Testing quantum superpositions of the gravitational field with Bose-Einstein condensates

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We consider the gravity field of a Bose-Einstein condensate in a quantum superposition. The gravity field then is also in a quantum superposition, which is in principle observable. Hence we have "quantum gravity" far away from the so-called Planck scale.

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The existence of macroscopically distinguishable superpositions in Bose-Einstein condensates (BEC's) has been discussed by several authors [1–3]. Such a superposition may be achieved as the ground state of two interacting BEC's in a double-well potential [1] or by a continuous quantum measurement on a condensate trapped in such a potential [2].

We now consider the gravitational field of a BEC of total mass M in such a double-well potential and we scatter a beam of particles of mass m in the middle of the double potential well. (We assume that the potential that affects the BEC does not affect these particles or that we know how to correct for its effect in our calculations.)

In order to avoid decoherence, the density matrix of the scattered particles must be factorized in the form $\rho=\rho_{\text{int}} \otimes \rho_{\text{c.m.}}$ where ρ_{int} stands for the density matrix of the internal degrees of freedom and $\rho_{\text{c.m.}}$ for the density matrix of the external degrees of freedom.

Let us treat classically the interaction of the center of mass of the scattered particles with the gravitational field of the BEC. Suppose that we know for certain that the BEC condensate is localized within the left well or the right well (this may happen, for example, in the case of an infinite barrier in the double potential well). The scattered particles will be deflected from their course due to the gravitational attraction between them and the BEC. Here we also assume that all other interactions between the scattered particles and the BEC either do not exist or are negligible with respect to the gravitational interaction (which for itself is very weak). We shall set up our axis system so that the double potential well lies on the x axis (the maximum of the potential barrier is at $x=0$) and the scattered particles' initial trajectory is along the y axis, with $x=0$, so that their inital momentum is $p_0=(0,p_y,0)$. The distances between the two minima in the potential will be a, and the particles' trajectories are such that without the gravitational interaction they would pass at a distance a within each minima. In the crudest "undergraduate" approximation, the scattered particles will be under the influence of a gravitational force GmM/a^2 during a time interval of $a/v_y=am/p_y$. The scattered particles, initially with $p_x=0$, will have, after the interaction, a nonzero x component for their momentum $\pm GMm^2/ap_y$, where + is for the case in which the BEC is in the right well and − if it is in the left one. The deflection angle in each case will be small and can be written as $\theta \approx \pm p_x/p_y = GMm^2/ap_y^2 = GM/av_y^2$.

Now assume that the potential barrier between the two wells is finite and the BEC condensate is in a symmetric state $\psi_S=\psi_L+\psi_R$ where ψ_R and ψ_L are functions that are localized in the right and left wells, respectively. What is the result of scattering the particles from the gravitational field of this state? After the scattering, the state of the center of mass of the particles will be entangled with the state of the BEC and in a superposition of states, one being a result of a deflection from a BEC in the right well and the other being the result of deflection from a BEC in the left well. The state of the system can be written in the following manner:

$$\varphi(\mathbf{r}) = A_1\psi_L \otimes \exp[i/\hbar(p_x,p_y,0)\cdot\mathbf{r}] + A_2\psi_R \otimes \exp[i\hbar(-p_x,p_y,0)\cdot\mathbf{r}]. \quad (1)$$

Here it is essential that the initial momentum spread Δp of the BEC be much larger than the momentum kick due to the interaction.

We do not record the state of the BEC during the experiment, or in other words we are tracing over the BEC states. If ψ_L and ψ_R were orthogonal, tracing over the BEC state would prevent us from seeing any interference fringes. The situation is different if they are not orthogonal. Let $\xi = \langle \psi_L | \psi_R \rangle$. The density matrix of the center of mass of the scattered particles after tracing over the BEC states is

$$\begin{aligned}\rho(\mathbf{r},\mathbf{r}') = {} & |A_1|^2 \exp[i/\hbar(p_x,p_y,0)\cdot(\mathbf{r}-\mathbf{r}')] \\ & + |A_2|^2 \exp[i/\hbar(-p_x,p_y,0)\cdot(\mathbf{r}-\mathbf{r}')] \\ & + 2\,\mathrm{Re}(A_1A_2^*\xi \exp\{i/\hbar[(p_x,p_y,0)\cdot\mathbf{r} \\ & + (p_x,-p_y,0)\cdot\mathbf{r}']\}). \end{aligned} \quad (2)$$

Assuming $A_1=A_2$, it is easy to see that $\rho(\mathbf{r},\mathbf{r})$ will be maximal when $\cos(p_x x/\hbar)=1$ or $p_x x/\hbar = 2\pi m$, so the distance between fringes will be

$$\Delta x = 2\pi\hbar/p_x = (hav_y)/(GMm). \quad (3)$$

Let us try to see the parameters needed in such an experiment in order for it to be feasible. As we shall soon see, the distance between fringes tends to be very large, so let us choose parameters to make it as small as possible (while difficult, it does not violate any law of physics). A BEC usually contains up to 10^7 Rb atoms, which give a mass of

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about 10^{-18} kg. For the scattered particles, let us take big particles (grains) with a mass of about a nanogram or 10^{-12} kg. The constants h and G have values

$$h \approx 6 \times 10^{-34}\ \mathrm{J\,s},$$

$$G \approx 6 \times 10^{-11}\ \mathrm{Kg}^{-1}\,\mathrm{m}^3\,\mathrm{s}^{-2}.$$

We need to get the product av_y as small as possible. The distance between the minima in the double potential well cannot be much smaller than about 1 μ or 10^{-6} m. The question now is, how slow can the incoming particles be and still scatter coherently only from the gravitational field of the BEC? Introducing all the above numbers, we have

$$\Delta x \approx 10 v_y\ [\mathrm{m}]. \quad (4)$$

So in order to achieve a distance between fringes that is on the order, say, of cm, so that the experiment will be feasible, we need the incoming particles' speed to be of the order 10^{-3} m/s. These are slow speeds, but nevertheless, there is no physical law that prevents them. We can also try to take heavier particles for the scattered particles. We can write

$$\Delta x \approx 10^{-11}(v_y/m)\ [\mathrm{m}]. \quad (5)$$

The ratio v_y/m will have to be of the orders of 10^9 m/(kg s).

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