$

where A and B are written to emphasize which qubits are with Alice and with Bob and the states $|\phi_j\rangle$ are defined in Table II.

Alice now makes a generalized Bell measurement (G measurement) obtaining with equal probabilities one of the 16 G states. By G measurement we mean that Alice makes a joint measurement in the two qubits she wants to teleport plus the two qubits from the G state shared with Bob. Then she sends to Bob a classical message of four bits to inform him which G state she has measured. With this information Bob knows what unitary operation (Table II) he must apply on his two qubits to faithfully recover the teleported state. After applying the unitary operation the protocol is finished and Alice has succeeded in teleporting her arbitrary two-qubit state.

Paying attention to the 16 unitary operations which Bob should apply on his two qubits, we see that they all can be written as $U = U_5 \otimes U_6$. Here 5 and 6 refer to the fifth and sixth qubit respectively, i.e., Bob's two qubits. This means that we need only single-qubit gates to implement all the 16 unitary operations. More elaborate two-qubit gates, such as the CNOT gate, are not necessary. This fact possibly simpli-

TABLE II. The first column shows the $|\phi_j\rangle$ states. The third column depicts the local unitary operations Bob must perform on his qubits, conditioned on Alice's measurement given in the second column, to finish the teleportation protocol.

$\|\phi_j\rangle$	Alice's result	Bob's operation
$\|\phi_1\rangle=\|\phi\rangle$	$\|g_1\rangle$	I
$\|\phi_2\rangle=\sigma_1^z\|\phi\rangle$	$\|g_2\rangle$	σ_1^z
$\|\phi_3\rangle=\sigma_2^z\|\phi\rangle$	$\|g_3\rangle$	σ_2^z
$\|\phi_4\rangle=\sigma_1^z\sigma_2^z\|\phi\rangle$	$\|g_4\rangle$	$\sigma_2^z\sigma_1^z$
$\|\phi_5\rangle=\sigma_2^x\|\phi\rangle$	$\|g_5\rangle$	σ_2^x
$\|\phi_6\rangle=\sigma_2^x\sigma_1^z\|\phi\rangle$	$\|g_6\rangle$	$\sigma_1^z\sigma_2^x$
$\|\phi_7\rangle=\sigma_2^x\sigma_2^z\|\phi\rangle$	$\|g_7\rangle$	$\sigma_2^z\sigma_2^x$
$\|\phi_8\rangle=\sigma_2^x\sigma_2^z\sigma_1^z\|\phi\rangle$	$\|g_8\rangle$	$\sigma_1^z\sigma_2^z\sigma_2^x$
$\|\phi_9\rangle=\sigma_1^x\|\phi\rangle$	$\|g_9\rangle$	σ_1^x
$\|\phi_{10}\rangle=\sigma_1^x\sigma_1^z\|\phi\rangle$	$\|g_{10}\rangle$	$\sigma_1^z\sigma_1^x$
$\|\phi_{11}\rangle=\sigma_1^x\sigma_2^z\|\phi\rangle$	$\|g_{11}\rangle$	$\sigma_2^z\sigma_1^x$
$\|\phi_{12}\rangle=\sigma_1^x\sigma_1^z\sigma_2^z\|\phi\rangle$	$\|g_{12}\rangle$	$\sigma_2^z\sigma_1^z\sigma_1^x$
$\|\phi_{13}\rangle=\sigma_1^x\sigma_2^x\|\phi\rangle$	$\|g_{13}\rangle$	$\sigma_2^x\sigma_1^x$
$\|\phi_{14}\rangle=\sigma_1^x\sigma_2^x\sigma_1^z\|\phi\rangle$	$\|g_{14}\rangle$	$\sigma_1^z\sigma_2^x\sigma_1^x$
$\|\phi_{15}\rangle=\sigma_1^x\sigma_2^x\sigma_2^z\|\phi\rangle$	$\|g_{15}\rangle$	$\sigma_2^z\sigma_2^x\sigma_1^x$
$\|\phi_{16}\rangle=\sigma_1^x\sigma_2^x\sigma_1^z\sigma_2^z\|\phi\rangle$	$\|g_{16}\rangle$	$\sigma_2^z\sigma_1^z\sigma_2^x\sigma_1^x$

TABLE III. The first column shows the magic states. The second column represents the corresponding G states, and the third column the F states. The elements of each row should be read as $a=b=c$.

Magic states	G states	F states
$\|e_1\rangle$	$\|g_1\rangle$	$\|f_1\rangle$
$\|e_2\rangle$	$i\|g_2\rangle$	$i\|f_2\rangle$
$\|e_3\rangle$	$\|g_4\rangle$	$\|f_3\rangle$
$\|e_4\rangle$	$i\|g_3\rangle$	$i\|f_4\rangle$
$\|e_5\rangle$	$\|g_6\rangle$	$\|f_5\rangle$
$\|e_6\rangle$	$i\|g_5\rangle$	$i\|f_6\rangle$
$\|e_7\rangle$	$\|g_7\rangle$	$\|f_7\rangle$
$\|e_8\rangle$	$i\|g_8\rangle$	$i\|f_8\rangle$
$\|e_9\rangle$	$\|g_{10}\rangle$	$\|f_9\rangle$
$\|e_{10}\rangle$	$i\|g_9\rangle$	$i\|f_{10}\rangle$
$\|e_{11}\rangle$	$\|g_{11}\rangle$	$\|f_{11}\rangle$
$\|e_{12}\rangle$	$i\|g_{12}\rangle$	$i\|f_{12}\rangle$
$\|e_{13}\rangle$	$\|g_{13}\rangle$	$\|f_{13}\rangle$
$\|e_{14}\rangle$	$i\|g_{14}\rangle$	$i\|f_{14}\rangle$
$\|e_{15}\rangle$	$\|g_{16}\rangle$	$\|f_{15}\rangle$
$\|e_{16}\rangle$	$i\|g_{15}\rangle$	$i\|f_{16}\rangle$

fies future experimental realizations of this protocol. It is worth mentioning that we can construct other sets of 16 orthogonal states which faithfully teleport any two qubits, but now depending on Alice's outcome, Bob will need to implement a CNOT gate to complete the protocol.

If we use quantum channels with generalized Greeneberger-Horne-Zeilinger (GHZ) states [7], $|\mathrm{GHZ}\rangle = 1/\sqrt{2}(|0000\rangle+|1111\rangle)$, the protocol does not work. It is impossible to teleport an arbitrary two qubit state using a GHZ state. Only special classes of two-qubit states such as $b|01\rangle + c|10\rangle$ can be teleported [8].

The technical difficulties that we need to circumvent to experimentally test this protocol are not trivial. Three benchmarks must be reached to realize this protocol. First, Alice and Bob should have a source of four maximally entangled states (G states). Second, Bob should be able to implement the 16 local unitary operations, and Alice should somehow realize a G measurement. The first two benchmarks are very close [9,10], but we still need an efficient way to discriminate the 16 G states.

The previous protocol can be generalized to teleport an N-qubit state. For this purpose Alice needs to share a $2N$ entangled G state with Bob. Then she realizes a $2N$ joint G measurement with the N qubit to be teleported and half of the shared G state and sends a $2N$-bit classical message to Bob informing him of the measurement outcome. Bob finishes the protocol performing at most $2N$ single-qubit gates to obtain the teleported state. The number of unitary operations Bob needs to apply on his N qubits is conditioned on Alice's outcome. The N-qubit teleportation protocol can be rigorously constructed as follows. (1) Generate the seed G state $|s_0\rangle=(2^{-N/2})\Sigma_{j=0}^{M}|x_j\rangle A|x_j\rangle B$, where $M=2^N-1$ and x_j is the binary representation of the number j. In the two-qubit teleportation protocol, $x_0=00$, $x_1=01$, $x_2=10$, and $x_3=11$. Zeros must be added to make all x_j with the same number of bits (N bits). This G state is our quantum channel and it is composed of $2N$ qubits. (2) Using the seed G state it is possible to obtain all the G states locally operating on its first N qubits, $|s_j\rangle=\otimes_{k=1}^{N}(\sigma_k^z)^{j_{2k-1}}(\sigma_k^x)^{j_{2k}}|s_0\rangle$. Now j_k represents the kth bit (from right to left) of the number $0\leqslant j\leqslant 2^{2N}-1$, which is written in binary notation and again zeros should be added to leave all j's with the same number of bits ($2N$ bits). The subindex k indicates on which qubit the Pauli matrices σ^x and σ^z should operate. For the two-qubit protocol shown above, $|s_0\rangle=|g_1\rangle$, $|s_1\rangle=|g_2\rangle$, $|s_2\rangle=|g_9\rangle$, $|s_3\rangle=|g_{10}\rangle$, and so forth. (3) Alice makes a joint G measurement with the N qubits to be teleported and with her N qubits of the shared G state. She then sends to Bob a $2N$-bit classical message informing Bob of the measurement outcome. (4) With this information Bob applies the corresponding unitary operation on his N qubits. These operations are given by $U_j=\otimes_{k=1}^{N}(\sigma_k^z)^{j_{2k-1}}(\sigma_k^x)^{j_{2k}}$ and for the two-qubit teleportation protocol they are shown in Table II.

We now define the generalized magic basis ($|e_j\rangle$) and an auxiliary basis ($|f_j\rangle$), which help us in the calculations that follow. See Table III.

In terms of the F states, a general pure two-qubit state can be written as $|\Psi\rangle=\Sigma_{j=1}^{16}\alpha_j|f_j\rangle$. From this state we define $|\tilde{\Psi}\rangle=\Sigma_{j=1}^{16}\alpha_j^*|f_j\rangle$, where α_j^* means the complex conjugate of α_j. With the previous definitions, we can present the generalized concurrence [11] as

$$C(\Psi)=\left|\langle\tilde{\Psi}|\sigma_y^{\otimes 4}|\Psi\rangle\right|, \tag{22}$$

where $\sigma_y^{\otimes 4}=\sigma_1^y\sigma_2^y\sigma_3^y\sigma_4^y$. Since $\sigma_y^{\otimes 4}|\Psi\rangle=\Sigma_{j=1}^{16}(-1)^{j+1}\alpha_j|f_j\rangle$,

$$C(\Psi) = \left| \sum_{j=1}^{16} (-1)^{j+1} \alpha_j^2 \right|. \tag{23}$$

But in the magic basis $|\Psi\rangle = \Sigma_{j=1}^{16} \beta_j |e_j\rangle$ and using the relations between the F states and the magic states in Table III we can show that $\alpha_j = \mathrm{i}^{(j+1)\oplus 2} \beta_j$, where $a \oplus 2 = 0$ if a is even and 1 if a is odd. This implies that

$$C(\Psi) = \left| \sum_{j=1}^{16} \beta_j^2 \right|. \tag{24}$$

Using Eq. (24) we can show that Eq. (22) satisfies the same properties as the original Wootters concurrence [6]. (1) If all β_j are real then $C=1$; (2) $0 \leqslant C \leqslant 1$; (3) in the magic basis, $C = |\langle \tilde{\Psi} | \Psi \rangle|$; (4) every two-qubit state with $C=1$ can be written, up to a global phase, as a combination of the magic states with real coefficients; and (5) all the magic states have $C=1$. Noting that $\sigma_y^{\otimes 4}$ is the spin flip operator, we can show that if $|\Psi\rangle$ is separable then $C=0$.

In order to quantify the usefulness of a four-qubit state to teleport two qubits we introduce the entanglement of teleportation (E_T):

$$E_T(\Psi) = \frac{1}{16} \sum_{j=1}^{L} C(\Psi_j), \tag{25}$$

where $|\Psi_j\rangle$ are all the $L \leqslant 16$ orthogonal states that can be obtained from $|\Psi\rangle$ using the 16 unitary operations U_j listed in the third column of Table II. It should be noted that Eqs. (22) and (25) can easily be extended to a $2N$-qubit state:

$$C(\Psi) = \left| \langle \tilde{\Psi} | \sigma_y^{\otimes 2N} | \Psi \rangle \right|, \tag{26}$$

$$E_T(\Psi) = \frac{1}{2^{2N}} \sum_{j=1}^{L} C(\Psi_j), \tag{27}$$

where now $|\Psi_j\rangle$ are the $L \leqslant 2^{2N}$ orthogonal states obtained from $|\Psi\rangle$ using the 2^{2N} unitary operations $U_j = \otimes_{k=1}^{N} (\sigma_k^z)^{j_{2k-1}} (\sigma_k^x)^{j_{2k}}$. E_T satisfies some interesting properties. (1) It can discriminate the generalized W, GHZ, and G states, i.e., $E_T(W) < E_T(\mathrm{GHZ}) < E_T(g_j)$; (2) it can be seen as a measure of the efficiency with which $2N$ qubits can teleport N qubits; (3) all G states have $E_T=1$; (4) all separable states have $E_T=0$.

In order to illustrate the above properties, let us return to the four-qubit case. Consider the G state $|g_1\rangle$. Applying the U_j operations we obtain 16 orthogonal states (the G basis) with $C=1$. Therefore, $E_T=1$. Now let us study the generalized GHZ state $|\mathrm{GHZ}^+\rangle = (1/\sqrt{2})(|0000\rangle + |1111\rangle)$. The 16 unitary operations U_j produce only eight orthogonal states with $C=1$,

$$|\mathrm{GHZ}^\pm\rangle = \frac{1}{\sqrt{2}} (|0000\rangle \pm |1111\rangle), \tag{28}$$

$$|G^\pm\rangle = \frac{1}{\sqrt{2}} (|0100\rangle \pm |1011\rangle), \tag{29}$$

$$|H^\pm\rangle = \frac{1}{\sqrt{2}} (|1000\rangle \pm |0111\rangle), \tag{30}$$

$$|Z^\pm\rangle = \frac{1}{\sqrt{2}} (|1100\rangle \pm |0011\rangle), \tag{31}$$

which implies that $E_T = 1/2$. Repeating the same procedure with the generalized W state $|W\rangle = (1/2)(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle)$ we obtain eight orthogonal states with $C=0$, i.e., $E_T=0$. We can get a physical picture of the meaning of these values for E_T by noting that using G states ($E_T=1$) we can deterministically teleport any two qubits, using GHZ states ($E_T=1/2$) only special classes of two qubits can be teleported [8], and for W states even these special classes cannot be deterministically teleported. And more, we can show that the GHZ states can deterministically teleport one qubit but the W states accomplish the same task only probabilistically.

We have explicitly shown a teleportation protocol that allows Alice to faithfully teleport an arbitrary two-qubit state to Bob by sending to him a four-bit classical message. We generalized this protocol for the N-qubit case, where Alice can teleport N qubits to Bob by sending to him a $2N$-bit classical message. They must also share a $2N$ entangled state, which is lost at the end of the protocol. We also presented a way to quantify the efficiency with which a $2N$ entangled state can faithfully teleport N qubits, defining the entanglement of teleportation (E_T). E_T is able to quantify the generalized Bell states, the GHZ states, and W states in terms of their teleportation capacities. We think that this characterization of entanglement will be useful in the study of multipartite states, since it was shown that it has a simple physical interpretation, is operational, and is easily generalized to states with an even number of qubits.

Note added in proof. Recently we became aware of an interesting publication of Ikram *et al.* [12] showing how to experimentally teleport N qubits using N high-Q cavities.

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