

Adiabatic pulse propagation in coherent atomic media with the tripod level configuration

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We investigate the problem of propagation of three-component resonant light pulses with adiabatically varying amplitudes through a medium consisting of atoms with the tripod level configuration. By means of both analytic and numerical methods we find the two modes of shape-preserving pulse propagation. The pulse propagation velocity of the fast mode is equal to the speed of light in vacuum, whereas the group velocity of the other (slow) mode is significantly slowed down. These two modes represent a general asymptotic solution of the problem of adiabatic pulse propagation, i.e., a pulse of any shape, which is consistent with the adiabaticity conditions, and a finite duration evolves at large propagation distances (and, correspondingly, at large times of interaction with the medium) to a well-separated pair of fast and slow pulses. The experimental requirements for adiabatic pulse propagation in a tripod medium are similar to that needed for observation of slow light propagation in a medium with the Λ configuration of levels. However, the tripod scheme offers a different possibility, which is absent in the Λ -medium case: collisions of fast and slow pulses. It is found numerically that after such a collision the shapes of the pulses change, so these pulses do not match the classical definition of a soliton.

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I. INTRODUCTION

Coherent population trapping (CPT) is a well-known phenomenon of preparation of atoms in a coherent superposition of ground or metastable state sublevels (so-called dark state), which is immune to excitation by a two-component laser radiation under the two-photon resonance condition [1]. Since the laser radiation is not scattered by atoms in the dark state, the radiation absorption is dramatically reduced. This effect is called electromagnetically induced transparency (EIT) and is actively studied since the early 1990's [2]. One of the most striking features of EIT is the possibility of shape-preserving propagation of light pulses with slowly (adiabatically) varying amplitudes at the group velocity significantly reduced with respect to the speed of light in vacuum c [3]. Shape-preserving electromagnetic pulses propagating in a coherent atomic medium at the reduced group velocity were called in Ref. [3] "adiabatons." Experimental observations of light a pulse group velocity less by many orders of magnitude than c has been repeatedly reported [4]. Slowing down the laser light followed by spatial compression of the pulses provides a unique possibility for the design of nonlinear-optical devices operating on a few-photon level [5]. Extreme sensitivity of CPT and EIT to deviations from the two-photon resonance allowed us to observe experimentally large Kerr nonlinearity [6] and absorptive optical switching [7] in cold rubidium vapor. Such nonlinear optical phenomena, along with the possibility of reversible conversion of a photonic excitation to a collective spin excitation [8] and trapping light in a medium with the photonic band gap induced by a periodic modulation of the EIT resonance [9], are of great importance for quantum information storage and processing.

A different direction in CPT and EIT studies is related to the systems admitting more than one dark state for the given real Rabi frequencies $|\Omega_j|$ and phases χ_j associated with the

resonantly driven transitions. The simplest scheme of such a kind is the tripod scheme displayed in Fig. 1. Stimulated Raman adiabatic passage (STIRAP) in such an optically thin atomic medium with the tripod level scheme was investigated theoretically [10] and demonstrated experimentally [11] by Bergmann and co-workers. A proposal to use the tripod scheme as a physical implementation of a qubit has been made recently [12]. A five-level scheme being the extension of the tripod scheme was considered in Ref. [13].

The very specifics of the tripod scheme is that during adiabatically slow change of the external field parameters, transitions between the two dark state occur. These transitions are described by a non-Abelian phase matrix [14], which is a generalization of a geometric (Berry) phase [15] to the case of degenerate eigenstates of an adiabatic Hamiltonian. One may expect that these transitions give rise to rich and complicated dynamics of laser pulse propagation in an optically dense medium with the tripod level configuration. However, only few theoretical works on EIT in such media are available. Paspalakis and Knight [16] considered parametric frequency generation for the case of time-independent fields at the medium entrance and calculated the group ve-

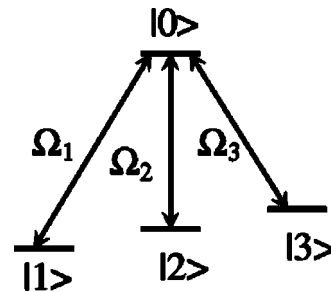


FIG. 1. Tripod scheme of levels driven by resonant electromagnetic fields.

locity of a weak probe field. Petrosyan and Malakyan [17] investigated theoretically EIT in a tripod medium as a tool for optical cross-phase modulation and high-precision magnetometry in the weak probe field limit. The value of the group velocity obtained in Refs. [16,17] is strongly reduced with respect to c in the same way as in the standard case of EIT in a three-level medium [3]. In the theoretical interpretation of the experiment on four-wave mixing in a solid-state system with the tripod level configuration [18] and other numerical calculations by Ham [19] related to that system, small optical density of the medium was assumed.

The aim of the present paper is to study pulse propagation in a medium with the tripod level scheme (hereafter briefly called “tripod medium”) for a general case, in which none of the three resonant electromagnetic fields is assumed to be weak compared to others. The paper is organized as follows. In Sec. II we present the set of equations treating pulse propagation in the adiabatic regime in a tripod medium beyond the weak probe approximation. In Sec. III the analytic solutions describing slow and fast pulse propagation are obtained. Section IV contains the results of our numerical calculations and their interpretation. Section V deals with some particular regimes of propagations. Section VI is devoted to conclusive remarks.

II. BASIC EQUATIONS

If the three electromagnetic fields are tuned exactly in resonance with the corresponding transitions $|j\rangle \leftrightarrow |0\rangle$, $j = 1, 2, 3$, the Hamiltonian in the interaction representation reads as

$$\hat{H} = -\hbar \sum_{j=1}^3 \Omega_j |0\rangle \langle j| + \text{H.c.}, \quad (1)$$

$\Omega_j \equiv |\Omega_j| e^{i\chi_j} = d_{0j} E_j / \hbar$, where d_{0j} is the dipole moment matrix element of the given transition. The electric field in the j th laser wave is $E_j \exp[ik_j(z - t/c)] + \text{c.c.}$, k_j being the radiation wave number. The complex amplitude E_j is a slowly varying function of z and t . Expanding the atomic wave function as $|\psi\rangle = a_0 |0\rangle + \sum_{j=1}^3 a_j |j\rangle$, we obtain the Schrödinger equation for the probability amplitudes:

$$i\dot{a}_j = -\Omega_j^* a_0, \quad j = 1, 2, 3, \quad (2)$$

$$i\dot{a}_0 = -\sum_{j=1}^3 \Omega_j a_j.$$

The set of shortened Maxwell equations for slowly varying field amplitudes can be written as

$$\left(\frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \Omega_j = iG_j a_0 a_j^*, \quad j = 1, 2, 3, \quad (3)$$

where $G_j = k_j n d_{0j}^2 / (2\hbar \epsilon_0)$, where ϵ_0 is the dielectric permittivity of vacuum in Système International (SI) units, and n is the atomic number density. Taking into consideration propagation effects described by Eq. (3) is the essence of the theory developed in the present section, in contrast to the

theory of Ref. [10], which applies to the case of a refractively thin medium.

Hereafter we assume that all the matter-field coupling constants are equal:

$$G_1 = G_2 = G_3 \equiv G. \quad (4)$$

Violation of this assumption leads to adiabaticity breakdown during the pulse propagation and subsequent pulse front steepening [20]. Thermal motion of atoms leads, besides reduction of the effective number density of atoms in resonance with the laser radiation, to a similar effect of pulse front steepening [21]. However, the pulse shape distortion effects manifest themselves at propagation distances much larger than the typical propagation distance associated with an adiabatic pulse formation [20,21]. Therefore we can neglect both the differences of oscillator strengths associated with the three laser-driven transitions and thermal motion of atoms. We also neglect the radiative decay of the excited state $|0\rangle$ since it plays no role in the adiabatic regime, because of negligible population of the excited state [3].

We parametrize the Rabi frequencies by introducing a generalized Rabi frequency $\Omega = (\sum_{j=1}^3 |\Omega_j|^2)^{1/2}$, two angular variables φ and ϑ , and three phases χ_j :

$$\begin{aligned} \Omega_1 &= \sin \vartheta \cos \varphi e^{i\chi_1} \Omega, \\ \Omega_2 &= \cos \vartheta \cos \varphi e^{i\chi_2} \Omega, \\ \Omega_3 &= \sin \varphi e^{i\chi_3} \Omega. \end{aligned} \quad (5)$$

There are two mutually orthogonal nonabsorbing (dark) states associated with the Hamiltonian of Eq. (1):

$$\begin{aligned} |\Phi^{(1)}\rangle &= \cos \vartheta e^{-i\chi_1} |1\rangle - \sin \vartheta e^{-i\chi_2} |2\rangle, \\ |\Phi^{(2)}\rangle &= \sin \vartheta \sin \varphi e^{-i\chi_1} |1\rangle + \cos \vartheta \sin \varphi e^{-i\chi_2} |2\rangle \\ &\quad - \cos \varphi e^{-i\chi_3} |3\rangle. \end{aligned} \quad (6)$$

An atom initially prepared in either of these two states remains unexcited since

$$\hat{H} |\Phi^{(s)}\rangle = 0, \quad s = 1, 2. \quad (7)$$

An atom also remains unexcited if the parameters of the laser radiation vary in time slowly enough to satisfy the adiabaticity conditions

$$\dot{\vartheta} \ll \Omega, \quad \dot{\varphi} \ll \Omega, \quad (8)$$

and

$$\dot{\chi}_j \ll \Omega, \quad j = 1, 2, 3. \quad (9)$$

However, there are adiabatic transitions [14] between the dark states defined by Eqs. (6), where the instantaneous values of the varying angles φ and ϑ and phases χ_j enter. If at $t \rightarrow -\infty$ an atom was in the s th dark state, its wave function at subsequent instants of time is

$$|\Psi^{(s)}\rangle = \sum_{s'=1}^2 B_{ss'}(t) |\Phi^{(s')}\rangle, \quad (10)$$

where the matrix \hat{B} obeys the equation

$$\dot{B}_{ss'}(t) + \sum_{s''} B_{ss''}(t) A_{s's''}(t) = 0 \quad (11)$$

with the obvious initial condition

$$B_{ss'}(-\infty) = \delta_{ss'}, \quad (12)$$

and

$$A_{s's''}(t) = \langle \Phi^{(s')} | \frac{\partial}{\partial t} | \Phi^{(s'')} \rangle. \quad (13)$$

Now we recall briefly the derivation [14] of Eqs. (11) and (13). First of all, the Hamiltonian (1) does not couple the state (10) directly to the excited state $|0\rangle$. And if the adiabaticity conditions (8) and (9) are satisfied, the coupling of the state (10) to the most absorbing state $|\Xi\rangle = \sum_{j=1}^3 (\Omega_j^*/\Omega) |j\rangle$, which is orthogonal to both the dark states, becomes negligibly small. This means that adiabatic evolution of an arbitrary superposition of the atomic dark states is confined to a two-dimensional subspace of the three-dimensional Hilbert space with basis vectors $|1\rangle, |2\rangle, |3\rangle$. The subspace comprising an arbitrary dark state evolves in time analogously to rotation of a plane in a three-dimensional Cartesian space. In a general case the right-hand side of Eq. (13) is nonzero for $s'=1, s''=2$. This means that transitions between the two orthogonal dark states occur even in the adiabatic regime. However, the two orthogonal states defined by Eq. (10) have the property

$$\langle \Psi^{(1)} | \frac{\partial}{\partial t} | \Psi^{(2)} \rangle = 0, \quad (14)$$

provided that Eqs. (11) and (13) hold. It means that adiabatic transitions between $|\Psi^{(1)}\rangle$ and $|\Psi^{(2)}\rangle$ are absent, and the time derivative of each of these two state vectors is always orthogonal to both of them (i.e., is collinear with the most absorbing state).

Explicitly, Eq. (13) takes the form

$$A_{11} = -i(\dot{\chi}_1 \cos^2 \vartheta + \dot{\chi}_2 \sin^2 \vartheta),$$

$$A_{12} = \dot{\vartheta} \sin \varphi - i(\dot{\chi}_1 - \dot{\chi}_2) \sin \vartheta \cos \vartheta \sin \varphi,$$

$$A_{21} = -A_{12}^*,$$

$$A_{22} = -i[(\dot{\chi}_1 \sin^2 \vartheta + \dot{\chi}_2 \cos^2 \vartheta) \sin^2 \varphi + \dot{\chi}_3 \cos^2 \varphi]. \quad (15)$$

It is easy to show (we do it in the end of the present section) that if the phases of the laser fields are kept constant at the medium entrance, then

$$\dot{\chi}_j = 0, \quad j = 1, 2, 3 \quad (16)$$

in the whole tripod medium. The opposite is not true. If the absolute values of the field amplitudes are constant at the

medium entrance, but the phases are modulated, then the absolute values of the fields amplitudes and hence ϑ and φ become time dependent inside the medium. In the present paper we consider only the case when Eq. (16) holds. In this case Eqs. (15) are reduced to

$$A_{11} = A_{22} = 0, \quad A_{12} = -A_{21} = \dot{\nu}, \quad (17)$$

where

$$\dot{\nu} = \dot{\vartheta} \sin \varphi \quad (18)$$

and $\nu(z, -\infty) = 0$. Then Eqs. (11) and (12) yield the following result [10]:

$$B_{11} = B_{22} = \cos \nu, \quad B_{12} = -B_{21} = \sin \nu. \quad (19)$$

We assume that the tripod medium occupies the half space $z > 0$. Initially, at $t \rightarrow -\infty$, all the atoms in the medium are in the coherent superposition of the dark states,

$$|\Psi(-\infty)\rangle = \cos \beta |\Phi^{(1)}\rangle + \sin \beta |\Phi^{(2)}\rangle. \quad (20)$$

The boundary conditions for the fields at the medium entrance $\Omega(0, t) = \Omega_0(t)$, $\vartheta(0, t) = \vartheta_0(t)$, and $\varphi(0, t) = \varphi_0(t)$ are consistent with Eq. (8). Thus the adiabatic regime of the laser radiation propagation inside the medium is ensured. It is convenient to introduce new variables $\zeta = z$ and $\tau = t - z/c$, as in Ref. [3]. Respectively, the derivatives over the new variables are $\partial/(\partial\tau) = \partial/(\partial t)$ and $\partial/(\partial\zeta) = \partial/(\partial z) + c^{-1}\partial/(\partial t)$.

Now we can solve self-consistently the set of Schrödinger-Maxwell equations (2) and (3). First of all, we note that in the adiabatic regime a_0 is very small, and the probability amplitudes of the low-energy states ($j=1, 2, 3$) are, according to Eqs. (10), (19), and (20),

$$a_j = \cos(\nu + \beta) \langle j | \Phi^{(1)} \rangle + \sin(\nu + \beta) \langle j | \Phi^{(2)} \rangle. \quad (21)$$

Then we find easily that, similarly to the case of adiabatic pulse propagation in a Λ medium [3],

$$\frac{\partial}{\partial\zeta} \Omega = 0, \quad (22)$$

i.e., $\Omega = \Omega_0(\tau)$. Then we use the trick first applied in Ref. [3]: We express the small probability amplitude of the excited state as $a_0 = -(i/\Omega_j^*) \partial a_j / (\partial\tau)$ and substitute this expression into the shortened Maxwell equations (3). We get

$$\frac{\partial}{\partial\zeta} \left| \frac{\Omega_j}{\Omega} \right|^2 = \frac{G}{\Omega^2} \frac{\partial}{\partial\tau} |a_j|^2, \quad j = 1, 2, 3, \quad (23)$$

or, explicitly,

$$\begin{aligned} \frac{\partial}{\partial\zeta} (\sin \vartheta \cos \varphi)^2 &= \frac{\partial}{\partial w} [\cos(\nu + \beta) \cos \vartheta \\ &+ \sin(\nu + \beta) \sin \vartheta \sin \varphi]^2, \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial\zeta} (\cos \vartheta \cos \varphi)^2 &= \frac{\partial}{\partial w} [-\cos(\nu + \beta) \sin \vartheta \\ &+ \sin(\nu + \beta) \cos \vartheta \sin \varphi]^2, \end{aligned}$$

$$\frac{\partial}{\partial \zeta} \sin^2 \varphi = \frac{\partial}{\partial w} [\sin(\nu + \beta) \cos \varphi]^2. \quad (24)$$

Here we introduced, instead of τ , a new variable (nonlinear time)

$$w = \frac{1}{G} \int_{-\infty}^{\tau} \Omega_0^2(\tau') d\tau', \quad (25)$$

which has the dimension of length. Then Eq. (18) takes the form

$$\frac{\partial \nu}{\partial w} = \frac{\partial \vartheta}{\partial w} \sin \varphi. \quad (26)$$

All the initial conditions set at $t \rightarrow -\infty$ apply now to $w=0$.

Only two of Eqs. (24) are independent. After some tedious calculations they are reduced to

$$\begin{aligned} \frac{\sin(\nu + \beta)}{\cos \varphi} \left(\frac{\partial}{\partial \zeta} + \frac{\partial}{\partial w} \right) \varphi - \cos(\nu + \beta) \left(\frac{\partial}{\partial \zeta} + \frac{\partial}{\partial w} \right) \vartheta = 0, \\ \frac{\cos(\nu + \beta)}{\cos \varphi} \frac{\partial \varphi}{\partial \zeta} + \sin(\nu + \beta) \frac{\partial \vartheta}{\partial \zeta} = 0. \end{aligned} \quad (27)$$

It is convenient now to change the variables to $u_1 = \zeta - w$ and $u_2 = w$. The set of Eqs. (26) and (27) takes the form

$$\frac{\cos(\nu + \beta)}{\cos \varphi} \frac{\partial \varphi}{\partial u_1} + \sin(\nu + \beta) \frac{\partial \vartheta}{\partial u_1} = 0, \quad (28)$$

$$\frac{\sin(\nu + \beta)}{\cos \varphi} \frac{\partial \varphi}{\partial u_2} - \cos(\nu + \beta) \frac{\partial \vartheta}{\partial u_2} = 0, \quad (29)$$

$$\frac{\partial \nu}{\partial u_1} - \frac{\partial \nu}{\partial u_2} = \left(\frac{\partial \vartheta}{\partial u_1} - \frac{\partial \vartheta}{\partial u_2} \right) \sin \varphi. \quad (30)$$

Let us now return to explanation of Eq. (16). In the way similar to that resulting in derivation of Eq. (23), we can obtain an equation describing the evolution of the field phases inside the medium in a general case:

$$\frac{\partial}{\partial \zeta} \chi_j = \frac{G}{|\Omega_j|^2} \text{Im} \left(a_j^* \frac{\partial}{\partial \tau} a_j \right), \quad j = 1, 2, 3. \quad (31)$$

If initially (at $\tau \rightarrow -\infty$) and at the medium boundary ($\zeta=0$) the field phases are set to zero, and the medium is in a superposition of the two dark states with real coefficients, then for all τ and ζ inside the medium the right-hand side of Eq. (31) is identically zero, since the coefficients a_j remain real. It results in Eq. (16).

Of course, to avoid any doubt in physical applicability of Eq. (16), one needs to perform also linear stability analysis of solutions of the set of Eqs. (28)–(30) with respect to fluctuations of the pulse phases. This problem in its general form is to be addressed in a separate publication, along with a theoretical study of adiabatic pulse propagation in a case of nontrivial boundary and initial conditions for χ_j . Now we mention the results of perturbative treatment in a particular case of unperturbed fields independent on both τ and ζ [16,17], where no instability is found. Correspondingly, we

can guess that in the general case of real, τ -, and ζ -dependent pulse amplitudes spontaneous generation of field phases due to instability is also impossible.

III. SLOW AND FAST PULSES: ANALYTIC SOLUTION

The set of Eqs. (28)–(30) is especially convenient for searching analytic solutions in a case when the unknown functions φ and ϑ depend on only one of the variables u_1, u_2 . We find two classes of solutions.

The first one is the class of slow pulses. In this case the unknown functions depend only on $u_1 = \zeta - w$. The group velocity v_g of pulses of such has the same form as that of adiabats in a Λ medium [3]: $v_g = (c^{-1} + G/\Omega^2)^{-1}$ and can be much less than c . All the derivatives over u_2 vanish, thus making Eq. (29) an identity. The two remaining equations (28) and (30) become ordinary differential equations, yielding the general solution in the parametric form:

$$|\cos \varphi| = \frac{C_1}{|\cos(\nu + \beta)|}, \quad |\sin(\vartheta - C_2)| = \frac{|\sin(\nu + \beta)|}{\sqrt{1 - C_1^2}}. \quad (32)$$

Here C_1, C_2 are arbitrary constants, and one may take for $\nu = \nu(\zeta - w)$ any function of $\zeta - w$ compatible with the adiabaticity conditions (8). Any solution of the form of Eq. (32) satisfies the equations $[\partial/(\partial z) + v_g^{-1} \partial/(\partial t)] \varphi = 0$ and $[\partial/(\partial z) + v_g^{-1} \partial/(\partial t)] \vartheta = 0$ thus making obvious the statement that v_g defined above is the group velocity of a slow pulse.

Similarly, we find a general solution for the class of fast pulses, obeying the equations $[\partial/(\partial z) + c^{-1} \partial/(\partial t)] \varphi = 0$ and $[\partial/(\partial z) + c^{-1} \partial/(\partial t)] \vartheta = 0$, i.e., propagating at the speed of light:

$$|\cos \varphi| = \frac{C_3}{|\sin(\nu + \beta)|}, \quad |\sin(\vartheta - C_4)| = \frac{|\cos(\nu + \beta)|}{\sqrt{1 - C_3^2}}. \quad (33)$$

Here C_3, C_4 are arbitrary constants, and a particularly chosen form of $\nu = \nu(w)$ must be compatible with Eq. (8).

Although the set of Eqs. (28)–(30) looks rather simple and symmetric, our attempts to find its general solution in the case of dependence of φ and ϑ on both u_1 and u_2 have been unsuccessful. However, we can prove that a time-dependent solution in the parametric form

$$\varphi = \varphi(\nu), \quad \vartheta = \vartheta(\nu) \quad (34)$$

does not exist if

$$\frac{\partial \nu}{\partial u_1} \neq 0, \quad \frac{\partial \nu}{\partial u_2} \neq 0. \quad (35)$$

Indeed, Eqs. (28) and (29) can be considered as linear homogeneous algebraic equations for $\sin(\nu + \beta)$ and $\cos(\nu + \beta)$. They have a solution if

$$\frac{\partial \vartheta}{\partial u_1} \frac{\partial \vartheta}{\partial u_2} + \frac{1}{\cos^2 \varphi} \frac{\partial \varphi}{\partial u_1} \frac{\partial \varphi}{\partial u_2} = 0. \quad (36)$$

But if we make an assumption given by Eq. (34) then Eq. (36) results in

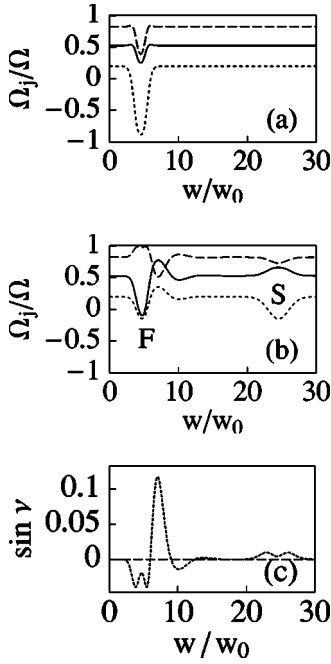


FIG. 2. Splitting of the incident pulse into the fast (F) and slow (S) pulses. The normalized pulse amplitudes Ω_j/Ω for $j=1$ (solid line), 2 (dashed line), and 3 (dotted line) versus scaled nonlinear time, w/w_0 (the same line styles are reserved for the pulse amplitudes in the subsequent figures) are displayed for (a) $\zeta=0$ (the boundary conditions at the medium entrance) and (b) $\zeta=20w_0$ (numerical simulation results). (c) Sine of the mixing angle ν vs w/w_0 for $\zeta=0$ (dashed line) and $20w_0$ (dotted line).

$$\frac{\partial \nu}{\partial u_1} \frac{\partial \nu}{\partial u_2} \left[\left(\frac{d\vartheta}{d\nu} \right)^2 + \frac{1}{\cos^2 \varphi} \left(\frac{d\varphi}{d\nu} \right)^2 \right] = 0. \quad (37)$$

If Eq. (35) holds, it follows from Eq. (37) that $\varphi=\text{const}$ and $\vartheta=\text{const}$, i.e., there is no variation of the electromagnetic fields in space and time.

The fact that we have not found other pulse group velocities than c and $(c^{-1}+G/\Omega^2)^{-1}$ is in full agreement with the results of perturbative approach [16,17].

IV. NUMERICAL SOLUTIONS

It is natural to expect that any pulse of finite duration evolves in the medium into pairs of fast and slow pulses, which become more and more separated in space due to the difference of their group velocities. Indeed, our numerical simulations confirm such an expectation, so that Eqs. (32) and (33) appear to be general asymptotic solutions of the set of Eqs. (28)–(30) in the limit of large ζ or large w . An example is shown in Fig. 2. The quantity w_0 used for normalization of the horizontal axes of the plots in Fig. 2 and subsequent determines the order of magnitude of $\dot{\varphi}$ and $\dot{\vartheta}$, which are $\sim \Omega^2/(Gw_0)$. The adiabaticity condition (8) results in the following restriction: $w_0 \gg \Omega/G$. One can see that the incident pulse evolves into a well separated pair of fast (F) and slow (S) pulses, and the mixing angle ν describing transitions between the two dark states emerges (the incident pulse is chosen in such a form that $\nu \equiv 0$ at the medium

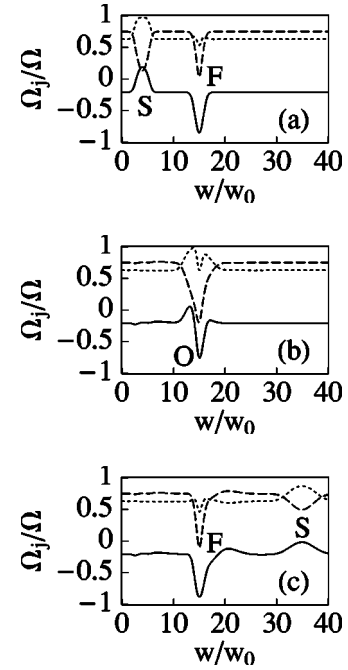


FIG. 3. Collision of pulses. (a) The normalized pulse amplitudes vs w/w_0 at (a) $\zeta=0$ (the boundary conditions), (b) $\zeta=10w_0$ (numerical results), and (c) $\zeta=30w_0$ (numerical results). See the text for more detailed description.

entrance). The fast and slow pulses at large propagation distances can be excellently fitted with formulas (33) and (32), respectively. E.g., fitting with Eq. (33), $C_1=0.88$, $C_2=1.75$, for $w/w_0 < 15.5$ and Eq. (32), $C_3=0.43$, $C_4=-0.90$, for $w/w_0 > 15.5$ yields the plots practically indistinguishable from the numerical results displayed in Fig. 2(b).

We also investigated numerically collisions between fast and slow pulses. The results are presented in Fig. 3. The pulse sequence is organized in such a way that the pulse of a shape satisfying Eq. (32) enters the medium first. After some time delay the next pulse obeying Eq. (33) enters the medium. The first pulses propagates at the slow group velocity whereas the second one propagates at the speed of light. The distance between them decreases, and at certain ζ the two pulses overlap [this is marked by O in Fig. 3(b)]. Their nonlinear interaction leads to strong distortion of their shapes, which becomes apparent at larger propagation distances. Thus adiabatic pulses in a tripod medium cannot be called solitons in the exact sense of soliton definition by Zabusky and Kruskal [22]. Note that it is impossible to arrange a collision of two adiabats in a Λ medium.

The initial conditions used in the numerical simulations imply ζ -independent laser fields in the medium at $w=0$. Of course, the boundary and initial conditions are consistent, i.e., they give the same values of Ω_j , $j=1, 2, 3$, at the point $(\zeta, w)=(0, 0)$. From the initial conditions for the pulse amplitudes one can infer from Eq. (5) the initial values for ϑ and φ . The initial atomic state also does not depend on ζ and is defined by the constant parameter β , which is equal to 1.12 for Fig. 2 and 1.87 for Fig. 3.

V. PARTICULAR REGIMES OF PROPAGATION

There are a few particular regimes of adiabatic pulse propagation admitting analytic treatment. The first one occurs if atoms are prepared initially in a statistical mixture described by the density matrix $\hat{\rho} = \frac{1}{2}(|\Phi^{(1)}\rangle\langle\Phi^{(1)}| + |\Phi^{(2)}\rangle\langle\Phi^{(2)}|)$, rather than in a pure state. Such a mixed state remains invariant under the action of the slowly varying electromagnetic fields: $\hat{B}\hat{\rho}\hat{B}^\dagger = \hat{\rho}$, where \hat{B} is given by Eq. (19). Statistical averaging over $\hat{\rho}$ is equivalent to averaging over the parameter β uniformly distributed between 0 and 2π , without any correlation with the instantaneous values of φ and ϑ . The result of statistical averaging of Eqs. (24) is

$$\frac{\partial\varphi}{\partial\zeta} = -\frac{1}{2}\frac{\partial\varphi}{\partial w}, \quad \frac{\partial\vartheta}{\partial\zeta} = -\frac{1}{2}\frac{\partial\vartheta}{\partial w}. \quad (38)$$

Equations for φ and ϑ become decoupled. Their solution $\varphi = \varphi_0(w - \frac{1}{2}\zeta)$, $\vartheta = \vartheta_0(w - \frac{1}{2}\zeta)$ describes independent propagation of perturbations of ϑ and φ at the same group velocity $v_g = [c^{-1} + G/(2\Omega^2)]^{-1}$.

We may hazard a conjecture what occurs if atoms are prepared in a mixed state with the density matrix $\hat{\rho}' = q|\Phi^{(1)}\rangle\langle\Phi^{(1)}| + (1-q)|\Phi^{(2)}\rangle\langle\Phi^{(2)}|$, $0 \leq q \leq 1$. It is likely that there are always two classes of pulses with well defined group velocities. If q grows from 0 to 0.5, one of these velocities decreases whereas the other increases. At $q=0.5$ they achieve the same value mentioned in the previous paragraph, and then again restore their values c and $(c^{-1} + G/\Omega^2)^{-1}$, as q approaches 1. At least, it can be proven easily in the perturbative regime, when the changes of both φ and ϑ are small.

Another interesting regime is related to particular initial conditions $\beta=0$ or $\beta=\pi/2$. Let all the atoms be pumped initially into the state $|3\rangle$. The fields are switched on in the following order, which is a generalization of the counterintuitive pulse order for a Λ medium [3]: Initially, at $w=0$ only the field driving the empty transition $|2\rangle \leftrightarrow |0\rangle$ is present, i.e., $\varphi = \vartheta = 0$. Obviously, $\beta=0$. Then the field driving the transition $|1\rangle \leftrightarrow |0\rangle$ is switched on adiabatically, so that ϑ grows and then is kept constant at a certain level. Finally, the field driving the transition $|3\rangle \leftrightarrow |0\rangle$ is switched on.

When ϑ changes, $\sin\varphi=0$. Then, according to Eq. (18), v remains zero, and Eqs. (27) are reduced to $\partial\varphi/(\partial u_1)=0$, $\partial\vartheta/(\partial u_2)=0$. Such a propagation regime occurs unless the front of the ϑ pulse, propagating at the slow group velocity, approaches the front of the φ pulse, propagating at c .

Thus one has a possibility of preparation of a tripod medium in any desired coherent superposition of low-energy states. Numerical results presented in Fig. 3 illustrate this conclusion: Finally, atoms in the region $0 < z < 20w_0$ are prepared in the state $-0.29|1\rangle - 0.53|2\rangle - 0.80|3\rangle$, as can be derived from the values $\varphi = -0.65$, $\vartheta = 0.50$ at $w = 40w_0$. Then one can suddenly change the laser radiation parameters in such a manner that this state will correspond to a coherent superposition of the two dark states defined with respect to the new values of the Rabi frequencies, thus obtaining a new value for the parameter β .

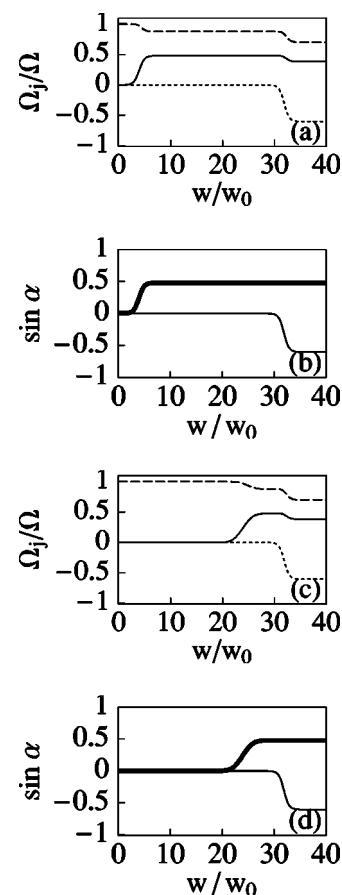


FIG. 4. Adiabatic pulse propagation for the particular initial condition $\beta=0$. The boundary conditions ($\zeta=0$): (a) the normalized pulse amplitudes and (b) sine functions of the angles $\alpha = \vartheta$ and φ vs w/w_0 . The numerical results at $\zeta=20w_0$ for (c) Ω_j/Ω and (d) $\sin\vartheta$, $\sin\varphi$.

The case of $\beta = \pi/2$ is physically equivalent to the previous one, differing only in notation of the states and electromagnetic fields.

VI. CONCLUSION

Requirements for experimental implementation of adiabatic pulse propagation in a tripod medium should not differ from that for slow light propagation in Λ media [2,4,8]. A method for initial preparation of a tripod medium in any desired superposition state was outlined in the previous section. For example, consider a tripod medium with the following parameters: $d_{0j} \approx 0.4ea_B$, e being the elementary charge and a_B being the Bohr radius, $k_j \approx 10^5 \text{ cm}^{-1}$, $n \approx 10^{12} \text{ cm}^{-3}$. Let the total laser intensity be of about 2.6 mW/cm^2 (close to the atomic transition saturation limit). Hence $\Omega \approx 2.4 \times 10^6 \text{ s}^{-1}$ and $\Omega^2/G \approx 10^4 \text{ cm/s}$ (i.e., the group velocity of the slow pulses is $v_g \approx 0.3 \times 10^{-6}c$). The value $w_0 = 0.1 \text{ cm}$ is thus large enough to provide adiabaticity. Therefore the processes illustrated in Fig. 2–4 can be observed in a gas cell of a length ranging from 1 cm [Fig. 3(b)] to 3 cm [Fig. 3(c)]. The time delay between the fast and slow pulses is of about

0.2 ms, therefore the lifetime of coherence between the states $|j\rangle$, $j=1,2,3$, should be 1 ms or longer. It is achievable in coated cells or cells with a buffer gas.

To provide an illustrative example, we reconsider Figs. 2–4. If the total laser intensity is kept constant during the pulse, when the ratios between the amplitudes of the three components vary, then the variable w becomes a linear function of time $w/w_0=t/t_0$. Accepting the set of parameters described above we get $t=0.01$ ms. Likewise, in terms of the electric-field amplitudes E_j we get $\Omega_j/\Omega=E_j/E_{max}$, where $E_{max}=70$ V/m.

To observe the slow propagation mode in the absence of the fast one, it is necessary first to choose adiabatically slow varying function $\nu(z-v_g t)$ and the two constants, C_1 and C_2 . Then the variables φ and ϑ should be obtained using Eq. (32). Then Eq. (5) should be used to determine the pulse slowly varying amplitudes Ω_j and hence E_j , $j=1,2,3$. By setting $z=0$, one obtains the formula for the incident pulse shape. As concerns experiment, one has to pump all the atoms into a certain sublevel of the ground state, say, into $|1\rangle$, then to apply a pulse sequence as described in Sec. V, to prepare the medium in the initial state consistent with the pre-chosen initial values of φ and ϑ . At the final stage, the amplitudes of the three laser fields must be rapidly changed (only if one wants to perform an experiment for the value of β other than 0 or $\pi/2$) and then slowly modulated to create an incident pulse of the desired form. The time interval between the medium preparation stage and sending the pulse must not exceed the lifetime of coherence between the

ground-state sublevels. The procedure for achieving the fast propagation mode only is completely analogous.

To conclude, we have investigated electromagnetic pulse propagation in a coherent atomic medium with the tripod configuration of levels in the adiabatic regime. The propagation equations (28)–(30) are derived and their solutions in the form of slow [Eq. (32)] and fast [Eq. (33)] pulses are obtained analytically. Our numerical simulations confirm that these solutions are general asymptotic solutions for any incident pulse of a finite duration. We have suggested a method of preparation of a tripod medium in an arbitrary superposition of the low-energy states based on switching on the laser fields in a counterintuitive order. The tripod scheme provides two interesting features in comparison to the Λ scheme. The first one is adiabatic pulse propagation in a medium prepared in a statistical mixture of the two dark states. The second one is the possibility of collisions between the slow and fast pulses revealing that they change their shapes after nonlinear interaction and thus do not satisfy the classical definition of a soliton [22].

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