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Simple experimental scheme of preparing a four-photon entangled state for the teleportation-based realization of a linear optical controlled-NOT gate

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We propose a simple experimental scheme of preparing a four-photon entangled state, which is a useful resource for two-qubit quantum operations, e.g., the controlled-NOT gate. The scheme can successfully postselect the events of the desired four-photon entangled state in the coincidence basis and can be constructed with four photons from parametric down conversion, linear optical devices, and conventional photon detectors, all of which are available in current technology. We also show an experimental scheme for a controlled-NOT gate based on teleportation with the prepared four-photon state.

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I. INTRODUCTION

In linear optical quantum information processing, there has been much interest in multipartite entangled states, because they are considered as resources of quantum computation and communication. One example of multipartite entangled states is a four-qubit entangled state

$$|\chi\rangle = \tfrac{1}{2}[(|00\rangle + |11\rangle)|00\rangle + (|01\rangle + |10\rangle)|11\rangle], \qquad (1)$$

whose importance has been pointed out by Gottesman and Chuang [1]. They showed that a teleportation scheme, which uses $|\chi\rangle$ as a resource, can perform the controlled-NOT (CNOT) gate on two qubits [1] (see also Sec. II of [2]). Furthermore, Raussendorf and Briegel [3] have recently proposed a measurement-based quantum computation using special multipartite entangled states called "cluster states," and some schemes of optical quantum computation with cluster states have also been proposed in [4,5]. The state $|\chi\rangle$ is also important in the sense that $|\chi\rangle$ is equivalent to a four-qubit cluster state under a local unitary transformation. Several schemes [6–10] can be utilized to prepare $|\chi\rangle$.

In this paper, we propose a simple experimental scheme for preparing the four-photon entangled state $|\chi\rangle$, which has fewer requirements and/or a greater yield compared to existing methods. Our scheme can be constructed from four photons produced by parametric down conversion, polarizing beam splitters, half-wave plates, and conventional photon detectors, that can only discriminate the vacuum from one or more photons. The successful events of preparing $|\chi\rangle$ are selected by coincidence detection. Our scheme does not require the optical paths to be stable to subwavelength order. In order to show the power of $|\chi\rangle$ as a resource, we further propose an experimental scheme of a CNOT gate via teleportation.

This paper is organized as follows: In Sec. II, we first discuss the preparation of the four-photon entangled state $|\chi\rangle$ using existing methods, then describe an alternate simple scheme for preparing $|\chi\rangle$. In Sec. III, we show an experimental scheme of a teleportation-based CNOT gate using $|\chi\rangle$. Finally, we describe our conclusions in Sec. IV.

II. PREPARING THE FOUR-PHOTON ENTANGLED STATE $|\chi\rangle$

The four-photon entangled state $|\chi\rangle$ can be generated by applying a CNOT gate between two entangled photon pairs as shown in Fig. 3 of [1]. Therefore, one can prepare $|\chi\rangle$ by applying probabilistic CNOT gates in the coincidence basis [7–10] between two entangled photon pairs from parametric down conversion (PDC). When the scheme [7–9] is used, the success probability is $1/9$ on the condition that two entangled photon pairs are generated. This scheme requires the optical paths to be stable to subwavelength order for interferometric stability. If we use another probabilistic CNOT gate in the coincidence basis [10], the success probability increases to $1/8$. The success probability further increases to $1/4$ by performing feed-forward control [10,11]. The scheme [10] does not need the interferometric stability of subwavelength order, unlike the above scheme. However, an ancilla photon is necessary to perform the gate; therefore one needs five photons to prepare $|\chi\rangle$. Another scheme of preparing $|\chi\rangle$ with only four photons has been proposed in [6] in the context of probabilistic realization of a CNOT gate in the manner of [1]. The required resources are one entangled photon pair and two single photons. The preparation of $|\chi\rangle$ is achieved by applying beam splitter operations on both sides of the entangled photon pair with single photons and by postselecting the events of coincidence detection. This scheme does not need the interferometric stability of subwavelength order either. However, to perform this scheme in a real experiment, if we substitute the two single photons with another pair of photons from PDC, multiphoton generation on the same optical path causes errors, even if we allow for postselection.

Here we propose an alternative simple scheme for preparing $|\chi\rangle$ (Fig. 1), which has fewer requirements and/or a

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greater yield compared to the existing methods. Our scheme requires only four photons from PDC. This scheme starts with one entangled photon pair and adds two other single photons by applying beam splitter operations. Unlike the scheme [6], the beam splitter operations are applied on only one side of the initial entangled pair; thus, even if we substitute the two single photons with another pair of photons from PDC, multiphoton generation errors from PDC can be eliminated by postselection, as described later. Furthermore, our scheme does not require the optical paths to be stable to subwavelength precision [12]. The successful events are postselected in the coincidence basis as in other four-photon experiments [13–20]. The success probability is 1/4 provided that an entangled photon pair and two single photons are generated.

Let us describe the details of our scheme. First, we assume that a polarization entangled photon pair is prepared in spatial modes 1 and 2, and two single photons are also prepared in spatial modes 3 and 4 as follows:

$$\frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes\frac{1}{\sqrt{2}}(|H\rangle_3+|V\rangle_3)\otimes\frac{1}{\sqrt{2}}(|H\rangle_4+|V\rangle_4). \quad (2)$$

Here, for example, $|H\rangle_1$ represents the state of a photon with horizontal polarization in spatial mode 1, and $|V\rangle_1$ represents the state of a photon with vertical polarization in spatial mode 1. Since a polarizing beam splitter (PBS) transmits horizontal polarization and reflects vertical polarization, the state of photons in modes 1, 2, and 3 is transformed into

$$\tfrac{1}{2}(|H\rangle_1|H\rangle_s|H\rangle_t+|V\rangle_1|V\rangle_s|V\rangle_t) \quad (3)$$

after passing through PBS1 and by keeping only the terms having one photon in each of the output modes 1, s, and t. The half-wave plates (HWPs) rotate the polarizations of the three photons in modes 1, s, and t by 45°, i.e., $|H\rangle_1\to(|H\rangle_1+|V\rangle_1)/\sqrt{2}$, $|V\rangle_1\to(|H\rangle_1-|V\rangle_1)/\sqrt{2}$, resulting in

$$\tfrac{1}{2}(|H\rangle_1|H\rangle_s|H\rangle_t+|V\rangle_1|V\rangle_s|V\rangle_t)\to\frac{1}{2\sqrt{2}}(|H\rangle_{1'}|H\rangle_{s'}|H\rangle_{t'}+|V\rangle_{1'}|V\rangle_{s'}|H\rangle_{t'}+|H\rangle_{1'}|V\rangle_{s'}|V\rangle_{t'}+|V\rangle_{1'}|H\rangle_{s'}|V\rangle_{t'}). \quad (4)$$

After the photons in modes t' and 4 pass through PBS2, we obtain the four-photon entangled state

$$\tfrac{1}{4}[|H\rangle_5|H\rangle_6|H\rangle_7|H\rangle_8+|V\rangle_5|V\rangle_6|H\rangle_7|H\rangle_8+|H\rangle_5|V\rangle_6|V\rangle_7|V\rangle_8+|V\rangle_5|H\rangle_6|V\rangle_7|V\rangle_8]=\tfrac{1}{4}[(|H\rangle_5|H\rangle_6+|V\rangle_5|V\rangle_6)|H\rangle_7|H\rangle_8+(|H\rangle_5|V\rangle_6+|V\rangle_5|H\rangle_6)|V\rangle_7|V\rangle_8] \quad (5)$$

by keeping only the terms having one photon in each of the output modes 5, 6, 7, and 8. This state is equivalent to state (1). The successful events of obtaining $|\chi\rangle$ can be postselected by four-photon coincidence detection. The success probability is 1/4 on the condition that an entangled photon pair in modes 1 and 2 and two single photons in modes 3 and 4 are provided.

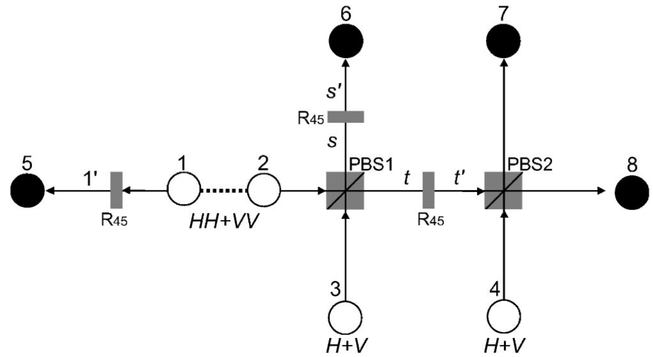

FIG. 1. Schematic diagram of preparing a four-photon polarization entangled state $|\chi\rangle$.

Next, we consider the case in which we use PDC to generate the photons for the input modes 1, 2, 3, and 4. In PDC, the photon pair generation rate per pulse γ is $\sim 10^{-4}$ in typical experiments [13,21,22]. The successful events of preparing $|\chi\rangle$ are obtained only when an entangled photon pair is generated in modes 1 and 2 by PDC, and two single photons are generated in modes 3 and 4 by another PDC. Such events occur with a rate $O(\gamma^2)$. On the other hand, with a rate of the same order $O(\gamma^2)$, two-photon pairs are generated in modes 1 and 2 (or in modes 3 and 4) with modes 3 and 4 (1 and 2) being left in the vacuum. This contribution could lead to errors, but we can eliminate these failure events by postselection as follows. If two-photon pairs are produced in modes 1 and 2, then three photons never exist in output modes 6, 7, and 8. If two-photon pairs are produced in modes 3 and 4, then no photon will exist in output mode 5. The errors only occur when three (or more) photon pairs are produced by PDC, with a small rate of $O(\gamma^3)$. The dark counting rate of conventional photon detectors is also quite low for fourfold coincidence detection (see [23]).

III. TELEPORTATION-BASED CNOT GATE USING $|\chi\rangle$

The importance of $|\chi\rangle$ was shown in [1] to be a resource for teleporting two qubits through a CNOT gate. As described in [6,24], a teleportation-based CNOT gate using $|\chi\rangle$ can be implemented by adding two photons as an input and by applying probabilistic Bell measurements using beam splitting operations. In this case, we need six photons for the demonstration, and the success probability is 1/4 on the condition that $|\chi\rangle$ and the input state of two photons are provided. It does not require the optical paths to be stable to subwavelength order.

Here, we present another experimental scheme of the teleportation-based CNOT gate (Fig. 2), which uses only four photons. The teleportation deterministically succeeds in principle on the condition that $|\chi\rangle$ is given, though the input state of the CNOT gate is limited to a known product state (or a separable mixed state in general) and the scheme requires the optical paths to be stable to subwavelength precision. The basic idea of the teleportation in this scheme relies on

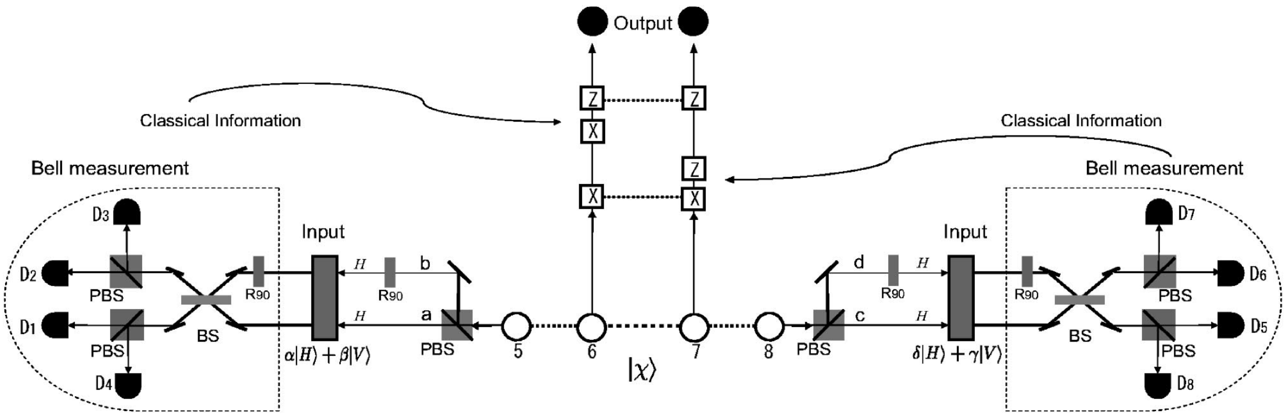

FIG. 2. Schematic diagram of a teleportation-based controlled-NOT gate. $|\chi\rangle$ can be produced by using the state preparation method of Sec. II. The input of the CNOT gate is a product state $(\alpha|H\rangle_5+\beta|V\rangle_5)(\gamma|H\rangle_8+\delta|V\rangle_8)$. This state is prepared by transforming the polarizations of photons 5 and 8. The parts surrounded by dotted lines correspond to Bell measurements.

[25,26], and we modify the scheme such that the teleportation can be simply applied after $|\chi\rangle$ is given.

Let us describe the detail of the scheme. We assume $|\chi\rangle$ is provided in polarization qubits of photons 5, 6, 7, and 8 as follows:

$$|\chi\rangle = \tfrac{1}{2}[(|H\rangle_5|H\rangle_6 + |V\rangle_5|V\rangle_6)|H\rangle_7|H\rangle_8 + (|H\rangle_5|V\rangle_6 + |V\rangle_5|H\rangle_6)|V\rangle_7|V\rangle_8]. \tag{6}$$

Photons 5 and 8 pass through PBSs, after which we obtain

$$\tfrac{1}{2}[(|a\rangle_5|H\rangle_5|H\rangle_6 + |b\rangle_5|V\rangle_5|V\rangle_6)|H\rangle_7|c\rangle_8|H\rangle_8 + (|a\rangle_5|H\rangle_5|V\rangle_6 + |b\rangle_5|V\rangle_5|H\rangle_6)|V\rangle_7|d\rangle_8|V\rangle_8], \tag{7}$$

where $|a\rangle_5|H\rangle_5$ ($|c\rangle_8|H\rangle_8$) represents the state of a photon with horizontal polarization in path a (c), and $|b\rangle_5|V\rangle_5$ ($|b\rangle_8|V\rangle_8$) represents the state of a photon with vertical polarization in path b (d). Labels 5 and 8 now indicate double channels that lead to Bell measurements. The half-wave plates in path b and d rotate the polarizations of the photons by 90°. We then obtain

$$\tfrac{1}{2}[(|a\rangle_5|H\rangle_6 + |b\rangle_5|V\rangle_6)|H\rangle_7|c\rangle_8 + (|a\rangle_5|V\rangle_6 + |b\rangle_5|H\rangle_6)|V\rangle_7|d\rangle_8]|H\rangle_5|H\rangle_8. \tag{8}$$

Now that the positions of photons 5 and 8 and the polarizations of photons 6 and 7 are entangled [which is equivalent to Eq. (1)], and the polarizations of photons 5 and 8 are free from this entanglement, we can transform the polarizations of photons 5 and 8 into arbitrary product states as follows:

$$\tfrac{1}{2}[(|a\rangle_5|H\rangle_6 + |b\rangle_5|V\rangle_6)|H\rangle_7|c\rangle_8 + (|a\rangle_5|V\rangle_6 + |b\rangle_5|H\rangle_6)|V\rangle_7|d\rangle_8] \times(\alpha|H\rangle_5 + \beta|V\rangle_5)(\gamma|H\rangle_8 + \delta|V\rangle_8). \tag{9}$$

Here, the state $(\alpha|H\rangle_5+\beta|V\rangle_5)(\gamma|H\rangle_8+\delta|V\rangle_8)$ is assumed to be the input to the CNOT gate. After that, photon 5 (8) passes through the optical elements and should be detected by one of the detectors D_1, D_2, D_3, and D_4 (D_5, D_6, D_7, and D_8). Here, the optical elements and the detectors surrounded by the dotted lines in Fig. 2 correspond to Bell measurements (see [26]). Applying a unitary transformation $I\otimes I$, $\sigma_x\otimes\sigma_x$, $I\otimes\sigma_z$, or $\sigma_x\otimes\sigma_z\sigma_x$ to photons 6 and 7 according to detection at D_5, D_6, D_7, or D_8, and applying a unitary transformation $I\otimes I$, $\sigma_x\otimes I$, $\sigma_z\otimes\sigma_z$, or $\sigma_z\sigma_x\otimes\sigma_z$ to photons 6 and 7 according to detection at D_1, D_2, D_3, or D_4, we always obtain the same output state

$$\alpha\gamma|H\rangle_6|H\rangle_7 + \alpha\delta|V\rangle_6|V\rangle_7 + \beta\gamma|V\rangle_6|H\rangle_7 + \beta\delta|H\rangle_6|V\rangle_7. \tag{10}$$

This is exactly the state that is obtained by applying the CNOT gate to the target qubit in state $\alpha|H\rangle_5+\beta|V\rangle_5$ and the control qubit in $\gamma|H\rangle_8+\delta|V\rangle_8$.

This scheme can be demonstrated in current technology by combining the scheme of Sec. II for preparing $|\chi\rangle$. We should provide a few remarks about the implementation. First, as we mentioned above, the input of the CNOT gate is limited to be a known product state. Second, since the preparation of $|\chi\rangle$ by the scheme of Sec. II probabilistically succeeds in the coincidence basis, the successful events of the CNOT gate should be finally selected by coincidence detection. Third, although the scheme of Fig. 1 does not require the optical path to be stable to subwavelength order, the Bell measurement parts of Fig. 2 should be carefully aligned to subwavelength precision in order to achieve the desired projection of the four Bell states.

IV. CONCLUSIONS

We have proposed a simple experimental scheme for preparing the four-photon entangled state $|\chi\rangle$. This scheme can be realized with four photons produced by parametric down conversion, linear optical devices, and conventional photon detectors. It does not require the optical paths to be stable to subwavelength precision. The successful events are selected by coincidence detection, and the main errors from multiphoton emissions by parametric down converters are also eliminated by postselection.

Since our scheme probabilistically succeeds in the coincidence basis, it is not scalable for quantum computation as it is. However, such a probabilistic scheme is also useful. For example, it could be used for the investigation of the nonlocality and Bell inequality of cluster states [27], or the demonstration of the basic quantum gates of certain quantum computational models [1,3,28,29]. Actually, the experimental scheme for the CNOT gate in Sec. III can be thought of as a simple scheme for demonstrating deterministic linear optical quantum computation [29] in the sense that (8) is the resource (called a "linked state") of a quantum circuit that has only the CNOT gate in this model. Furthermore, the probabilistic preparation of resource states in the coincidence basis could become scalable for quantum computation, if postselection is done without losing photons, which could be achieved by using quantum nondemolition measurement techniques, e.g. [30,31], and it is combined with the near-deterministic teleportation scheme [28] or the measurement-based optical quantum computation approach using cluster states [4,5].

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