Combining Jaynes-Cummings and anti-Jaynes-Cummings dynamics in a trapped-ion system driven by a laser

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We show that, if one combines the Jaynes-Cummings and anti-Jaynes-Cummings dynamics in a trapped-ion system driven by a laser, additional series of collapses and revivals of the vibrational state of the ion can be generated.

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I. INTRODUCTION

The Jaynes-Cummings model (JCM) [1] has been a subject of continuous theoretical studies already for the last 40 years [2] and it has more recently been realized in the laboratory [3]. Still, probably the most famous feature of this system, namely, the revival of oscillations of the atomic population inversion [2], is difficult to observe in Cavity QED experiments [4] because of the presence of dissipation that produces a rapid decay of atomic oscillations. Revivals can also occur in quite different physical systems such as trapped ions interacting with laser fields, and, because dissipation does not play an important role in this system, they may be experimentally observed. Recently Morigi *et al.* [5] have shown a method to accelerate the revivals, undoing the dynamics by a suitable manipulation of the two-level system, more specifically by a quasi-instantaneous change of its phase.

In this paper we propose a method to accelerate the revival of Rabi oscillations of the atomic inversion by switching from the JCM-type interaction to an anti-JCM one. This sudden change can be implemented in trapped ions by switching the frequencies of the lasers and in cavities by interacting the two-level atom with an external classical field (see for instance [7]).

II. REVERSING DYNAMICS IN TRAPPED IONS

The Hamiltonian governing the dynamics of a single ion trapped in a harmonic potential in interaction with laser light in the (optical) rotating-wave approximation has the form

$$\hat{H} = \tfrac{1}{2}[\hat{p}^2 + \nu^2\hat{x}^2] + \hbar\omega_{01}\hat{A}_{11} + \hbar\mu[E^{(-)}(\hat{x},t)\hat{A}_{01} + \text{H.c.}], \tag{1}$$

where $\hat{A}_{11},\hat{A}_{00}$ are the electronic level population operators, $\hat{A}_{01},\hat{A}_{10}=\hat{A}_{01}^{\dagger}$ describe transitions with frequency ω_{01} between electronic levels $|0\rangle$ ($\hat{A}_{00}|0\rangle=|0\rangle$) and $|1\rangle$ ($\hat{A}_{11}|1\rangle=|1\rangle$) and satisfy the commutation relations $[\hat{A}_{11},\hat{A}_{00}]=0$, $[\hat{A}_{00},\hat{A}_{01}]=-\hat{A}_{01}$, $[\hat{A}_{11},\hat{A}_{01}]=\hat{A}_{01}$, ν is the trap frequency, μ is the electronic coupling matrix element, and

$$E^{(-)}(\hat{x},t) = E_0 e^{-i(\vec{k}\hat{x}-\omega t)} \tag{2}$$

is the negative part of the classical electric field of the driving laser beam. The operators $\hat{x}$ and $\hat{p}$ are the position and momentum of the center of mass of the ion. In the rotating frame the Hamiltonian (1) takes the form

$$\hat{\mathcal{H}} = \hbar\nu\left(\hat{n} + \frac{1}{2}\right) + \hbar\frac{m\nu}{2}\hat{A}_{11} + \hbar\Omega[e^{-i(\hat{a}+\hat{a}^{\dagger})\eta}\hat{A}_{01} + \text{H.c.}], \tag{3}$$

where $\Omega=E_0\mu$, $\hat{a}=\sqrt{\nu/2\hbar}\hat{x}+i\hat{p}/\sqrt{2\hbar\nu}$, $\hat{n}=\hat{a}^{\dagger}\hat{a}$, $\eta=|\vec{k}|\sqrt{\hbar/2\nu}$ is the Lamb-Dicke parameter, and the field frequency satisfies the resonant condition $\omega_{01}-\omega=m\nu$, where m is an integer. In the case $\Omega\ll\nu$ we may neglect rapidly rotating terms in the Hamiltonian (3) leading to a JCM-type interaction when $m=1$:

$$\hat{H}_{JC} = \hbar\nu\hat{n} + \hbar\frac{\nu}{2}\hat{A}_{11} + \hbar\eta\Omega[\hat{a}^{\dagger}\hat{A}_{01} + \hat{a}\hat{A}_{10}], \tag{4}$$

and by choosing $m=-1$ it takes the anti-JCM form

$$\hat{H}_{AJC} = \hbar\nu\hat{n} - \hbar\frac{\nu}{2}\hat{A}_{11} + \hbar\eta\Omega[\hat{a}\hat{A}_{01} + \hat{a}^{\dagger}\hat{A}_{10}]. \tag{5}$$

Let us suppose that the applied laser field is redshifted, $m=1$, during the time t_1 (JCM regime) and then is switched to the blue regime, $m=-1$, emulating the anti-JCM-type interaction for the time t_2. The evolution of an arbitrary initial electronic and vibrational state of the ion can be found exactly; however, we will study the effect of changing the JCM to anti-JCM dynamics for highly excited coherent vibrational states $|\alpha\rangle$, $\bar{n}=|\alpha|^2\gg1$, in approximate form (nevertheless the numerical results we will show are exact).

It is well known that in the usual JCM (describing interaction of a two-level atom with a single mode of a quantized field) there is a single series of revivals, which appears at time instants $t_r=2\pi\sqrt{\bar{n}}k/(\eta\Omega), k=1,2,\ldots$. The appearance of the revival structure of the atomic population can be nicely explained in terms of the evolution of the so-called semiclassical [8] states $|\underline{p}\rangle$, $p=0,1$, eigenstates of $\hat{A}_x=(\hat{A}_{01}+\hat{A}_{10})/2$ operators,

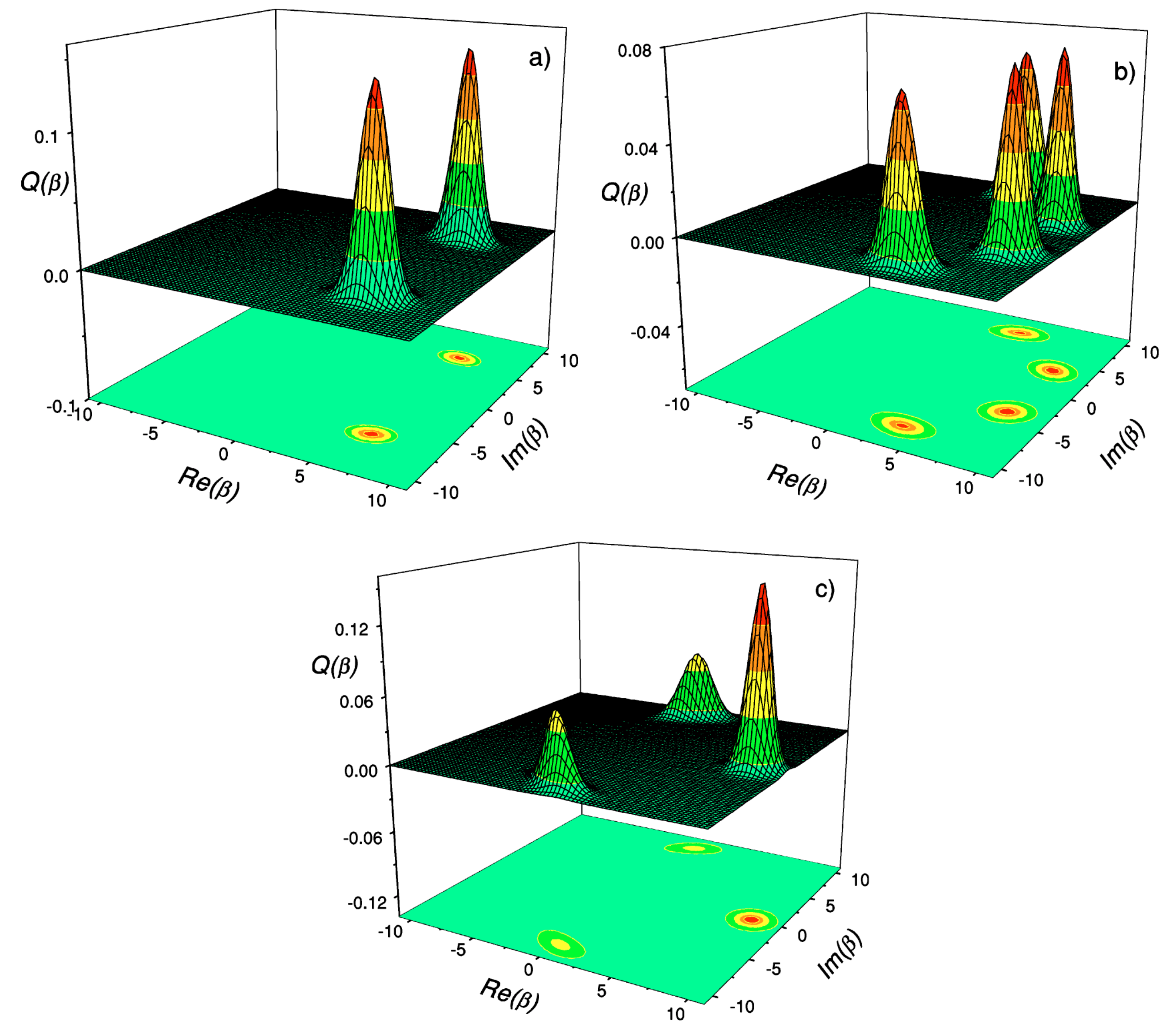

FIG. 1. Surface and contour plots of the Q function at different times $t=t_1+t_2$, for a switch time equal to one-fourth of the revival time, i.e., $t_1=t_r/4$. $t_2=$ (a) 0, (b) $t_r/8$, (c) $t_r/4$.

$$\hat{A}_x|\underline{p}\rangle = \lambda_p|\underline{p}\rangle, \quad \lambda_p = p - 1/2. \tag{6}$$

In particular, the evolution of the initial semiclassical state $|\underline{p}\rangle|\alpha\rangle$ leads to an approximately factorized form of the wave function [9]:

$$|\Psi(t)\rangle \simeq |A_p(t)\rangle \otimes |\Phi_p(t)\rangle, \tag{7}$$

with

$$|A_p(t)\rangle = \exp\left[-i\frac{\omega_p t}{2}\left(\hat{A}_{11} - \hat{A}_{00} + 1\right)\right]|\underline{p}\rangle,$$

$$|\Phi_p(t)\rangle = \exp[-i\eta\Omega t\lambda_p\sqrt{\hat{n}}]|\alpha\rangle,$$

where

$$\omega_p = \frac{\eta\Omega\lambda_p}{2\sqrt{\bar{n}}} = \frac{\lambda_p\pi}{t_r}.$$

We will show below that in the field (or in our case, vibrational mode) phase space each factorized state can be described by its Q function, which has the shape of a single hump revolving around a circle of radius $\sqrt{\bar{n}}$ with an angular velocity ω_p. The distribution of excitations for these states is always Poissonian, but they spread in phase, due to an intensity-dependent phase shift [9]. Any initial atomic (electronic) state can be expanded in the basis of the semiclassical states as

$$|\text{in}\rangle = \sum_p c_p|\underline{p}\rangle, \tag{8}$$

and, correspondingly, the state of the total system can be rewritten as a superposition of the factorized states:

$$|\Psi(t)\rangle = \sum_p c_p |A_p(t)\rangle \otimes |\Phi_p(t)\rangle. \tag{9}$$

Hence, a generic initial state causes the appearance of two humps revolving around a circle of radius $\sqrt{\bar{n}}$ in the field (vibration mode) phase space with angular velocities ω_p. The motion of the humps in the phase space of the field (vibration mode) determines the behavior of the atomic (electronic) inversion. When the humps are well separated, there are no Rabi oscillations (i.e., we have the collapse region), while the collision of two humps at t_r leads to the revival of Rabi oscillations.

The effect of switching from the JCM to the anti-JCM regime can also be studied using the idea of wave function factorization. Taking into account that Hamiltonians (4) and (5) can be represented as

$$\hat{H}_{JC} = \hat{Q}(\hbar\nu\hat{n} + 2\hbar\eta\Omega\sqrt{\hat{n}}\hat{A}_x)\hat{Q}^\dagger, \tag{10}$$

$$\hat{H}_{AJC} = \hat{Q}^\dagger(\hbar\nu\hat{n} + 2\hbar\eta\Omega\sqrt{\hat{n}}\hat{A}_x)\hat{Q}, \tag{11}$$

where $\hat{Q}$ is defined as [10]

$$\hat{Q} = \exp[i\hat{\phi}\hat{A}_{11}] = |0\rangle\langle 0| + |1\rangle e^{i\hat{\phi}}\langle 1|, \tag{12}$$

and $\exp[\pm i\hat{\phi}]$ are the Susskind-Glogower phase operators [11], the evolution operator describing the JCM dynamics during a time t_1 and consecutive anti-JCM dynamics during a time t_2 has the form

$$\hat{U}(t_1+t_2) = \hat{Q}^\dagger \exp[-it_2(\nu\hat{n} + 2\eta\Omega\sqrt{\hat{n}}\hat{A}_x)]\hat{Q}^2 \times \exp[-it_1(\nu\hat{n} + 2\eta\Omega\sqrt{\hat{n}}\hat{A}_x)]\hat{Q}^\dagger. \tag{13}$$

The initial state $|\underline{p}\rangle|\alpha\rangle, p=0,1$ (for simplicity we consider an initial vibrational state with zero phase, $\alpha=\sqrt{\bar{n}}$), under the action of (13) evolves according to

$$|\psi_p(t)\rangle = \hat{U}(t_1+t_2)|\underline{p}\rangle|\alpha\rangle = \hat{Q}^\dagger e^{-i(t_1+t_2)\nu\hat{n}} e^{-2i\eta\Omega\sqrt{\hat{n}}(\hat{A}_x t_2 + \lambda_p t_1)} e^{-2it_1(\omega_p+\nu)\hat{A}_{11}}|\underline{p}\rangle|\alpha\rangle, \tag{14}$$

where we have taken into account (6) and the following property [10] of the operator (12):

$$\hat{Q}|\alpha = \sqrt{\bar{n}}e^{i\varphi}\rangle|\underline{1}\rangle = e^{i\varphi}|\alpha\rangle|\underline{1}\rangle + O(1/\sqrt{\bar{n}}).$$

In particular, we obtain from Eq. (14)

$$|\psi_0(t)\rangle = \hat{U}(t_1+t_2)|\underline{0}\rangle|\alpha\rangle = \hat{Q}^\dagger[\cos\omega t_1 e^{i\eta\Omega\sqrt{\hat{n}}(t_1+t_2)}|\underline{0}\rangle - i\sin\omega t_1 e^{i\eta\Omega\sqrt{\hat{n}}(t_1-t_2)}|\underline{0}\rangle]|\alpha\rangle \tag{15}$$

and

$$|\psi_1(t)\rangle = \hat{U}(t_1+t_2)|\underline{1}\rangle|\alpha\rangle = \hat{Q}^\dagger[\cos\omega t_1 e^{-i\eta\Omega\sqrt{\hat{n}}(t_1+t_2)}|\underline{1}\rangle + i\sin\omega t_1 e^{-i\eta\Omega\sqrt{\hat{n}}(t_1-t_2)}|\underline{0}\rangle]|\alpha\rangle, \tag{16}$$

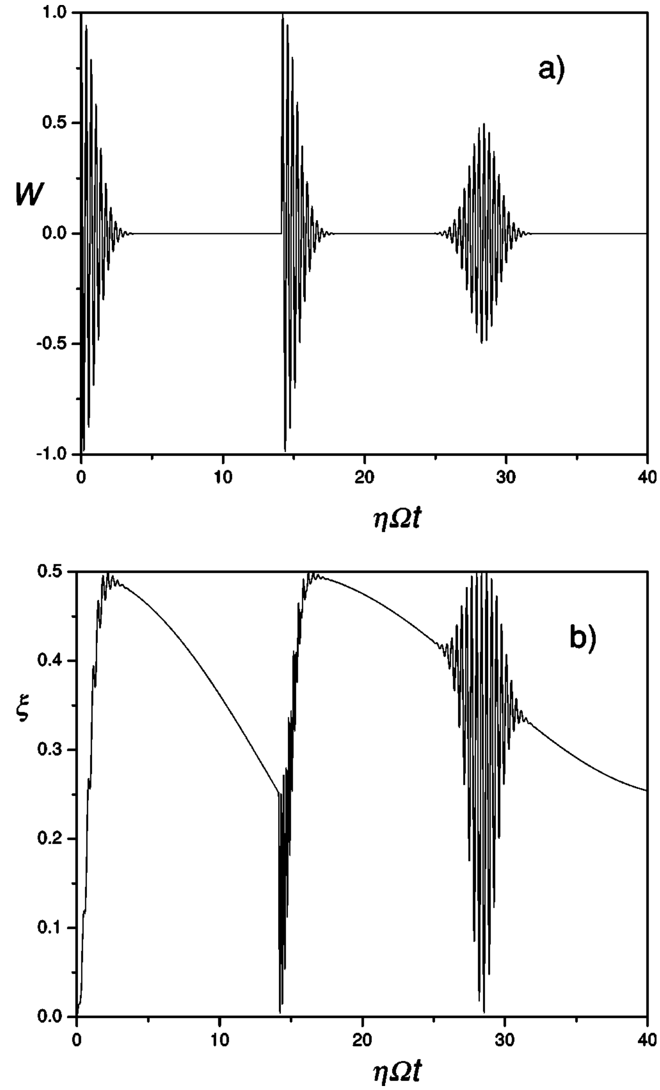

FIG. 2. (a) Atomic population inversion $W=2P_e-1$, with $P_e=\mathrm{Tr}(|e\rangle\langle e|)$, and (b) purity function, $\xi=1-\mathrm{Tr}\{\rho_v^2\}$, as a function of the scaled time $\eta\Omega t$, with a switch time equal to one-fourth of the revival time, $t_1=t_r/4$.

where $\omega=\eta\Omega/(2\sqrt{\bar{n}})$ and we consider stroboscopic times, $\nu t_{1,2}=2\pi k$. The corresponding density matrices of the vibrational state have the form

$$\rho^{vm}_{0,1}(t) = \tfrac{1}{2}e^{i\hat{\phi}}\hat{U}_{11}|\alpha\rangle\langle\alpha|\hat{U}_{11}^\dagger e^{-i\hat{\phi}} + \tfrac{1}{2}\hat{U}_{22}|\alpha\rangle\langle\alpha|\hat{U}_{22}^\dagger, \tag{17}$$

with $\hat{U}_{11}=\cos\omega t_1 e^{\pm i\eta\Omega\sqrt{\hat{n}}(t_1+t_2)} \mp i\sin\omega t_1 e^{\pm i\eta\Omega\sqrt{\hat{n}}(t_1-t_2)}$ and $\hat{U}_{22}=\cos\omega t_1 e^{\pm i\eta\Omega\sqrt{\hat{n}}(t_1+t_2)} \pm i\sin\omega t_1 e^{\pm i\eta\Omega\sqrt{\hat{n}}(t_1-t_2)}$. Thus, the Husimi Q functions corresponding to the initial electronic states $|\underline{0}\rangle$ and $|\underline{1}\rangle$ take the following approximate forms $[Q(\beta)=\langle\beta|\rho|\beta\rangle]$:

$$Q_{0,1}(\beta) \approx \cos^2\omega t_1|\langle\beta|e^{\pm i\eta\Omega\sqrt{\hat{n}}(t_1+t_2)}|\alpha\rangle|^2 + \sin^2\omega t_1|\langle\beta|e^{\pm i\eta\Omega\sqrt{\hat{n}}(t_1-t_2)}|\alpha\rangle|^2. \tag{18}$$

This means that depending on the time t_1, the initial distribution of vibrational quanta corresponding to a single initial semiclassical electronic state is split after changing the

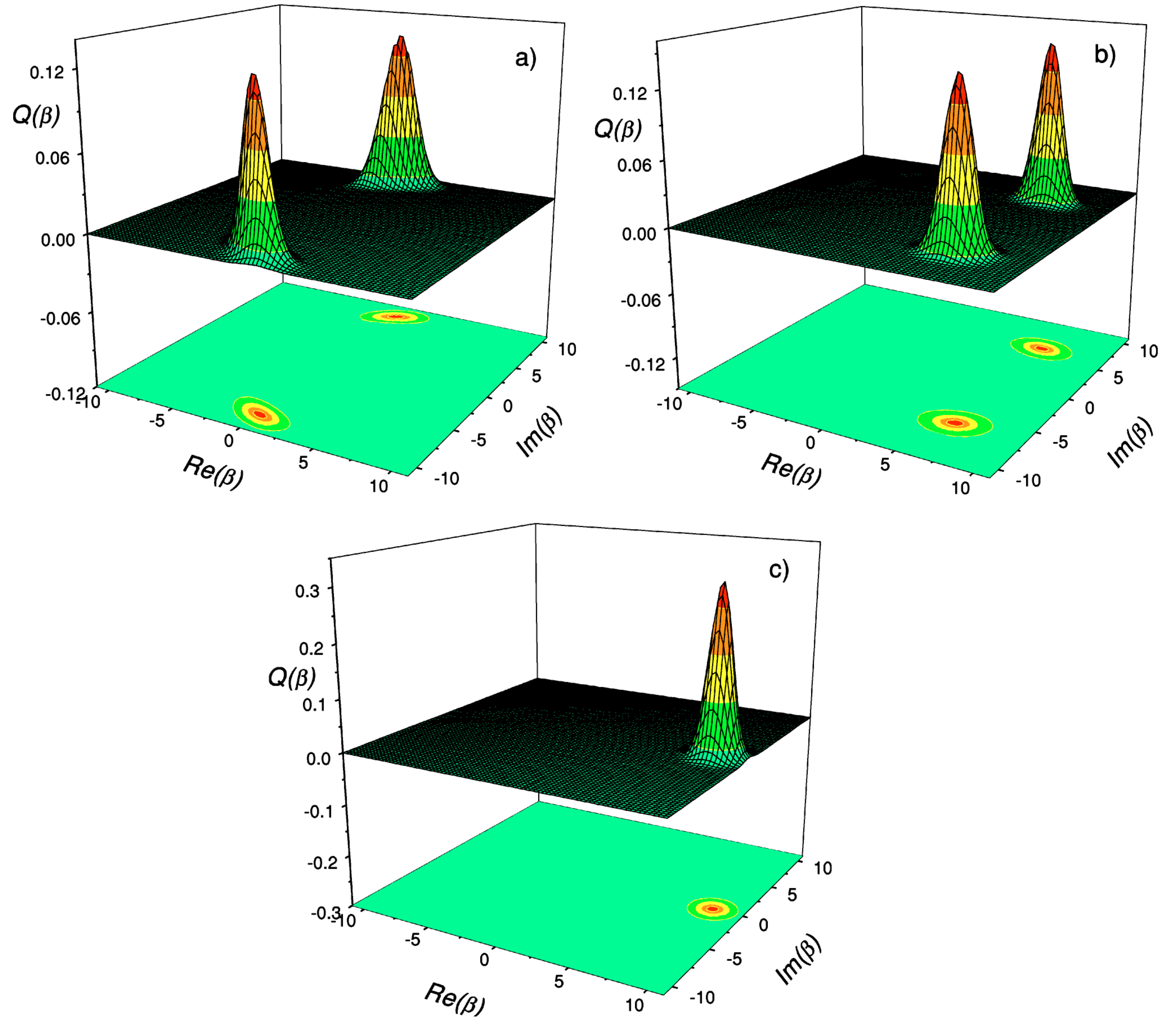

FIG. 3. Surface and contour plots of the Q function at different times $t=t_1+t_2$, for a switch time equal to one-half of the revival time, $t_1=t_r/2$. $t_2=$ (a) 0, (b) $t_r/4$, (c) $t_r/2$.

frequency of the laser field from $m=1$ to -1, in general, into two distributions, except for two particular instants: (a) if $\omega t_1=\pi k$ the initial state continues moving as a single distribution in the same direction; (b) if $\omega t_1=\pi(k+1/2)$ the distribution only changes the direction of its rotation to the opposite one.

Then, taking into account (8) the evolution of the initial excited electronic state and coherent vibrational state can be represented as

$$|\Psi(t)\rangle = U(t_1+t_2)|1\rangle|\alpha\rangle = \frac{1}{\sqrt{2}}\hat{Q}^{\dagger}[|\underline{1}\rangle|\Phi_1(t)\rangle + |\underline{0}\rangle|\Phi_0(t)\rangle], \tag{19}$$

where

$$|\Phi_{0,1}(t)\rangle = [\cos \omega t_1 e^{\pm i[\eta\Omega\sqrt{\hat{n}}(t_1+t_2)+\omega t_1]} \pm i \sin \omega t_1 e^{\mp i[\eta\Omega\sqrt{\hat{n}}(t_1-t_2)+\omega t_1]}]|\alpha\rangle. \tag{20}$$

The evolution of the inversion of electronic levels can be written as

$$\langle\Psi(t)|(\hat{A}_{22}-\hat{A}_{11})|\Psi(t)\rangle = \mathrm{Re}\langle\Phi_0(t)|\Phi_1(t)\rangle. \tag{21}$$

To estimate the revival times we neglect the effect of deformation of vibrational quanta distribution [a consequence of nonlinear evolution of the vibrational mode (18); see the discussion in [9]] and approximate the above overlap integral as follows:

$$\langle\Phi_0(t)|\Phi_1(t)\rangle \approx \cos^2 \omega t_1\langle\alpha e^{i\omega(t_1+t_2)}|\alpha e^{-i\omega(t_1+t_2)}\rangle + \sin^2 \omega t_1\langle\alpha e^{-i\omega(t_1-t_2)}|\alpha e^{i\omega(t_1-t_2)}\rangle. \tag{22}$$

It is thus clear that there exists a revival series associated

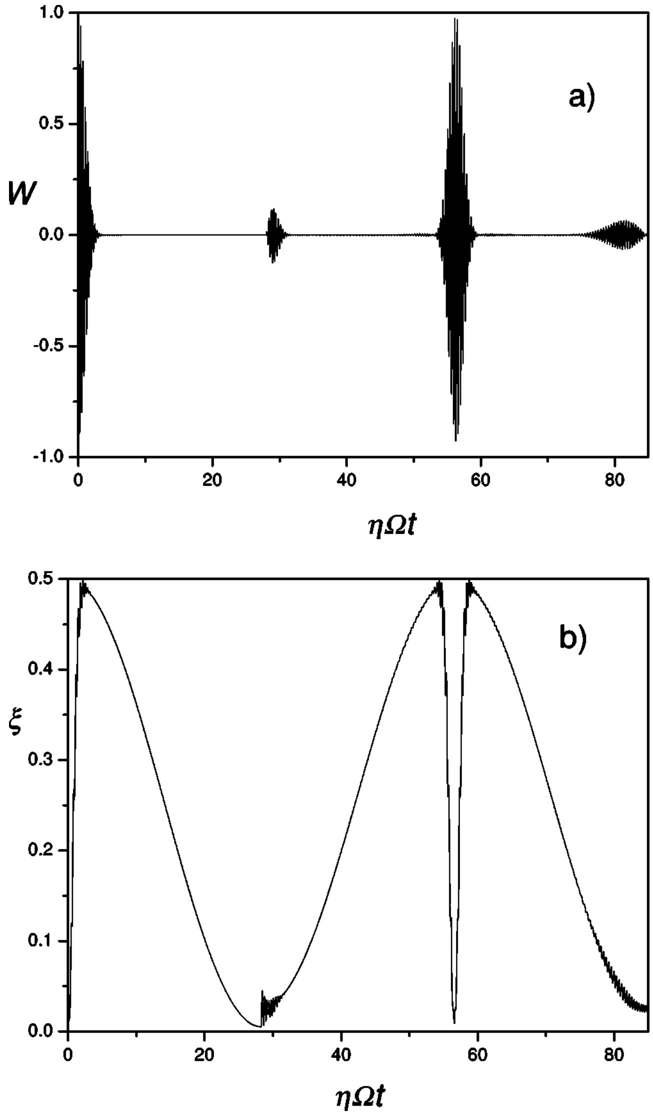

FIG. 4. (a) Atomic population inversion W and (b) purity function ξ as a function of the scaled time $\eta\Omega t$, with a switch time equal to one-half of the collapse time, $t_1 = t_r/2$.

with the rotation of the distribution corresponding to the interference of the semiclassical states $|\underline{1}\rangle$ and $|\underline{0}\rangle$ at time instants $\omega(t_1+t_2)=\pi k, k=1,2,\ldots$, which are the usual revival times. Nevertheless, the second revival series appears due to the splitting of the semiclassical states at the moment of switching from JCM to anti-JCM dynamics. This happens at instants $\omega(t_2-t_1)=\pi l, l=0,1,2,\ldots$. The amplitudes of these revivals depend on the switching time t_1 and are equal to $\cos^2 \omega t_1$ and $\sin^2 \omega t_1$ respectively if $\omega t_1 \neq \pi k/2, k=1,2,\ldots$. If the switching moment is such that $\omega t_1 = \pi k/2$, then revivals with unitary amplitude (i.e., complete revivals) happen at $\omega t_2 = \pi(l-k/2), k,l=1,2,\ldots$. This has the following phase space interpretation. In the course of the JCM dynamics the initial vibrational quanta distribution is split into two parts, each of which is associated with a corresponding semiclassical electronic state. If the switching moment is such that $\omega t_1 \neq \pi k/2$, then both distributions are split again into two components, one pair that rotates in the same direction as before the switching took place and another that changes its direction to the opposite one; the revivals are associated with overlapping of only one of those pairs (which happens at different times). If instead the switching moment coincides with half the revival time, then $\omega t_1 = \pi(k+1/2)$, and both parts of the distribution change direction to the opposite one and the revival occurs due to its overlapping (and interference) at the starting point of the phase space. If the switching moment coincides with the revival time $\omega t_1 = \pi k$, basically nothing happens and we have the standard JCM evolution.

In Figs. 1–4 we show the evolution of the Q function, the electronic inversion, and the purity evolution $[\xi(t)=1-\mathrm{Tr}\{\rho_v^2\}$ with ρ_v the vibrational (field) density matrix] for different switching instants. The initial state is taken excited for the electronic degree of freedom and coherent for the vibrational one. One can observe two possible scenarios of evolution.

(1) Switching at an arbitrary moment, but such that $t_1 \neq \pi\sqrt{\bar{n}}/\eta\Omega$, leads to a splitting of each distribution of vibrational quanta, corresponding to semiclassical electronic states $|\underline{0}\rangle$ and $|\underline{1}\rangle$, into two parts. One of them continues to move in the same direction as the initial distribution, whereas the other changes its direction to the opposite. Switching at a particular moment $t_1 = \pi\sqrt{\bar{n}}/2\eta\Omega$ leads to a splitting of that distribution into equal parts. Two series of revivals are present in this case.

(2) Switching at the moment $t_1 = \pi\sqrt{\bar{n}}/\eta\Omega$ (half revival moment) leads to a complete reversing of the dynamics, i.e., both parts of the distribution change direction of rotation into the opposite one. Obviously, only a single series of revivals survives with the revival time being the same as in the standard JCM case.

It is worth noting that switching at the moment $t_1 = 2\pi\sqrt{\bar{n}}/\eta\Omega$ (revival time) does not change the dynamics of the system at all.

III. CQED CASE

The same effect can be produced in the case of a quantized field interacting with a two-level atom in a high-Q cavity. It has been shown that the (complete) interaction of an ion with a laser field is completely equivalent to the atom-field interaction in CQED with an extra atomic driving term [6,7]. Solano *et al.* [7] have demonstrated that it is indeed possible to engineer anti-JC interactions in CQED. Below we use counter-rotating terms to produce such interaction. We start with the Hamiltonian

$$\hat{H} = \hbar\omega\hat{n} + \hbar\frac{\omega_0}{2}\hat{\sigma}_z + \hbar\lambda(\hat{a}+\hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-), \tag{23}$$

where ω and ω_0 are the field and atomic transition frequencies, respectively, λ is the interaction constant, and $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{\sigma}_-$ ($\hat{\sigma}_+$) are the lowering operators for the cavity field and the atom, respectively. Under the rotating-wave approximation (RWA) the above Hamiltonian (23) takes the standard JCM form:

$$\hat{H}_{JC} = \hbar\omega\hat{n} + \hbar\frac{\omega_0}{2}\hat{\sigma}_z + \hbar\lambda(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-). \tag{24}$$

Now, suppose we apply a Ramsey pulse (injecting a strong classical field for a short time inside the cavity) to the atom. The system state $|\psi(t)\rangle$ abruptly changes to

$$|\psi'(t)\rangle = \hat{\sigma}_y|\psi(t)\rangle. \tag{25}$$

This is equivalent to transforming the Hamiltonian (23) to the following form:

$$H_R = \hat{\sigma}_y H\hat{\sigma}_y = \hbar\omega\hat{n} - \hbar\frac{\omega_0}{2}\hat{\sigma}_z - \hbar\lambda(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-), \tag{26}$$

which after application of the RWA takes the anti-JCM form

$$H_{AJC} = \hbar\omega\hat{n} - \hbar\frac{\omega_0}{2}\hat{\sigma}_z - \hbar\lambda(\hat{a}\hat{\sigma}_- + \hat{a}^\dagger\hat{\sigma}_+). \tag{27}$$

IV. CONCLUSIONS

In conclusion, we have shown that the possibility of combining JCM and anti-JCM dynamics in trapped ions and/or CQED leads to the appearance of a second series of collapses and revivals in the process of interaction of two-level systems with a quantized field. On the other hand, it is notable that there are two particular switching instants at (a) the half revival time, leading to the complete reversing of the dynamics; and (b) the revival moment, when the dynamics of the system does not change. In particular, this means that in the first case, the field (vibrational excitations) distributions run over only half of the phase space, whereas in the second case, these distributions make a whole (or complete) revolution in the phase space. Finally, we should stress that our approach is different from that of Morigi *et al.* [5] as they manipulate the JC dynamics via an external pulse that produces a quasi-instantaneous phase change; this manipulation produces a JC evolution operator multiplied by the atomic inversion operator ($\hat{\sigma}_z$). Here instead we exploit the possibility of turning our system from JC to anti-JC dynamics, which results in a JC evolution operator multiplied by the operator $\hat{\sigma}_y$ [see Eq. (25)] that allows the presence of revivals immediately after the switching takes place. In the case of the switching taking place at half the revival time, it is shown that the dynamics can be completely reversed.

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