Pulse transformation and time-frequency filtering with electromagnetically induced transparency

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A simple analytical solution for the propagation of a weak Gaussian pulse in a dense absorptive medium with electromagnetically induced transparency is found. This solution is applied to the analysis of three regimes: (1) and (2) the pulse spectrum is narrower than the transparency window [which is narrow (1) or wide (2) with respect to the width of the absorption line] and (3) the pulse spectrum is broader than the transparency window. It is shown that the pulse maintains its area in all three regimes and maintains its Gaussian shape but narrows in spectrum in regime 1. In regime 2, the pulse begins to distort after a certain distance. In regime 3, the pulse is split into two parts. One part is an adiabatic part with a spectrum defined by the effective width of the transparency window for a thick medium and the other is an oscillating nonadiabatic part of short duration. The adiabatic part propagates slowly and the nonadiabatic part propagates with a velocity close to the speed of light. Thus in regime 3, the medium acts as a time-frequency filter, separating the narrow and wide spectrum components of the pulse in time at the output of the absorber.

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I. INTRODUCTION

Electromagnetically induced transparency (EIT) [1–4], slow group velocity or stopping of light [5–10], and large refractive index without absorption [11–14] are the subjects of intense research in quantum optics, both theoretical and experimental. EIT has been successfully demonstrated under different experimental conditions: in continuous wave and pulsed regimes [3], with atomic and molecular gases [15,16], with solids doped by rare-earth ions [17], and with semiconductor quantum wells [18,19] for different wavelengths ranging from optics to microwaves. Recently [20,21], a variation of EIT, called level mixing induced transparency, has been found using gamma radiation.

In a very basic scheme of EIT, the transparency for the probe field is created by the drive field if both fields are in resonance with different, but adjacent, transitions of a three-level atom. The probe field is in resonance with the transition from the ground state (g) to the excited state (e) and the drive field couples the excited state (e) with an unpopulated metastable state (m); see Fig. 1. A key point for EIT is the slow decay rate γ of the coherence g-m induced by the probe and drive fields via a two-quantum process. If $\gamma \ll \Gamma$ and $\gamma\Gamma \ll \Omega_d^2$, where Γ is the decay rate of the coherence g-e, induced by the probe, and Ω_d is the coupling parameter of the drive, then a narrow hole appears in the absorption line for the probe. The absorption of the probe at the hole center is reduced by a factor of $\gamma\Gamma/\Omega_d^2$ with respect to the unperturbed line. The width of this hole for a single atom is estimated as Ω_d^2/Γ if $\Omega_d < \Gamma$. For an optically thick absorber, the hole narrows as $\Omega_d^2/\Gamma\sqrt{T}$, where T is the effective thickness of the sample (see, for example, [22]).

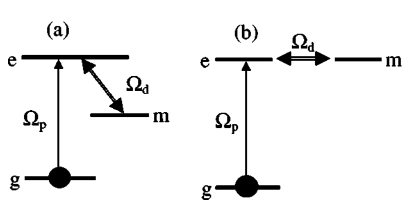

FIG. 1. Three-level atom excited by the probe Ω_p and drive Ω_d fields. The amplitude of the probe is time dependent and the amplitude of the drive is not. g, e, and m are ground, excited, and metastable states. (a) e and m states have different energies. (b) e and m are degenerate states and the drive is a dc field.

In this paper we develop an analytical theory describing the propagation of a weak Gaussian pulse in a thick resonant absorber with EIT window. Our aim is to consider the case when the approximation of adiabatic following of the dark state [2,3,23] is not applicable or violated to some extent, which is the case for a short pulse whose spectrum is much broader than the transparency window. We propose a time-frequency filter that is complementary to cw (or frequency domain) field filtering by EIT. Such a filter does not change the shape of the input pulse and the pulse area; however, it makes the pulse longer. The pulse area is an important parameter for the description of the resonant pulse interaction with atoms [see the definition of the pulse area in Eq. (27) below]. For example, the atomic polarization induced by a short resonant pulse in a two-level medium with long irreversible relaxation time is proportional to the sine function of the pulse area; see, for example, Ref. [24]. The polarization induced in an ensemble of two-level atoms with an inhomogeneously broadened line keeps the memory about the phase, duration, and shape of the pulse, which can be retrieved in the photon echo pulse if a second reading pulse is applied [24]. Since the pulse area specifies the value of the induced polarization, this pulse parameter defines the effectiveness of the imprinting of the information into the atomic coherence induced by the pulse. EIT filtering of the pulse conserves its area, i.e., the ability of the pulse to induce a certain amount of atomic coherence. Therefore, the EIT window can be used as a perfect filter to create a source field for an optical memory based on the photon echo phenomenon. This filter makes the pulse spectrum narrow but preserves the pulse area.

The paper is organized as follows. In Sec. II we present the general formalism employed in the description of the propagation of a small-amplitude, probe pulse in a medium containing three-level atoms driven by a cw coupling field. In Secs. III and IV we consider the adiabatic solution of the Maxwell-Bloch equations for the probe pulse propagation with a spectral width smaller than the width of the EIT window. In Sec. V, this solution is applied as part of the general solution to describe the pulse with spectral width larger than the width of the EIT window. In this section the nonadiabatic corrections are also found. In Sec. VI the interference phenomenon is considered if the probe frequency is detuned from the line center. The analytical expressions for the pulse distortion and nonadiabatic corrections are derived in Appendixes A and B.

II. PROPAGATION OF A SMALL-AMPLITUDE PULSE AT THE EIT CONDITION

We consider the interaction of two fields, the probe $E_p=E_{p0}(z,t)\exp(-i\omega_p t+ik_p z)$ and the drive $E_d=E_{d0}\exp(-i\omega_d t+ik_d z)$, with a three-level atom shown in Figs. 1(a) and 1(b). Scheme (a) assumes the laser pump as a drive field, while in scheme (b) the drive is a dc field that couples two closely spaced (or essentially degenerate) levels e and m and hence $\omega_d=0$ and $k_d=0$. In both schemes, the atom is initially in the ground state g. The coherence between the ground state g and the excited state e decays with a fast decay rate Γ. The coherence between the ground state g and the metastable state m decays with a much smaller decay rate γ ($\gamma \ll \Gamma$). In both schemes, the coupling parameter Ω_d for the drive is constant. In case (a), this corresponds to the cw pump of the initially empty states e and m.

We assume that the amplitude of the probe pulse E_p is small, that is, its coupling parameter Ω_p satisfies the conditions $\Omega_p \ll \Gamma$, Ω_d and $\Omega_p^2 \ll \Gamma\Gamma_e$, where Γ_e is the decay rate of the excited state e. In this case, one can take a linear response approximation where only two equations from the complete set of the matter equations have to be considered, i.e.,

$$\dot{\sigma}_{eg}=(i\delta-\Gamma)\sigma_{eg}+i\Omega_p N+i\Omega_d\sigma_{mg}, \tag{1}$$

$$\dot{\sigma}_{mg}=(i\delta-\gamma)\sigma_{mg}+i\Omega_d\sigma_{eg}. \tag{2}$$

Here $\sigma_{eg}(z,t)=\rho_{eg}(z,t)\exp(i\omega_p t-ik_p z)$, $\sigma_{mg}(z,t)=\rho_{mg}(z,t)\exp[i(\omega_p-\omega_d)t-i(k_p-k_d)z]$ are the slowly varying amplitudes of the nondiagonal components of the three-level atom density matrix ρ_{ij}. The frequency of the probe can be detuned from resonance on $\delta=\omega_p-\omega_{eg}$. The drive frequency ω_d is assumed to be always in resonance with the transition e-m: $\omega_d=\omega_{em}$. Here $\omega_{eg}=\omega_e-\omega_g$ and $\omega_{em}=\omega_e-\omega_m$ are the resonant frequencies of the transitions e-g and e-m between the states e, m, and g with the energies $\hbar\omega_e$, $\hbar\omega_m$, and $\hbar\omega_g$, respectively. The coupling parameters for the probe and drive are defined as real values $\Omega_p=d_{eg}E_{p0}(z,t)/\hbar$ and $\Omega_d=d_{em}E_{d0}/\hbar$, where d_{eg} and d_{em} are the dipole matrix elements for the transitions e-g and e-m, respectively. The value $N=\rho_{gg}-\rho_{ee}$ is assumed to be equal to unity throughout the excitation process since we neglect any saturation effect (weak probe condition).

The wave equation for the slowly varying amplitude of the probe $E_{p0}(z,t)$ is

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)E_{p0}(z,t)=i\frac{\alpha\hbar}{d_{eg}}\sigma_{eg}(z,t), \tag{3}$$

where the right-hand side is the response function of the atoms located in a plane with coordinate z, and $\alpha=2\pi N_c|d_{eg}|^2\omega_p/\hbar c$ is the resonant absorption coefficient of the sample with concentration N_c of the three-level atoms. This coefficient is defined such that $2\alpha/\Gamma$ is the Beer's constant for a monochromatic field resonant with the e-g transition. The probe pulse is considered as a plane wave with the wave vector $\mathbf{k}_p$ parallel to the z axis.

By means of the Fourier transform

$$F(\nu)=\int_{-\infty}^{+\infty}f(t)e^{i\nu t}dt, \tag{4}$$

Eqs. (1) and (2) are reduced to a set of algebraic equations that can be solved easily. The solution for $\sigma_{eg}(z,\nu)$ is

$$\sigma_{eg}(z,\nu)=ia(\nu)d_{eg}E_{p0}(z,\nu)/\hbar. \tag{5}$$

Here $E_{p0}(z,\nu)$ is the Fourier transform of $E_{p0}(z,t)$ and

$$a(\nu)=\frac{\gamma-i(\nu+\delta)}{[\Gamma-i(\nu+\delta)][\gamma-i(\nu+\delta)]+\Omega_d^2}. \tag{6}$$

The wave equation (3) for the Fourier transform $E_{p0}(z,\nu)$ can be rewritten as

$$\left(\frac{\partial}{\partial z}-\frac{i}{c}\nu+A(\nu)\right)E_{p0}(z,\nu)=0, \tag{7}$$

where $A(\nu)=\alpha a(\nu)$. This equation is integrated as

$$E_{p0}(z,\nu)=E_{p0}(0,\nu)\exp[(i\nu z/c)-A(\nu)z]. \tag{8}$$

If one takes the inverse Fourier transform, the resulting expression for the probe pulse envelope is

$$E_{p0}(z,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}E_{p0}(0,\nu)\exp\left[-i\nu\left(t-\frac{z}{c}\right)-A(\nu)z\right]d\nu. \tag{9}$$

This is the general solution for the propagation of the small probe pulse, $E_p(z,t)$ in a sample with arbitrary thickness containing three-level atoms driven by the coupling field $E_d(z,t)$. The drive field is supposed to have a constant amplitude that is homogeneous in space. Both fields, probe and drive, are considered as unidirectional plane waves.

III. ADIABATIC SOLUTION FOR THE GAUSSIAN PULSE

In the time domain, the solution of the matter equations (1) and (2) for $\sigma_{eg}(z,t)$, satisfying the linear response approximation, is

$$\sigma_{eg}(z,t) = i\frac{d_{eg}}{2\pi\hbar}\int_{-\infty}^{+\infty} a(\nu)E_{p0}(z,\nu)e^{-i\nu t}dt. \quad (10)$$

In this section (and also in Secs. IV and V), we consider the case when the probe field is in exact resonance with the transition e-g, i.e., $\delta=0$. Then one can expand the function $a(\nu)$ in a power series near $\nu=0$,

$$a(\nu) = \sum_{k=0}^{\infty}(-i\nu)^k a_k, \quad (11)$$

where the first four coefficients a_k are $a_0=\gamma/(\Omega_d^2+\gamma\Gamma)$, $a_1=(\Omega_d^2-\gamma^2)/(\Omega_d^2+\gamma\Gamma)^2$, $a_2=[\gamma^3-\Omega_d^2(\Gamma+2\gamma)]/(\Omega_d^2+\gamma\Gamma)^3$, and $a_3=[(\Gamma+\gamma)^2\Omega_d^2-(\Omega_d^2-\gamma^2)^2]/(\Omega_d^2+\gamma\Gamma)^4$. The function $a(\nu)$ in Eq. (6) can be also expressed as

$$a(\nu) = i\frac{\nu+i\gamma}{(\nu-\nu_+)(\nu-\nu_-)}, \quad (12)$$

where

$$\nu_\pm = -\frac{i}{2}[\Gamma+\gamma\pm\sqrt{(\Gamma-\gamma)^2-4\Omega_d^2}] \quad (13)$$

are two poles that help to define the convergence radius of the expansion (11). This radius is defined by the absolute value of the smallest pole $|\nu_\pm|$. If $\Gamma>\gamma$ and $2\Omega_d<(\Gamma-\gamma)$, the radius of the convergence is $R_c=|\nu_-|$, i.e.,

$$R_c = \frac{1}{2}[\Gamma+\gamma-\sqrt{(\Gamma-\gamma)^2-4\Omega_d^2}]. \quad (14)$$

If $2\Omega_d\ll(\Gamma-\gamma)$, we have an approximate expression for this radius,

$$R_c \approx \gamma+\frac{\Omega_d^2}{\Gamma-\gamma}, \quad (15)$$

which is close to the width of the transparency window Ω_d^2/Γ estimated in Ref. [22] for $\gamma\ll\Omega_d^2/\Gamma$.

If $E_{p0}(z,\pm\infty)=0$ and

$$\lim_{t\to\pm\infty}\frac{\partial^k E_{p0}(z,t)}{\partial t^k} = 0, \quad (16)$$

then substituting the expansion $a(\nu)$, Eq. (11), into the integral (10) and applying a well-known differentiation property of the Fourier transform we find

$$\sigma_{eg}(z,t) = i\frac{d_{eg}}{\hbar}\sum_{k=0}^{\infty} a_k\frac{\partial^k}{\partial t^k}E_{p0}(z,t). \quad (17)$$

This expression converges if $\Delta_{in}<R_c$, where Δ_{in} is the spectral half-width of the input probe pulse, specifying the value of time derivatives of $E_{p0}(z,t)$ for $z=0$. It is assumed that the pulse spectrum is not broadened with distance. Therefore, Δ_{in} sets the upper limit for the spectral width of $E_{p0}(z,t)$ at distance z.

It can be shown that the atom subject to the probe and drive fields evolves via higher-order dark states introduced in Ref. [25]. In this picture the appearance of the infinite sum of the field time derivatives of ascending order in the atomic response function is a result of such evolution; see, for example, Ref. [26]. Our solution Eq. (17) contains the information about this evolution and also takes into account all relaxation processes in a natural way. If the atom adiabatically follows such dark states, its evolution starts and terminates in the ground state $|g\rangle$ [26]. Then, one can expect that the probe pulse would propagate solitonlike because no excitation is left in the medium. To clarify this argument, we give a brief sketch of the high-order dark state formalism and the qualitative definition of the adiabatic following of these states.

The notion of a dark state (d_0) was first introduced by Arimondo in [2]. It is a particular superposition of states g and m: $|d_0\rangle=\cos\beta_0|g\rangle-\sin\beta_0|m\rangle$, where the state mixing angle β_0 satisfies the condition $\tan\beta_0=\Omega_p/\Omega_d$. The dark state is an eigenstate of the three-level system interacting with the cw probe and cw drive field. Therefore, if the three-level atom is in the dark state it stays there and will not be excited to state $|e\rangle$. In our case the drive field amplitude does not change in time while the amplitude of the probe does, so that one has to consider the dark state $|d_0\rangle=|d_0(t)\rangle$ with a time-dependent state mixing angle $\beta_0(t)$. Since $\Omega_p(\pm\infty)=0$, the ground state $|g\rangle$ coincides with the dark state $|d_0\rangle$ by definition before the probe pulse arrives. If the atom adiabatically follows the dark state, then the probe pulse does not populate the excited state $|e\rangle$, but induces the coherence g-m, populating the metastable state $|m\rangle$ with probability amplitude $\sin\beta_0=\Omega_p/\Omega_0$, where $\Omega_0=\sqrt{\Omega_d^2+\Omega_p^2}\approx\Omega_d$.

This is a simplified picture, ignoring the fact that the interaction Hamiltonian is time dependent. Any time-dependent Hamiltonian cannot be diagonalized similarly to a time-independent Hamiltonian (see, for example, Ref. [26]). If one takes this time dependence into account, then the dark state becomes coupled with the so-called bright state $|b_0\rangle=\sin\beta_0|g\rangle+\cos\beta_0|m\rangle$, which is orthogonal to $|d_0\rangle$ and $|e\rangle$. The coupling strength is $i\dot{\beta}_0(t)$. In the basis $|d_0\rangle$, $|b_0\rangle$, $|e\rangle$ we have again a three-level system, where $|d_0\rangle$ and $|b_0\rangle$ are coupled by $i\dot{\beta}_0(t)=i\dot{\Omega}_p\Omega_d/\Omega_0^2\approx i\dot{\Omega}_p/\Omega_d$, and $|b_0\rangle$ and $|e\rangle$ are coupled by $\Omega_0\approx\Omega_d$; see Fig. 2(b). Now, one can introduce a new combination of dark, bright, and common intermediate states in the two-quantum process, i.e., d_1, b_1, and c_1. State c_1 coincides with state b_0. State d_1 and state b_1 are superpositions of states d_0 and e, or explicitly $|d_1\rangle=\cos\beta_1|d_0\rangle+i\sin\beta_1|e\rangle$, $|b_1\rangle=i\sin\beta_1|d_0\rangle+\cos\beta_1|e\rangle$, where the state mixing angle is $\beta_1=\tan^{-1}(\dot{\beta}_0/\Omega_0)$. $|d_1\rangle$ can be called the first-order dark state, if $|d_0\rangle$ is the zeroth-order dark state. One can continue in the same way introducing the second-order dark state, etc., since every new dark state is coupled with its partner, the new bright state, due to the time variation of the field amplitude; see Figs. 2(a)–2(d).

If relaxation processes are ignored (which is the case of the short probe pulse), the transparency window is defined by the value of Ω_d. In case of a weak probe ($\Omega_p\ll\Omega_d$) and constant drive, one can show that the mixing angles satisfy the condition $\beta_k\approx\Omega_p^{(k)}/\Omega_d^{k+1}$ where $\Omega_p^{(k)}$ is the kth time derivative of Ω_p. If Δ_{in} is smaller than the width of the transparency window, Ω_d, we have $\beta_{k+1}/\beta_k<1$ and with the increase of k the mixing angle β_k decreases and the coupling $\dot{\beta}_k(t)$ becomes smaller and smaller with respect to Ω_d. In this

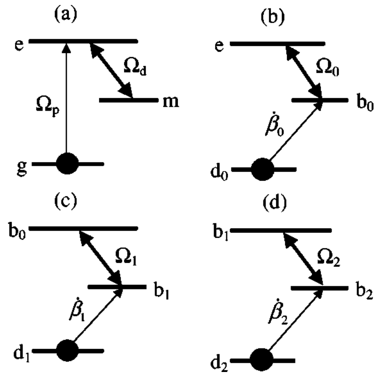

FIG. 2. Interaction of two fields, probe and drive, with the three-level atom in various representations. (a) Schrödinger representation. (b) Zero-order dark state representation. $\Omega_0=\sqrt{\Omega_p^2+\Omega_d^2}$ and $\beta_0=\tan^{-1}(\Omega_p/\Omega_d)$. (c) First-order dark state representation. $\Omega_1=\sqrt{\Omega_0^2+\dot\beta_0^2}$ and $\beta_1=\tan^{-1}(\dot\beta_0/\Omega_0)$. (d) Second-order dark state representation. $\Omega_2=\sqrt{\Omega_1^2+\dot\beta_1^2}$ and $\beta_2=\tan^{-1}(\dot\beta_1/\Omega_1)$. In the nth-order dark state representation, the state mixing angle is $\beta_n=\tan^{-1}(\dot\beta_{n-1}/\Omega_{n-1})$. The position of the levels in the diagrams (b)–(d) is schematic, not related to their energy.

case some higher-order dark state (for example, $|d_j\rangle$) can be approximately considered as the uncoupled state. Then the atom is assumed to follow such a state adiabatically. Its evolution starts and terminates in the ground state $|g\rangle$ because $\beta_j(\pm\infty)=0$ and $|d_j(\pm\infty)\rangle=|g\rangle$ for any j. In this case, one can expect that no excitation is left in the medium. For a discussion of the nonadiabatic corrections see, for example, Ref. [26].

Such a higher-order dark state, which the atom follows adiabatically, can be presented as a superposition of the ground, metastable, and excited states,

$$|d_n\rangle = C_g|g\rangle + C_m|m\rangle + C_e|e\rangle, \tag{18}$$

with the coefficients

$$C_e \approx i\sum_{n=0}(-1)^n\beta_{2n+1}, \tag{19}$$

$$C_m \approx \sum_{n=0}(-1)^{n+1}\beta_{2n}, \tag{20}$$

$$C_g \approx 1-\frac{1}{2}\left[\left(\sum_{n=0}(-1)^n\beta_{2n+1}\right)^2+\left(\sum_{n=0}(-1)^{n+1}\beta_{2n}\right)^2\right], \tag{21}$$

where the sum is specified by the condition $2n+1\leqslant j$; see, for example, Ref. [26]. For infinite j ($j\to\infty$), the sums in Eqs. (19)–(21) are finite if Δ_{in} is smaller than the transparency window. From Eqs. (19) and (21), one can calculate the density matrix element $\sigma_{eg}=C_eC_g^*$, which gives the atomic response to the probe. Simple algebra shows that such a calculated response coincides with the result given in Eq. (17) if we take $\Gamma=\gamma=0$ in the expressions for a_k (which is the case of the short pulse) and keep in $C_eC_g^*$ only the linear terms proportional to the time derivatives of Ω_p (linear response approximation).

This is only a qualitative argument explaining the adiabatic following of the higher-order dark states. Below we use Eq. (17), which takes into account the relaxation processes, and we show that the first four terms of expansion (17) are sufficient to describe the pulse propagation if $\Delta_{in}<R_c$.

For the input pulse at $z=0$ we take a pulse with a Gaussian envelope $E_{p0}(0,t)=E_{p0}\exp(-r^2t^2)$. This pulse shape is typical for pulsed lasing obtained by phase locking of many modes [27]. The Fourier transform of this pulse is $E_{p0}(0,\nu)=(E_{p0}\sqrt{\pi}/r)\exp[-(\nu/2r)^2]$ and hence the half-width of its spectrum is $\Delta_{in}=2r$. If we take into account only three terms of the expansion of the atomic response function $a(\nu)\approx a_0-ia_1\nu-a_2\nu^2$, the integral (9) for the Gaussian pulse can be easily calculated,

$$E_{pA}(z,t)=E_{p0}\left(\frac{\Delta_{out}}{\Delta_{in}}\right)\exp\left[-T_{EIT}-\frac{1}{4}\Delta_{out}^2(t-t_{dA})^2\right], \tag{22}$$

where the subscript A designates that this is the adiabatic part of the solution, $T_{EIT}=\alpha a_0 z$, $\Delta_{out}=\Delta_{in}/\sqrt{1-\Delta_{in}^2\alpha a_2 z}$ [a_2 must be negative, which corresponds to the condition $\Omega_d^2(\Gamma+2\gamma)>\gamma^3$], and $t_{dA}=\alpha a_1 z$. Time t in Eq. (22) and below is actually the local time $t-z/c$. The parameters T_{EIT}, Δ_{out}, and t_{dA} are z dependent. They have a significance for the description of the pulse propagation. Therefore, we give their explicit expressions and meanings below.

The parameter

$$T_{EIT}=T\frac{\gamma\Gamma}{\Omega_d^2+\gamma\Gamma}, \tag{23}$$

is the EIT reduced effective length of the absorber for the central, resonant component of the probe pulse spectrum. Without EIT (the drive is off, $\Omega_d=0$), the effective length is $T=\alpha z/\Gamma$, where $2\alpha/\Gamma$ is the Beer's constant. T_{EIT} comes from the first term of the $a(\nu)$ expansion, a_0, which defines the absorption exactly at the center of the EIT hole, ω_{eg}. This absorption is strongly reduced by a factor $\gamma\Gamma/\Omega_d^2$ with respect to the value for the unperturbed line if $\Omega_d^2\gg\gamma\Gamma$. The a_0 term is not really adiabatic. It describes the process of population leakage from the dark state $|d_0\rangle$ due to the γ decoherence.

The parameter

$$t_{dA}=T\Gamma\frac{\Omega_d^2-\gamma^2}{(\Omega_d^2+\gamma\Gamma)^2} \tag{24}$$

describes the pulse delay t_{dA} due to the steep dispersion at the center of the EIT window, which results in the slow group velocity of the pulse $V_g=c/(1+\alpha a_1 c)$. This parameter comes from the second term of the $a(\nu)$ expansion, a_1, approximating the real part of the atomic susceptibility $\chi'(\nu)$ by the linear function $\sim\nu$. If the EIT hole is deep ($\Omega_d^2\gg\gamma\Gamma$ or $\gamma\to 0$), we have $t_{dA}\approx T\Gamma/\Omega_d^2$. The a_1 term of the expansion maintains the population of the dark state of the first order d_1. Therefore, it is an adiabatic term and does not contribute to the dissipation of the pulse E_p.

The parameter

$$\Delta_{out} = \frac{\Delta_{in}}{\sqrt{1 + (\Delta_{in}/\Delta_{eff})^2}}, \tag{25}$$

describes the spectral half-width of the output pulse, where

$$\Delta_{eff} = \sqrt{\frac{(\Omega_d^2 + \gamma\Gamma)^3}{T\Gamma[\Omega_d^2(\Gamma + 2\gamma) - \gamma^3]}} \tag{26}$$

is the effective half-width of the EIT window for a thick sample ($T > 1$). If $\Omega_d^2 \gg \gamma\Gamma$, the effective half-width is $\Delta_{eff} \approx \Delta_h/\sqrt{T}$, where $\Delta_h = \Omega_d^2/\Gamma$ is the half-width of the EIT window for one atom. For a thick sample the effective half-width narrows as $\sim 1/\sqrt{T}$. According to Eq. (25), the half-width of the output pulse, Δ_{out}, also narrows and tends to Δ_{eff} for large T if $\Delta_{in} \gg \Delta_{eff}$. The parameters Δ_{out} and Δ_{eff} come from the third term, a_2, of the expansion $a(\nu)$, which approximates the imaginary part of the atomic susceptibility $\chi''(\nu)$ by a parabolic function $\sim \nu^2$.

The a_2 term produces pulse broadening in time or its spectrum narrowing with distance. This process is adiabatic with respect to the central frequency Fourier component of the probe pulse, which is the pulse area. Therefore, the process of the pulse spectrum narrowing or pulse broadening in time with distance preserves the pulse area, which is explicitly the time integral of the pulse amplitude, i.e.,

$$\theta_A(z) = \frac{2d_{eg}}{\hbar}\int_{-\infty}^{+\infty} E_{pA}(z,t)dt = \theta(0)e^{-T_{EIT}}, \tag{27}$$

where $\theta(0) = 2\sqrt{\pi} d_{eg} E_{p0}/r\hbar$ is the pulse area at the input ($z = 0$). Usually the pulse area is defined as a dimensionless parameter. The pulse area reduces with distance only due to the residual absorption at the bottom of the EIT window. One can obtain this result for any pulse whose central frequency coincides with the center of the EIT hole. This is because the pulse area by definition is the zero-frequency Fourier component of the spectrum of the pulse envelope. If we take into account the carrier frequency of the pulse, ω_p, then we find that this Fourier component is just the spectral component of the pulse coinciding with the center of the EIT window.

The energy of the output pulse or its time integrated intensity $I_{pA}(z,t) = |E_{pA}(z,t)|^2$ is reduced due to the pulse broadening by the EIT window as

$$U_A(z) = \int_{-\infty}^{+\infty} I_{pA}(z,t)dt = U(0)\frac{\Delta_{out}}{\Delta_{in}}e^{-2T_{EIT}}, \tag{28}$$

where $U(0) = \sqrt{\pi/2}|E_{p0}|^2/r$ is the input pulse energy. The energy of the pulse corresponds to the central frequency Fourier component of the pulse intensity, which can be expressed as

$$\int_{-\infty}^{+\infty} I_{pA}(z,t)dt = \int_{-\infty}^{+\infty} E_{pA}(z,\nu)E_{pA}^*(z,\nu)d\nu. \tag{29}$$

Since the integral at the right-hand side contains all the frequency components of the pulse amplitude, this value is not preserved.

For $\gamma \to 0$ we can introduce the adiabaticity parameter $\varepsilon = \Delta_{in}/\Delta_h$, which is the ratio of the input pulse spectrum width and the width of the EIT hole for one atom. Then all terms of the adiabatic expansion Eq. (11), except the first one ($a_0 \approx 0$), are $\nu^k a_k \sim \varepsilon^k/\Gamma$. If $\varepsilon \ll 1$, expansions (11) and (17) converge quickly. According to Eq. (28), the pulse envelope broadening reduces the energy of the pulse by a factor $\Delta_{out}/\Delta_{in} \approx 1/\sqrt{1+T\varepsilon^2}$. To have the adiabatic propagation of the pulse, ε must be smaller than unity. If the sample is thick ($T \gg 1$) and the process is adiabatic ($\varepsilon < 1$), but $T\varepsilon^2 \gg 1$, the output energy of the pulse is reduced by a factor of $\Delta_h/\Delta_{in}\sqrt{T}$, which is Δ_{eff}/Δ_{in}. This is because only the $2\Delta_{eff}$ part of the spectral content of the input pulse, $2\Delta_{in}$, comes out of the sample. Below, we refer to the case $T\varepsilon^2 \gg 1$, $\varepsilon < 1$ as the case of adiabatic pulse propagation in a thick resonant absorber. In this case the half-width of the output pulse spectrum, Δ_{out}, tends to the constant value Δ_{eff}. This value is $\sqrt{T}$ times smaller than the half-width of the EIT hole in the absorption line of one atom.

The adiabatic solution (22) is a good approximation of the integral in Eq. (9) if (a) $\Omega_d^2 \gg \gamma\Gamma$, $\Gamma \gg \gamma$ and (b) $\Omega_d^2/\Gamma \gg \Delta_{in}$ (or $\varepsilon \ll 1$). Condition (a) specifies the presence of a deep, narrow EIT hole in the absorption line. Condition (b) demands that the spectral width of the pulse be much smaller than the width of the EIT hole Δ_h (not Δ_{eff}). Figure 3(a) shows a comparison of the analytical approximation $E_{pA}(z,t)$, Eq. (22), with the numerical integration for $E_{p0}(z,t)$, Eq. (9), where $A(\nu)$ is not approximated. The input pulse $E_{p0}(0,t)$ is shown by the bold line. The adiabaticity parameter is $\varepsilon = 0.4$ and $T = 30$. In this case $\Delta_{in}/\Delta_h = 0.4$ and $\Delta_{in}/\Delta_{eff} = 2.2$.

Figure 3(a) shows a small deviation of the analytical approximation Eq. (22) from the integral (9). The fit can be improved if one takes into account the fourth term of the expansion (11). Then $a(\nu) \approx a_0 - ia_1\nu - a_2\nu^2 + ia_3\nu^3$ and the output pulse is presented as a convolution of Eq. (22) and the Airy function $\mathrm{Ai}(x)$, i.e.,

$$E_{pA1}(z,t) = E_{p0}\frac{\Delta_{out}\Delta_{dist}}{\Delta_{in}}\int_{-\infty}^{+\infty} \mathrm{Ai}(\Delta_{dist}\tau)e^{-(1/4)\Delta_{out}^2(t - t_{dA} - \tau)^2 - T_{EIT}}d\tau, \tag{30}$$

where $\Delta_{dist} = (3\alpha a_3 z)^{-1/3}$ is the pulse distortion parameter. Here a_3 is assumed to be positive and hence $\Delta_{dist} > 0$. This expression originates from the integral representation of the Airy function [28]

$$\Delta_{dist}\mathrm{Ai}(\pm\Delta_{dist}\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp\left[-i\left(\nu\tau \pm \frac{\nu^3}{3\Delta_{dist}^3}\right)\right]d\nu, \tag{31}$$

and the convolution theorem

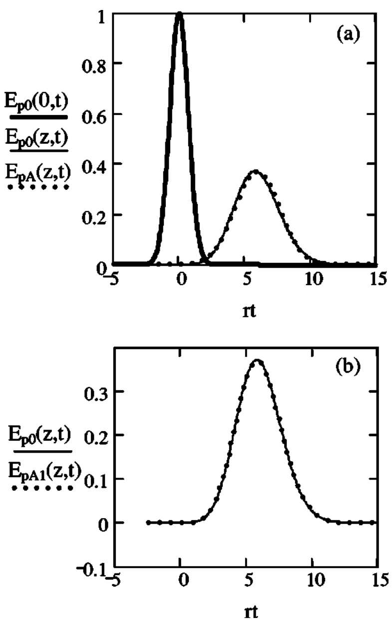

FIG. 3. (a) Comparison of the adiabatic solution $E_{pA}(z,t)$ for the output probe pulse Eq. (22), shown by dots, with the numerical calculation of $E_{p0}(z,t)$, Eq. (9), shown by the thin solid line. The solid bold line shows the probe pulse dependence without absorber. All plots are normalized by the maximum amplitude of the input pulse E_{p0}. The time scale is in units of the input pulse parameter r. The delay time of the output pulse is $t_{dA}=6/r$. The zero time is chosen for $t-z/c=0$. The effective thickness of the sample is $T=\alpha z/\Gamma=30$. Other parameters are $\Delta_{in}/\Delta_h=0.4$, $\gamma/\Gamma=10^{-3}$, $\Omega_d/\Gamma=0.5$, and $r/\Gamma=0.05$. In this case the spectral width of the input probe pulse is 2.5 times smaller than the width of the EIT hole. (b) Comparison of the numerical calculation of $E_{p0}(z,t)$, Eq. (9) (thin solid line) with the analytical approximation $E_{pA1}(z,t)$, given by Eq. (30) and shown by dots. The parameters are the same as for the plot (a).

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} f_1(\nu)f_2(\nu)e^{-i\nu t}d\nu = \int_{-\infty}^{+\infty} F_1(t-\tau)F_2(\tau)d\tau, \quad (32)$$

where $f_k(\nu)$ is the Fourier transform of $F_k(t)$, $k=1,2$. If $a_3<0$, then $\Delta_{dist}=(3\alpha|a_3|z)^{-1/3}$ and $\Delta_{dist}\mathrm{Ai}(\Delta_{dist}\tau)$ is replaced by $\Delta_{dist}\mathrm{Ai}(-\Delta_{dist}\tau)$ in Eq. (30). Figure 3(b) shows the comparison of the approximate solution (30) with the numerical integration (9). The fit is excellent. The parameter Δ_{dist} describes a ν^3 correction to the real part of the atomic susceptibility $\chi'(\nu)$, approximated in the first step by the linear function $\sim\nu$. Below we show that this parameter defines a border for the pulse breakup. Because of its importance, we give the explicit expression for this parameter:

$$\Delta_{dist} = \sqrt[3]{\frac{(\Omega_d^2+\gamma\Gamma)^4}{3T\Gamma[\Omega_d^2(\Gamma+\gamma)^2-(\Omega_d^2-\gamma^2)^2]}}, \quad (33)$$

which is approximated by $\Delta_{dist}\approx\Delta_h/\sqrt[3]{3T}$ if $\Omega_d^2\gg\gamma\Gamma$ and $\Gamma\gg\Omega_d\gg\gamma$.

The fourth term of the adiabatic expansion does not change the pulse area since

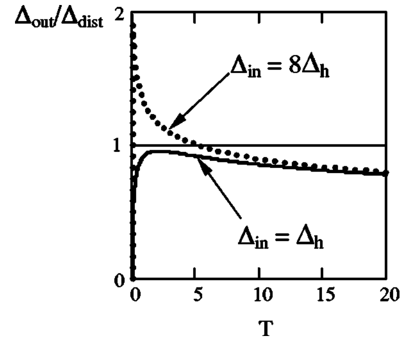

FIG. 4. The $\Delta_{out}/\Delta_{dist}$ parameter dependence on the optical thickness T. The solid line corresponds to the case if the spectral width of the input pulse coincides with the width of the transparency window Δ_h. The dotted line corresponds to the case if the spectral width of the input pulse is eight times larger than the width of the transparency window. The horizontal line corresponds to the border value of the parameter $\Delta_{out}/\Delta_{dist}$ below which the pulse distortion can be neglected and the adiabatic approximation Eq. (22) is applicable (see the text). The parameters of the absorber and the drive are $\Omega_d=\Gamma/2$ and $\gamma/\Gamma=10^{-3}$.

$$\Delta_{dist}\int_{-\infty}^{+\infty}\mathrm{Ai}(\Delta_{dist}\tau)d\tau = 1. \quad (34)$$

This is not surprising because the above mentioned argument about the zero-frequency Fourier component of the pulse envelope is valid for the pulse area irrespective of the particular form of the $a(\nu)$ function except the first term (a_0) of its expansion near $\nu=0$.

The presence of the fourth term of the adiabatic expansion does not change appreciably the shape of the pulse if $\Delta_{out}\ll\Delta_{dist}$. This can be proven if with the help of the substitution $x=\Delta_{dist}\tau$ Eq. (30) is presented as follows:

$$E_{pA1}(z,t) = E_{p0}\frac{\Delta_{out}}{\Delta_{in}}\int_{-\infty}^{+\infty}\mathrm{Ai}(x)\exp\left\{-\left(\frac{\Delta_{out}}{2\Delta_{dist}}\right)^2[\Delta_{dist}(t-t_{dA}) - x]^2 - T_{EIT}\right\}dx. \quad (35)$$

Then one finds that the maximum of the Gaussian pulse at $t=t_{dA}$ does not change its value $(\Delta_{out}/\Delta_{in})E_{p0}\exp(-T_{EIT})$, due to the convolution with the Airy function if $\Delta_{out}/2\Delta_{dist}\ll 1$. This is because the integral of the Airy function is unity [see Eq. (34)] and $\exp[-(\Delta_{out}/2\Delta_{dist})^2x^2]\sim 1$ over the domain of the variable x where $\mathrm{Ai}(x)$ gives the main contribution to the integral.

A numerical analysis of the integral (35) for an arbitrary value of $\Delta_{out}/\Delta_{dist}$ shows that the distortion of the Gaussian pulse due to the fourth term of the adiabatic expansion is small if $\Delta_{out}\leqslant\Delta_{dist}$. The pulse acquires an oscillatory shape if $\Delta_{out}=2\Delta_{dist}$. The condition $\Delta_{out}\leqslant\Delta_{dist}$ is well satisfied for all values of the effective thickness T if $\Gamma>\Omega_d>\gamma$ and $\Delta_{in}\leqslant\Delta_h$ (see Fig. 4). This is because $\Delta_{out}\rightarrow\Delta_{eff}$ and Δ_{eff}, the effective width of the EIT window, narrows with the thickness increase as $1/T^{1/2}$. The distortion (the breakup) border

Δ_{dist} narrows as $1/T^{1/3}$, i.e., Δ_{eff} narrows faster than Δ_{dist}. Therefore, in this case we have no breakup of the pulse with the increase of the thickness T.

If $\Delta_{in} \gg \Delta_{eff}$, $\Gamma \gg \Omega_d \gg \gamma$, and $T \gg 1$, we have $\Delta_{out}/\Delta_{dist} \approx \Delta_{eff}/\Delta_{dist} = \sqrt[3]{3}/T^{1/6}$. Therefore, even for a broadband input pulse ($\Delta_{in} \gg \Delta_h$) one obtains $\Delta_{out}/\Delta_{dist} \leqslant 1$ if $T \geqslant 9$ (see Fig. 4) and hence the adiabatic part of the pulse is not distorted.

In Appendix A we give an analytical approximation of the integral (9) if $a(\nu) \approx a_0 - ia_1\nu - a_2\nu^2 + ia_3\nu^3$, i.e., of the integral, which is a convolution of the Airy function with the Gaussian pulse, Eq. (30).

Summarizing, we conclude that a Gaussian pulse propagating through the EIT window in a thick resonant absorber is well described by the adiabatic solution Eq. (22), which takes into account only the first three terms of the expansion of the spectral function $a(\nu)$ near $\nu=0$. The fourth term of this expansion produces a small distortion of the pulse, which can be neglected if $\Omega_d < \Gamma$. These results are obtained if the spectral width of the pulse is smaller than the width of the EIT window Δ_h.

IV. A SHORT GAUSSIAN PULSE PROPAGATION WITHIN A POWER-BROADENED EIT WINDOW

In this section we consider the case of short pulse propagation if the adiabatic following condition of the dark states is satisfied. Short pulse means that its duration is much shorter than the decay time of the coherence g-e, i.e., $\Delta_{in} \gg \Gamma$. In this case the EIT window in the absorption spectrum can be created by a strong drive. One would expect that if the Autler-Townes splitting $2\Omega_d$ of the absorption line for the probe, which is created by the drive field [29], is larger than the spectral width of the input pulse Δ_{in}, i.e., $2\Omega_d \gg \Delta_{in}$, the absorption of the probe field is strongly reduced. These conditions on the input pulse width Δ_{in}, decoherence rate Γ, and EIT window $2\Omega_d$ give $2\Omega_d \gg \Delta_{in} \gg \Gamma$. This case was considered in our papers [23]. Here, we show that the condition of the adiabatic following of the dark states is violated starting from a certain value of the effective thickness. This violation results in a strong distortion of the pulse shape.

To simplify the analysis, we put $\Gamma \to +0$ and $\gamma \to +0$ in the definition of the coefficients a_k of the $a(\nu)$ expansion in a power series Eq. (11). Then, $a_0 \approx 0$, $a_1 \approx \Omega_d^{-2}$, $a_2 \approx 0$, and $a_3 \approx -\Omega_d^{-4}$. According to Eqs. (30) and (31), the output pulse is described by

$$E_{pA1}(z,t) = E_{p0}\int_{-\infty}^{+\infty} \Delta_{dist}\mathrm{Ai}(-\Delta_{dist}\tau) \times \exp\left[-\frac{1}{4}\Delta_{in}^2(t-t_{dA}-\tau)^2\right]d\tau, \qquad (36)$$

where $t_{dA} \approx \alpha z/\Omega_d^2$ and $\Delta_{dist} \approx \sqrt[3]{\Omega_d^4/(3\alpha z)}$. Here, the approximation $a_0 \approx 0$ and $a_2 \approx 0$, i.e., the absence of the absorption and of the pulse broadening ($\Delta_{in}/\Delta_{out} \approx 1$), is taken into account. As in the previous section, with the help of the substitution $x=-\Delta_{dist}\tau$ we reduce Eq. (36) to

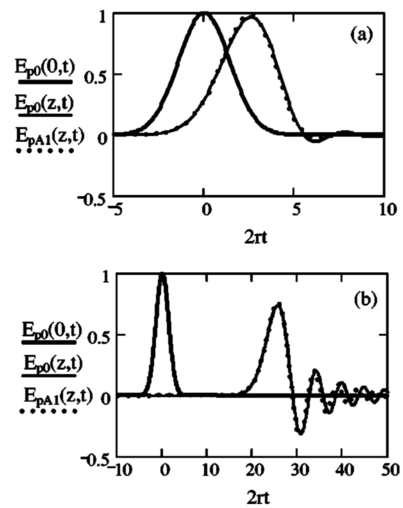

FIG. 5. Comparison of the numerical calculation of $E_{p0}(z,t)$, Eq. (9) (thin solid line) with the analytical approximation $E_{pA1}(z,t)$, given by Eq. (36) and shown by dots, for the short pulse propagation at the EIT condition. The solid bold line shows the input probe pulse dependence at $z=0$. The atom, probe pulse, and drive field parameters are $\Delta_{in}=\Omega_d/3$, $\Omega_d=300\Gamma$, and $\gamma=0$. For the analytical approximation Eq. (36), we take $\Gamma=\gamma=0$. Plot (a) corresponds to the case $\Delta_{in}/\Delta_{dist}=0.9$ and plot (b) corresponds to the case $\Delta_{in}/\Delta_{dist}=2$.

$$E_{pA1}(z,t) = E_{p0}\int_{-\infty}^{+\infty} \mathrm{Ai}(x)\exp\left\{-\left(\frac{\Delta_{in}}{2\Delta_{dist}}\right)^2 \times[\Delta_{dist}(t-t_{dA})+x]^2\right\}dx. \qquad (37)$$

The pulse distortion is small if $\Delta_{in}/\Delta_{dist} \leqslant 1$, or, explicitly, if $\Delta_{in} \leqslant \sqrt[3]{\Omega_d^4/(3\alpha z)}$. Since the width of the output pulse does not change appreciably and the distortion border narrows as $\Delta_{dist} \sim 1/z^{1/3}$, the condition $\Delta_{in}/\Delta_{dist} \leqslant 1$ starts to be violated from a certain distance z. Figure 5 shows a comparison of the probe pulse distortion for the case $\Delta_{in}/\Delta_{dist}=0.9$ [plot (a)] and for the case $\Delta_{in}/\Delta_{dist}=2$ [plot (b)]. For both plots $\Delta_{in}=\Omega_d/3$. For the numerical calculation of $E_{p0}(z,t)$, Eq. (9), we take $\gamma=0$ and $\Gamma=\Omega_d/300$. Γ is taken nonzero, but small, to avoid zero in the denominator of $A(\nu)$.

Thus, for the short pulse ($\Omega_d > \Delta_{in} \gg \Gamma$) the probe pulse is not distorted appreciably if certain constraints on the effective thickness $T=\alpha z/\Gamma$ and pulse width Δ_{in} are imposed. However, the pulse shape distortion gradually increases with distance and the condition $\Delta_{in} > \Delta_{dist} = \sqrt[3]{\Omega_d^4/(3\alpha z)}$ sets a border beyond which it cannot be neglected. As was shown in Sec. III, to preserve the Gaussian shape of the output pulse, the width of the output pulse Δ_{out} must be smaller than or limited by the value Δ_{dist}. For the case of a narrow transparency window, considered in Sec. III, the value Δ_{out} decreases with distance z as $\sim 1/\sqrt{z}$, and the distortion border shrinks as $\sim 1/\sqrt[3]{z}$, i.e., the pulse spectrum narrows faster with distance than the border of distortion narrows. Therefore, we have $\Delta_{out}/\Delta_{dist} \sim 1/\sqrt[6]{z} \to 0$ for large z. That is why the pulse satisfying the condition $\Delta_{in} \leqslant \Delta_h$ is not distorted for any distance in case of a narrow EIT window. In case of the propagation of a short pulse within a power-broadened EIT window, the contribution of the pulse spectrum narrowing is almost negligible since Γ and hence a_2 are small. However, the contribution of the term a_3 responsible for the pulse distortion is appreciable. Therefore, the pulse width Δ_{in} does not change with distance ($\Delta_{in} \approx \Delta_{out}$) but the pulse distortion border narrows as $\Delta_{dist} = \sqrt[3]{\Omega_d^4/(3\alpha z)}$. At the certain distance we have $\Delta_{in} > \Delta_{dist}$, and the pulse starts to experience a strong distortion. This point makes a qualitative difference between the case $\Omega_d < \Gamma$ and the case $\Omega_d \gg \Gamma$. In the case of $\Omega_d < \Gamma$, to have adiabatic pulse propagation, the spectral width of the input pulse must be smaller than the width of the EIT window (Δ_h), whatever the effective thickness of the sample. In the case of $\Omega_d \gg \Gamma$, there is a distance z where the condition of small distortion of the pulse shape, $\Delta_{in} \ll \Delta_{dist}$, is violated and then, starting from this distance, the pulse experiences a strong distortion of its shape.

V. BROADBAND PULSE PROPAGATION THROUGH A NARROW EIT WINDOW

In this section, we consider the Gaussian pulse propagation in a strongly absorptive medium if the pulse spectrum is much broader than the width of the narrow EIT window $\Delta_h = \Omega_d^2/\Gamma$ and $\Omega_d < \Gamma$. We set the only constraint on the pulse that its carrier frequency is exactly tuned to the center of the EIT hole.

If the spectral width of the input pulse is broader than the width of the EIT hole ($\Delta_{in} > \Omega_d^2/\Gamma$), the adiabatic expansion Eqs. (11) and (17) is not valid for all spectral components of the pulse. However, the part of the pulse spectrum that coincides with the EIT window can be transmitted through the absorber without appreciable absorption. This part may have a time dependence similar to the adiabatic solution presented in Sec. III. If this is the case, that part (which is adiabatic) spectrally narrows and delays in time.

The frequency content of the pulse that is outside the EIT hole is strongly reduced due to absorption in an optically thick medium. One can expect that this reduced part of the pulse will have a group velocity close to the speed of light in vacuum c. Such a filtering of the pulse through the EIT window can break up the broadband pulse into two components. One component must satisfy the adiabatic following condition. This adiabatic component is expected to be delayed and broadened in time. The other, nonadiabatic component originates from the part of the atomic response that does not follow the dark state. The nonadiabatic component is strongly reduced in amplitude, it could have almost no delay, and its duration is short. Therefore, the adiabatic and nonadiabatic components of the pulse must be well separated in time and space.

First we calculate the area of the pulse at the output of the absorber. This area is

$$\theta(z) = \frac{2d_{eg}}{\hbar}\int_{-\infty}^{+\infty} E_{p0}(z,t)dt. \tag{38}$$

Substitution of Eq. (9) into the integral (38) gives $\theta(z) = \theta_A(z) = \theta(0)\exp(-T_{EIT})$. This means that the output pulse area for a broadband Gaussian pulse coincides with the area of the input pulse reduced by a factor $\exp(-T_{EIT})$. Its reduction is defined by the absorption at the bottom of the EIT window ($\nu=0$) and the reduction is small if $T_{EIT} \ll 1$ [see Eq. (23) for the T_{EIT} definition].

The only way for the pulse to maintain its area almost unchanged is to broaden in time with reduction of its amplitude or to narrow in frequency content into a domain close to $\nu=0$. To estimate the temporal width of the output pulse, we hypothesize that its shape coincides with the adiabatic part of the solution $E_{pA}(z,t)$, Eq. (22). The spectral half-width of $E_{pA}(z,t)$ is $\Delta_{out} \approx \Omega_d^2/(\Gamma\sqrt{T})$ if $T(\Delta_{in}/\Delta_h)^2 \gg 1$. This width is $\Delta_{eff} = \Delta_h/\sqrt{T}$ and does not depend on the spectral width of the input pulse $E_{p0}(0,t)$.

The analysis of the nonadiabatic component $E_{pN}(z,t)$ and the approximate calculation of its shape is given in Appendix B. We assume that for a thick sample ($T \gg 1$) this part has nonzero amplitude mostly due to the far wings of the pulse spectrum, which are less absorbed due to the reduced absorption at the wings of the absorption line. The central part is strongly absorbed except the narrow part coinciding with the EIT hole. This narrow part is taken into account by the adiabatic part of the solution and hence, it can be ignored in the approximate calculation of $E_{pN}(z,t)$.

Figure 6 shows the result of the numerical calculation of the integral (9) (solid line), if the spectral width of the pulse is two times larger than the width of the unperturbed absorption line ($\Delta_{in} = 2\Gamma$) and eight times larger than the width of the EIT hole ($\Delta_{in} = 8\Delta_h$). The other parameters are the same as in Figs. 3(a) and 3(b) (see figure caption). We approximate the output pulse, Eq. (9), by the sum of the adiabatic and nonadiabatic parts, i.e.,

$$E_{an}(z,t) = E_{pA}(z,t) + E_{pN}(z,t), \tag{39}$$

shown by dots in Fig. 6. The nonadiabatic part (see Appendix B) is

$$E_{pN}(z,t) = E_{p0}M\cos[\omega_{wide}(t+t_{ph})]\exp\left[-T_{wide} - \frac{\Delta_{wide}^2(t-t_{dN})^2}{4}\right], \tag{40}$$

where

$$T_{wide} = \frac{\Gamma}{\Delta_{in}}\left(2\sqrt{T} - \frac{\Gamma}{\Delta_{in}}\right) \tag{41}$$

is the overall effective thickness of the sample for the broad, out of resonance, components of the pulse spectrum. The spectral half-width of the nonadiabatic part is $\Delta_{wide} = M\Delta_{in}/2$, where

$$M = \frac{1}{\sqrt{1-\Gamma/\sqrt{T}\Delta_{in}}}. \tag{42}$$

This approximation of the nonadiabatic part is valid if $\sqrt{T} > \Gamma/\Delta_{in}$. Therefore, $M > 1$ for finite T, and $M \to 1$ if $T \to +\infty$. Correspondingly, $\Delta_{wide} > \Delta_{in}/2$ if T is finite, and $\Delta_{wide} \to \Delta_{in}/2$ if $T \to +\infty$. The Gaussian shape of the nona-

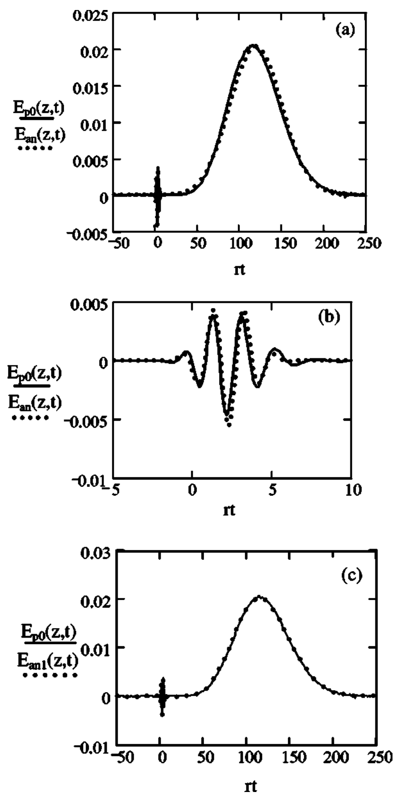

FIG. 6. (a) Comparison of the numerically calculated integral in Eq. (9), $E_{p0}(z,t)$ (solid line) with the analytical approximation $E_{an}(z,t)$, Eq. (39) (dots). $r=\Gamma$ and the other parameters are the same as for Fig. 3. (b) Magnified part of the plot (a) showing the evolution of the nonadiabatic part of the pulse in detail. All amplitudes are presented in units of E_{p0}. (c) Comparison of the numerically calculated integral in Eq. (9), $E_{p0}(z,t)$, (solid line) with the analytical approximation $E_{an1}(z,t)$, Eq. (44) (dots). The parameters are the same as for (a). The "blowup" of the nonadiabatic part is not shown since it is the same as in (b).

diabatic component of the pulse is modulated with a frequency

$$\omega_{wide} = \sqrt{\Delta_{in}\Gamma\sqrt{T} - \Gamma^2} \tag{43}$$

and this modulation has a phase shift determined by $t_{ph}=\sqrt{T}/\Delta_{in}$. The Gaussian envelope is centered at $t_{dN} =[\sqrt{T}-(2\Gamma/\Delta_{in})]/\Delta_{in}$.

The plots in Figs. 6(a) and 6(b) confirm what we expected. The pulse is broken in two parts, i.e., adiabatic and nonadiabatic. The adiabatic part delays and broadens in time. Its delay time $t_{dA}\approx T/\Delta_h$ depends on the sample parameters and does not depend on the pulse spectrum. The delay time t_{dA} of the adiabatic part of the pulse becomes much longer than the characteristic time $1/r$ of the input pulse $E_{p0}(0,t)$, i.e., $rt_{dA}\approx T\Delta_{in}/2\Delta_h \gg 1$, since for the broadband pulse we have $\Delta_{in}\gg\Delta_h$ and $T\gg 1$. For our numerical example, shown in Fig. 6, t_{dA} is 120 times longer than $1/r$. The temporal width of the adiabatic part is defined by the value $t_{width} =2/\Delta_{out}=r^{-1}\sqrt{1+(\Delta_{in}/\Delta_{eff})^2}$. For our numerical example, this temporal width is increased 43.8 times with respect to $1/r$. The spectral half-width of the adiabatic part of the pulse, $\Delta_{out}\approx\Delta_{eff}=\Delta_h/\sqrt{T}$, is reduced $\sim$40 times with respect to the half-width of the input pulse Δ_{in} and becomes 5.5 times narrower than the width of the transparency window Δ_h.

Figure 6(a) shows a slight deviation of the adiabatic part $E_{pA}(z,t)$ from the result of the numerical calculation of the integral (9). This deviation can be reduced if $E_{pA}(z,t)$ in Eq. (39) is replaced by $E_{pA1}(z,t)$, Eq. (30). Such a modified analytical approximation, i.e.,

$$E_{an1}(z,t) = E_{pA1}(z,t) + E_{pN}(z,t), \tag{44}$$

fits the numerical result perfectly; see Fig. 6(c). This means that the adiabatic part of the pulse satisfies the adiabatic following condition for any width of the input pulse. It is also remarkable that the first four terms of the expansion of the spectral function Eq. (11), $a(\nu)\approx a_0-ia_1\nu-a_2\nu^2+ia_3\nu^3$, fully describe the propagation properties of the adiabatic part of the pulse.

The nonadiabatic part has a very short delay t_{dN} compared with the delay time of the adiabatic part $t_{dN}\ll t_{dA}$, since $t_{dA}/t_{dN}\approx\Delta_{in}/\Delta_{eff}\gg 1$. The delay time t_{dN} is defined by the center of the Gaussian envelope of the cosine modulated pulse amplitude [see Eq. (40)]. For a thick absorber, $T\gg 1$, the temporal half-width of this Gaussian is only two times larger than the time half-width of the input pulse, since $\Delta_{wide}\approx\Delta_{in}/2=r$. The area of the nonadiabatic part $\theta_N(z) =2(d_{eg}/\hbar)\int_{-\infty}^{+\infty}E_{pN}(z,t)dt$ must be close to zero. This conjecture is supported by two arguments. First, if Eq. (44) is valid, then $\theta(z)=\theta_A(z)+\theta_N(z)$. Second, as shown above [see Eq. (38) and discussion immediately after it], we have the identity $\theta(z)\equiv\theta_A(z)$. The approximate expression for E_{pN}, Eq. (40), gives a slightly overestimated value of the area of the nonadiabatic part $\theta_N(z)\sim\theta(0)\exp(-6\sqrt{T}\Gamma/\Delta_{in})$. We assume that this value is nonzero because the nonadiabatic part sits on the far wing of the adiabatic solution and in this sense overlaps with it. The overlapping gives an exponentially small contribution to the area $\theta_N(z)$.

The time integrated intensity of the pulse is defined as

$$U(z) = \int_{-\infty}^{+\infty} I_{p0}(z,t)dt, \tag{45}$$

where $I_{p0}(z,t)=|E_{p0}(z,t)|^2$. For a Gaussian pulse, it can be reduced to

$$U(z) = \frac{I_{p0}}{2r^2}\int_{-\infty}^{+\infty}\exp\left\{-\frac{\nu^2}{2r^2} - 2z\,\mathrm{Re}[A(\nu)]\right\}d\nu, \tag{46}$$

where $I_{p0}=|E_{p0}|^2$. We can again apply the approximation

$$U_{an}(z) = U_A(z) + U_{Na}(z), \tag{47}$$

where $U_A(z)$ and $U_{Na}(z)$ are the adiabatic and nonadiabatic parts, respectively. The former is described by Eq. (28) and the latter is

$$U_{Na}(z) = U(0)M\exp(-2T_{wide}), \tag{48}$$

if $\sqrt{T}>\Gamma/\Delta_{in}$ (see Appendix B for the derivation of this expression). For large T the effective thickness for the nonadiabatic part is $T_{wide}\sim\sqrt{T}$ and $M\approx 1$; see Eqs. (41) and (42). Figure 7 shows the comparison of the numerical integration

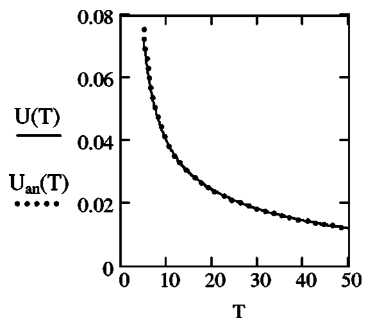

FIG. 7. Comparison of the numerically calculated time integrated intensity $U(z)$, Eq. (46), with the analytical approximation $U_{an}(z)$, Eq. (47). Both plots are normalized to $U(0)$. The parameters are $\gamma/\Gamma=10^{-3}$, $\Omega_d/\Gamma=0.5$, and $\Delta_{in}=2\Gamma$. $T=\alpha z/\Gamma$ is the effective thickness of the absorber.

of Eq. (46), $U(z)$, with the analytical approximation $U_{an}(z)$, Eq. (47). In a large range of the values for the effective thickness both functions are almost coincident. If $\gamma\to 0$, the adiabatic part of $U(z)$ decreases as $\sim 1/\sqrt{T}$.

Summarizing, we conclude that the EIT window in an optically thick resonant absorber allows transmission of a broadband pulse, which maintains its area. The pulse is time broadened and delayed. In spite of the large spectral width of the pulse with respect to the width of the EIT window, the propagation reveals adiabatic features typical for a pulse having a narrow spectrum considered in the previous section. Thus, the EIT window cuts out an "adiabatic" pulse from any input pulse and can be used as a perfect filter producing a well-defined output, almost independent of the parameters of the input pulse. The spectral half-width of the output pulse coincides with the effective half-width of the EIT window in an optically thick resonant absorber, $\Delta_{eff}=\Omega_d^2/\Gamma\sqrt{T}$. The output pulse amplitude is reduced by a factor Δ_{eff}/Δ_{in} with respect to the amplitude of the input pulse. The adiabatic and nonadiabatic parts of the pulse are well separated in time if $\Delta_{eff}t_{dA}/2>1$. This condition is well satisfied for an optically thick sample ($T\gg 1$) because it is reduced to $\sqrt{T}/2>1$ or $T>4$. The same condition is provided by the analysis of the minimum distortion of the adiabatic part due to the presence of the fourth term of the adiabatic expansion Eq. (17). The plot in Fig. 4, shown by dots, demonstrates that the parameter $\Delta_{out}/\Delta_{dist}$ satisfies the condition of negligible distortion of the Gaussian shape of the pulse, $\Delta_{out}/\Delta_{dist}<1$, if $T>7$. The latter inequality also indicates the fact that the adiabatic and nonadiabatic parts are separated or overlap very little since it satisfies the condition mentioned above ($T>4$). This means that, if the condition $\Delta_{out}/\Delta_{dist}<1$ is satisfied, the adiabatic part of the pulse is well separated from the nonadiabatic part, i.e., they do not overlap.

VI. PROBE PULSE TUNING

In this section, we consider the propagation of a Gaussian probe pulse if its carrier frequency is tuned from resonance ($\delta=\omega_p-\omega_{eg}\neq 0$). It is known that a Λ-type excitation is very sensitive to the condition of the dark resonance: $\omega_p-\omega_d=\omega_{mg}$, where ω_p and ω_d are the probe and drive field frequencies and ω_{mg} is the resonant frequency of the idle transition between the ground and metastable states [2,3]. If the drive is in resonance with the transition *m-e* ($\omega_d=\omega_{em}$) and the probe is detuned from resonance ($\delta\neq 0$), the two-photon resonance condition is violated. Below we show that a slight detuning of the probe from resonance does not change essentially its propagation properties, but produces a phase shift.

The condition of the adiabatic propagation demands that the half-width of the input pulse, Δ_{in}, must not exceed the half-width of the EIT hole for one atom, $\Delta_h=\Omega_d/\Gamma$. As was shown in Sec. II, the output probe field envelope is described by Eq. (9). If $\delta\neq 0$, we can make the substitution $x=\delta+\nu$ in the integral (9) and use the expansion of $A(\nu)$ near $x=0$ ($\nu=-\delta$) for the adiabatic pulse ($\Delta_{in}\leqslant\Delta_h$). Retaining only three terms of the expansion, we get $A_\delta(x)\approx\alpha(a_0-ia_1x-a_2x^2)$ for $A(\nu)$, where the subscript δ is introduced to distinguish this case from the one where $\delta=0$. Then the adiabatic solution for the output probe is

$$E_{pA\delta}(z,t)=\frac{E_{p0}}{2r\sqrt{\pi}}\int_{-\infty}^{+\infty}\exp[-B(x,z,t)]dx, \tag{49}$$

where

$$B(x,z,t)=\frac{(x-\delta)^2}{\Delta_{in}^2}+i(x-\delta)t+A_\delta(x)z, \tag{50}$$

t stands for $t-z/c$, and the subscript δ is again used to distinguish the nonresonant case. The integral in Eq. (49) can be easily calculated if one combines the terms in Eq. (50) as follows:

$$B(x,z,t)=ix(t-t_{dA})+\frac{[x-(\Delta_{out}^2/\Delta_{in}^2)\delta]^2}{\Delta_{out}^2}-i\delta t+\frac{\delta^2}{\Delta_{\rm eff}^2+\Delta_{in}^2}+T_{EIT}, \tag{51}$$

where t_{dA}, T_{EIT}, Δ_{out}, and Δ_{eff} are defined in Sec. III for the resonant probe pulse [see Eq. (22) and the discussion immediately after it]. Then the second substitution $y=x-\delta(\Delta_{out}/\Delta_{in})^2$ makes the integral (49) similar to the one that was calculated in Sec. III. Combining the results we obtain

$$E_{pA\delta}(z,t)=E_{pA}(z,t)\exp\left\{i\delta t-i\frac{\Delta_{out}^2}{\Delta_{in}^2}\delta(t-t_{dA})-\frac{\delta^2}{\Delta_{eff}^2+\Delta_{in}^2}\right\}, \tag{52}$$

where $E_{pA}(z,t)$ is defined in Eq. (22). The intensity of the output probe field, $I_{pA\delta}(z,t)=|E_{pA\delta}(z,t)|^2$, is almost identical to the value $I_{pA}(z,t)$ corresponding to zero detuning, $\delta=0$. The only difference is the exponential factor, i.e.,

$$I_{pA\delta}(z,t)=I_{pA}(z,t)\exp\left(-\frac{2\delta^2}{\Delta_{eff}^2+\Delta_{in}^2}\right). \tag{53}$$

For a thick sample ($T\gg 1$) and an adiabatic pulse ($\Delta_{in}=\Delta_h$), we have $\Delta_{in}\gg\Delta_{eff}$, and hence $I_{pA\delta}(z,t)\approx I_{pA}(z,t)\exp(-2\delta^2/\Delta_{in}^2)$. The factor $\exp(-2\delta^2/\Delta_{in}^2)$ describes the ratio of the intensity of the spectral component of the input pulse with frequency $\omega=\omega_p-\delta$, coinciding with the center of the EIT hole, ω_{eg}, to the intensity of the spectral component with central or carrier frequency of the pulse $\omega = \omega_p$. This result is obvious since the spectral half-width of the incoming radiation is much broader than the narrow effective half-width of the EIT hole for a thick sample, $\Delta_{eff} = \Delta_h/\sqrt{T}$. Therefore, only those spectral components of the pulse that coincide with the effective EIT hole go through the sample without appreciable absorption, while those that are out of the effective EIT window are strongly absorbed. Thus, the narrow EIT hole can be used as a frequency filter for frequency stabilization and spectrum narrowing of the pulsed field whose spectrum contains components coinciding with the effective EIT hole. The phase shift $i\delta t$ in the exponent of Eq. (52) confirms this argument because the carrier frequency of the probe, ω_p, becomes $\omega_p - \delta = \omega_{eg}$ at the output due to this phase shift.

The frequency filtering can be done for a pulse with spectral width comparable with or even broader than the width of the broad absorption line 2Γ. Then those components of the pulse that are out of the EIT window are absorbed if they coincide with the absorption part of the line. The components that are out of the absorption line or in the EIT region are transmitted. These last two components are separated in time because of the large difference in their group velocities. The narrow spectrum part from the EIT hole is delayed and the out of resonance broadband part has almost no delay and it is short.

For simplicity we consider the case if $\Delta_{in} > \Gamma$, but $\delta \ll \Gamma$. Then the nonadiabatic part of the output pulse, $E_{pN\delta}(z,t)$, can be approximated as

$$E_{pN\delta}(z,t) = E_{pN}(z,t)\exp(i\delta t), \tag{54}$$

where $E_{pN}(z,t)$ is defined in Eq. (40). The analytical approximation of the integral (9) in case of $\delta \neq 0$ is

$$E_{p\delta}(z,t) = E_{pA\delta}(z,t) + E_{pN\delta}(z,t). \tag{55}$$

Two parts of this analytical solution are well separated in time if $\sqrt{T}/2 \gg 1$ (see Sec. V).

Below we show that besides filtering the EIT hole allows us to realize a precise tuning of the carrier frequency of the pulse. One might expect that only a source of coherent radiation with very narrow spectrum can be used to define the position and the width of an EIT hole for a thick sample. This is obvious since even the adiabatic pulse with $\Delta_{in} = \Delta_h \gg \Delta_{eff}$ cannot be used directly for high-resolution spectroscopic studies. According to Eq. (53), the scanning of the carrier frequency of the pulse across the EIT hole does not change appreciably the output intensity if $\delta \sim \Delta_{eff} \ll \Delta_{in}$. However, because of the frequency-dependent phase shift of the pulse [see Eq. (52)] one can actually define the precise position of the EIT hole even with broadband pulsed radiation whose width Δ_{in} is much broader than the width of the EIT hole for one atom, Δ_h. Below we show that this position can be defined with an accuracy that exceeds far the effective width Δ_{eff} if a phase sensitive detection scheme is applied.

In a phase sensitive detection scheme, two fields are mixed and the intensity of the result is measured, i.e.,

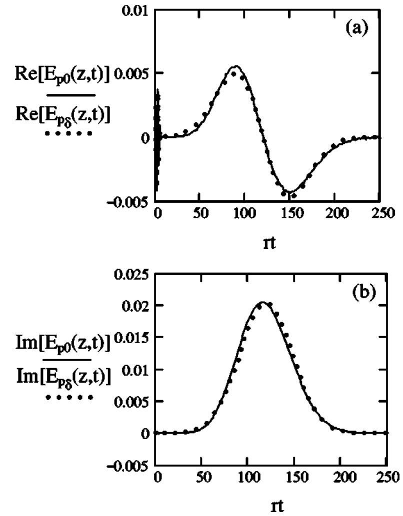

FIG. 8. Beat notes of the real (a) and imaginary (b) components of the probe pulse output for $\delta t_{dA} = \pi/2$. The thin solid line shows the numerical calculation of the integral (9) and the dots show the analytical approximation (55). The parameters are $\gamma/\Gamma = 10^{-3}$, $\Omega_d/\Gamma = 0.5$, $r = \Gamma$, and $T = 30$.

$$S(t) = |\mathbf{E}_R(z,t) + \mathbf{E}_{p\delta}(z,t)|^2, \tag{56}$$

where we assume that the output probe pulse is mixed with the cw reference field $\mathbf{E}_R(z,t) = \mathbf{E}_R\exp(-i\omega_R t + ik_R z)$ whose frequency ω_R is detuned from the frequency of the probe ω_p on the value of Δ_R, which is kept constant whatever is ω_p. Because $|\mathbf{E}_R| \gg |\mathbf{E}_{p\delta}(0,0)|$ and hence $|\mathbf{E}_R| \gg |\mathbf{E}_{p\delta}(z,t)|$, the detected signal is well approximated by

$$S(t) = |E_R|^2 + [E_R E^*_{p\delta}(z,t)e^{-i\Delta_R t + i(k_R - k_p)z} + \text{c.c.}], \tag{57}$$

where the vectors $\mathbf{E}_R$ and $\mathbf{E}_{p\delta}$ are substituted by their amplitudes E_R and $E_{p\delta}$. The interference term oscillates with frequency Δ_R. Amplifying the oscillating part of the signal (heterodyne technique), one can measure the time dependence of the probe field amplitude and its phase; see, for example, Ref. [30]. If the phase of the reference field is properly chosen, one can measure only the in-phase component of the output pulse, $\mathrm{Re}[E_{p\delta}(z,t)]$. This component is zero at the output pulse center $t = t_{dA}$ if $\delta t_{dA} = \pi/2$ [see Eq. (52)]. This zero is a first beat note, which takes place if $\delta = \delta_1$, where $\delta_1 = \pi\Delta_{eff}/2\sqrt{T}$. For $T \gg 1$, we have $\delta_1 \ll \Delta_{eff}$. In conventional spectroscopy the half-width of the transparency window for an optically thick sample, Δ_{eff}, defines the precision of the probe frequency tuning to the center of the EIT hole if the spectral half-width of the probe, Δ_{in}, is much smaller than Δ_{eff} or if cw monochromatic radiation with spectral width much smaller than Δ_{eff} is applied. Observation of the pulse envelope modulation containing only one period (tuning to δ_1) can provide an accuracy of the frequency tuning that is $2\sqrt{T}/\pi$ times higher than convention spectroscopy gives. The tuning we propose could be done for a broadband pulse. This method of frequency tuning can open new perspectives for high-resolution spectroscopy with a laser source of poor quality.

Figures 8(a) and 8(b) show beat notes of the real (a) and

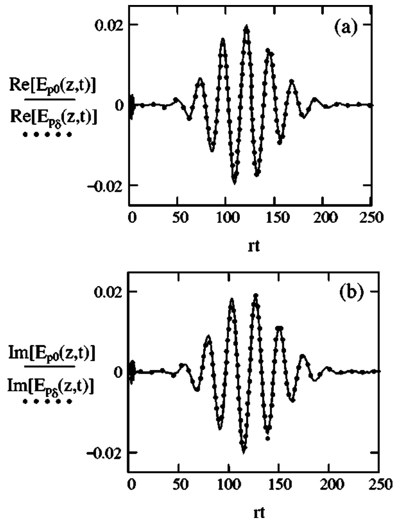

FIG. 9. Beat notes of the real (a) and imaginary (b) components of the probe pulse output for $\delta t_{dA}=10\pi$. The thin solid line shows the numerical calculation of the integral (9) and the dots show the analytical approximation (55). The other parameters are the same as for Fig. 8.

imaginary (b) (in-phase and out-of-phase) components of the output probe pulse for $\delta t_{dA}=\pi/2$. The solid line shows the result of the numerical calculation of the integral (9) and the analytical approximation, given by Eq. (55), is presented by dots.

If the probe frequency detuning δ is much larger than the width of the EIT window in a thick resonant absorber, Δ_{eff}, the output pulse contains many beat notes. Figure 9 shows the real (a) and imaginary (b) components of the output field amplitude if the bandwidth of the input field is large $(r=\Gamma)$ and the detuning $\delta=10\pi/t_{dA}$ is comparable with the half-width of the EIT hole for an individual atom (Δ_h) and six times larger than the half-width of the EIT hole for an optically thick sample, Δ_{eff}. Thus, changing the probe frequency detuning δ to reduce the number of beat notes as much as possible, one can make a fine tuning of the central frequency of the broadband field, or in other words, one can perform high-resolution spectroscopy with a probe radiation of poor quality. If the frequency of the probe is tuned such that no beat notes are present, one can get an even higher precision of the frequency tuning than $\delta_1=\pi\Omega_d^2/2\Gamma T$.

The phase sensitive detection of the probe frequency tuning gives the same precision for frequency resolution as the line narrowing and interference effects for the cw probe and drive fields considered in Ref. [22]. There it was found that heterodyne measurements of the absorption of a cw monochromatic probe reveal beat notes with a characteristic width $\Delta_{het}=\pi\Omega_d^2/\Gamma T$. These beat notes appear in the heterodyne measurements due to the contribution of the dispersive component of the atomic response χ' to the usual absorptive component χ''. The dispersive component contributes due to interference induced by a new field E_{new} arising from resonantly enhanced coherent Raman scattering. In Ref. [22] the drive couples the excited state to the ground and metastable states. Therefore, first, the probe and drive fields excite the coherence g-m via the two-quantum transition g-e-m. Then the drive via the g-e transition and the coherence ρ_{em}, excited in the first step, produce the new field E_{new} with a frequency $\omega_{new}=2\omega_d-\omega_p$. The amplitude of this field and its coupling strength Ω_{new} are proportional to Ω_d^2/ω_{mg}. They have nearly the same order as the amplitude of the probe and Ω_p. This is because in the experiment [22] $\Omega_p/\Omega_d\sim 10^{-2}$ and $\Omega_d/\omega_{mg}\sim 0.2\times 10^{-2}$ and hence $\Omega_p/\Omega_{new}=(\Omega_p/\Omega_d)/(\Omega_d/\omega_{mg})\sim 5$. Moreover, in the heterodyne detection scheme [22], the probe, drive, and new fields are mixed, producing an interference term oscillating with the microwave frequency $\omega_{mg}=6.83$ GHz.

In our scheme we consider the case when ω_{mg} belongs to the optical band, which makes the amplitude of the field $E_{new}\sim\Omega_d^2/\omega_{mg}$ extremely small. Therefore, the resonantly enhanced coherent Raman scattering can be disregarded. Having this difference between the two schemes, we, however, assume that some elements of the physics behind them are similar since they result in the same enhancement of the spectral resolution of the EIT window. What makes a difference is the possibility to perform high-resolution spectroscopy with a coherent source of poor spectral quality, which is phase locked with a cw reference field of the same quality. The coherence necessary to resolve the small frequency change of the order of kilohertz is introduced by the rf modulation.

VII. CONCLUSION

In this paper, we have considered the propagation of a small-amplitude pulse in an optically dense resonant medium if a narrow transparency window has been created at the center of the absorption line by a coupling field Ω_d. Two distinctive cases are analyzed.

The first case is realized if the spectral width of the pulse is smaller than the width of the transparency window of the individual atom, $\Delta_h=\Omega_d^2/\Gamma$. It is shown that in this case the pulse delays because of reduced group velocity. The delay time is $t_{dA}=T/\Delta_h$, where $T=\alpha z/\Gamma$ is the effective thickness of the sample (z is a propagation distance), Γ is the decay rate of the atomic polarization induced by the resonant pulse, and α is the resonant absorption coefficient of the sample. The pulse spectrum narrows inversely proportional to the square root of the propagation distance z, and tends to the value $\Delta_{eff}=\Delta_h/\sqrt{T}$. The process of narrowing of the pulse spectrum maintains the pulse area. If the pulse has a Gaussian shape, then its shape is also maintained.

The second, opposite, case is realized if the Gaussian pulse spectrum is much wider than the transparency window. It is shown that the pulse is broken in two parts. One is adiabatic and the other is not. The adiabatic part behaves similarly to the narrow bandwidth pulse. It is appreciably delayed, it has a Gaussian shape and its temporal width increases with distance. The spectral width of this part is determined by the width of the transparency window of the thick resonant absorber, Δ_{eff}. The pulse area of the adiabatic part coincides with the pulse area of the input pulse. The nonadiabatic part has a group velocity close to the phase velocity of light c and has almost no delay. It has an oscillatory time dependence with a Gaussian envelope. The adiabatic and nonadiabatic pulses are well separated in time and space if $t_{dA}\Delta_{eff}\approx\sqrt{T}\gg 1$.

We considered the case if $\Delta_{in}\gtrsim\Delta_h\gg\Delta_{eff}$ and the carrier frequency of the pulse is detuned from the center of the EIT hole. The pulse changes slightly if the frequency of the detuning is smaller than the spectral width of the pulse, Δ_{in}, hence, such a pulse cannot be used for high resolution spectroscopy. If a phase sensitive detection scheme is applied, one can find beat notes in the in-phase or out-phase components of the pulse. Minimizing the number of these beat notes, one can make a fine tuning of the pulse frequency. It enables the experimentalist to increase the resolution of the central frequency of the EIT window well below its half-width Δ_{eff}. The frequency resolution increases $\sqrt{T}$ times.

Summarizing, we conclude that in this paper we develop an accurate analytical theory of propagation of a pulse with arbitrary spectral width in a dense absorptive medium with a narrow EIT window. We show that the EIT window selects that part of the pulse spectrum that coincides with the EIT window and transforms it into a slowly propagating pulse. Therefore, in contrast to cw broadband excitation, the broadband pulse is broken up into two parts, separated in time at the output of the EIT medium, i.e., a broadband fast part and a slow part having a narrow spectrum. Therefore, such a medium works as a time-frequency filter.

ACKNOWLEDGMENTS

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APPENDIX A

In this section, we derive the analytical approximation of the adiabatic part of the integral Eq. (9), $E_{pA1}(z,t)$, if $A(\nu)\approx\alpha(a_0-ia_1\nu-a_2\nu^2+ia_3\nu^3)$. Such an approximation of the spectral function $A(\nu)$ describes well the transmission of the adiabatic part of the pulse whatever the spectral width of the input pulse $E(0,\nu)=(E_{p0}\sqrt{\pi}/r)\exp[-(\nu/2r)^2]$. Substituting the pulse spectrum and the approximate transmission function $A(\nu)$ into Eq. (9), we obtain the integral

$$E_{pA1}(z,t)=\frac{E_{p0}e^{-A_0z}}{2r\sqrt{\pi}}\int_{-\infty}^{+\infty}e^{-G(z,\nu)}d\nu, \tag{A1}$$

where

$$G(z,\nu)=i\nu\tau+\frac{\nu^2}{\Delta_{out}^2}+i\frac{\nu^3}{3\Delta_{dist}^3}, \tag{A2}$$

and $\tau=t-t_{dA}$. This integral can be calculated by the saddle-point method (see, for example, Ref. [31]). The saddle points

$$\nu_{1,2}=i\frac{\Delta_{dist}^3}{\Delta_{out}^2}\left(1\pm\sqrt{1+\frac{\Delta_{out}^4}{\Delta_{dist}^3}\tau}\right) \tag{A3}$$

are defined from the condition $G_\nu'(z,\nu)=0$. According to the method, the main contribution to the integral is given by the vicinity of the saddle points.

If $\Delta_{out}\tau>-(\Delta_{dist}/\Delta_{out})^3$, we have to take the negative branch of the square root in Eq. (A3) as the saddle point for the integral (A1) since the positive branch gives an exponentially increasing contribution to the integral. In this case, the integral (A1) is approximated by

$$E_{pA2}(z,t)=\frac{\Delta_{out}E_{p0}}{\Delta_{in}\sqrt{Q}}\exp\left[-T_{EIT}-\frac{\Delta_{dist}^6}{3\Delta_{out}^6}(2Q^3-3Q^2+1)\right], \tag{A4}$$

where $Q=\sqrt{1+(\Delta_{out}^4\tau/\Delta_{dist}^3)}$.

If $\Delta_{out}\tau<-(\Delta_{dist}/\Delta_{out})^3$, we have to take both roots of the equation $G_\nu'(z,\nu)=0$ into account since the result must be a real value. The calculation of the contribution of the integrand near these two saddle points gives the following approximation for the integral (A1):

$$E_{pA2}(z,t)=2\frac{\Delta_{out}E_{p0}}{\Delta_{in}\sqrt{Q}}\sin\left(\frac{2\Delta_{dist}^6}{3\Delta_{out}^6}Q^3+\frac{\pi}{4}\right)\times e^{-T_{EIT}-(\Delta_{dist}^6/\Delta_{out}^6)(Q^2+1/3)}, \tag{A5}$$

where $Q=\sqrt{-1-(\Delta_{out}^4\tau/\Delta_{dist}^3)}$.

These two functions Eqs. (A4) and (A5) describe quite well the small distortion of the output Gaussian pulse, Eq. (30), except at the vicinity of the point $t=t_{dA}-\Delta_{dist}^3/\Delta_{out}^4$ where $\Delta_{out}\tau=-(\Delta_{dist}/\Delta_{out})^3$. At this point and near it, the steepest descent method of the integral calculation is not applicable since the quadratic term of the expansion of the function $G(z,\nu)$ near the saddle point becomes smaller than the next term of the expansion.

Actually it is possible to calculate the integral (A1) if $\Delta_{out}\tau=-(\Delta_{dist}/\Delta_{out})^3$ just to show that there is not any singularity of the function $E_{pA1}(z,t)$ at this point of the time axis. Taking into account the cubic term of the expansion near the saddle point if $\tau=-\Delta_{dist}^3/\Delta_{out}^4$, we obtain an exact result for the integral

$$E_{pA2}(z,t_s)=\frac{E_{p0}2\sqrt{\pi}\Delta_{dist}}{3^{2/3}\Gamma\left(\frac{2}{3}\right)\Delta_{in}}\exp\left(-T_{EIT}-\frac{\Delta_{dist}^6}{3\Delta_{out}^6}\right), \tag{A6}$$

where $t_s=t_{dA}-\Delta_{dist}^3/\Delta_{out}^4$ and $\Gamma(2/3)\approx 1.354$ is the Gamma function. Figure 10 shows a comparison of a numerical calculation of the integral (A1) with the analytical approximation given in Eqs. (A4) and (A5).

If Δ_{dist} is negative, we can define it as a positive value: $\Delta_{dist}=(3\alpha|a_3|z)^{-1/3}$, which is the case considered in Sec. IV but we have to change sign of τ in Eqs. (A1)–(A6) and in the definition of Q. Then the time dependence of $E_{pA2}(z,t)$ is reversed with respect to the case of $a_3>0$.

APPENDIX B

In this section, we calculate the nonadiabatic components of the integrals in Eqs. (9) and (46), which are the functions $E_{pN}(z,t)$ and $U_{Na}(z)$ defined in Eqs. (39) and (47).

For a thick sample ($T\gg 1$), part of the broadband spectrum of the pulse, which coincides with the central part of

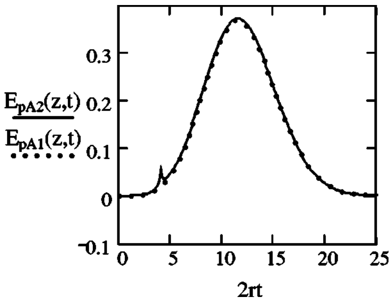

FIG. 10. Comparison of the result of a numerical calculation of the integral in Eq. (A1) (dotted line) with its analytical approximation given by Eqs. (A4) and (A5) (solid line). The parameters are $\Omega_d/\Gamma=0.5$, $r/\Gamma=0.05$, $\gamma/\Gamma=10^{-3}$, and $T=30$. The small bump on the left wing of the curve is artificial and it results from the saddle-point approximation. The true value of the function for the argument $t=t_s$, where the position of the bump is, is defined by Eq. (A6).

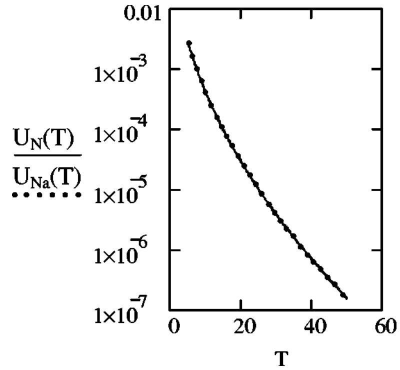

FIG. 11. Comparison of the numerically calculated integral (B2), solid line, with the analytical approximation (48), shown by dots. Both plots are normalized to $U(0)$. The parameters of the pulse and the sample are the same as for Fig. 6. $T=\alpha z/\Gamma$ is the optical thickness of the absorber.

the absorption line, is strongly absorbed. The boundaries of this part are defined by $(\omega_{eg}-\Gamma,\omega_{eg}+\Gamma)$. The wings of the pulse spectrum are less absorbed and their contribution to the integral (9) can be appreciable even for a thick sample. If an EIT hole is present, it changes only the central part of the absorption line $(\omega_{eg}-\Gamma,\omega_{eg}+\Gamma)$ while the wings are almost not affected. Therefore, only a narrow EIT hole of width Δ_h introduces a difference in the integral (9) with respect to the case $\Omega_d=0$. This difference can be taken into account by calculation of the pulse spectrum in the domain $2\Delta_h$, which is the adiabatic solution $E_{pA}(z,t)$ (see Sec. III). Therefore, for a thick sample $(T\gg1)$ the nonadiabatic part $E_{pN}(z,t)$ can be calculated simply by ignoring the presence of the EIT hole, i.e., by taking $\Omega_d=0$.

First, we consider the time integrated intensity $U(z)$ and replace $A(\nu)$ in the integral (46) by

$$A(\nu)=\frac{\alpha}{\Gamma-i(\nu+\delta)}, \tag{B1}$$

where $\Omega_d^2/[\gamma-i(\nu+\delta)]$ is disregarded in the denominator. Then the nonadiabatic part of this integral is expressed as

$$U_N(z)=\frac{I_{p0}}{2r^2}\int_{-\infty}^{+\infty}\exp[-2f(\nu)]d\nu, \tag{B2}$$

where

$$f(\nu)=\frac{\nu^2}{4r^2}+\frac{T\Gamma^2}{\nu^2+\Gamma^2}. \tag{B3}$$

In this case the saddle points $\nu_s=\pm\Gamma\sqrt{\eta-1}$ are defined from the condition $f'(\nu)=0$, where $\eta=2r\sqrt{T}/\Gamma$. These points are located on the real axis if $\eta>1$. For a thick resonant absorber $(T>1)$ and broadband input pulse $(r\sim\Gamma)$, the saddle points are on the wings of the absorption line (close to or beyond the line half-width if $\eta\geqslant2$). The contribution from these points is expressed via the integral

$$U_{Na}(z)=\frac{I_{p0}}{r}\int_{-\infty}^{+\infty}\exp[-2x^2(1-\eta^{-1})-2\xi^2(2\eta-1)]dx, \tag{B4}$$

where $\xi=\Gamma/2r$ and $x=\nu/r$. The value of the integral is

$$U_{Na}(z)=U(0)\frac{\exp[-\Gamma^2(2\eta-1)/2r^2]}{\sqrt{1-\eta^{-1}}}, \tag{B5}$$

which corresponds to Eq. (48). For the analysis of the result, we introduce in Eq. (48) the parameter $M=\sqrt{\eta/(\eta-1)}$. Figure 11 shows the comparison of the numerically calculated integral (B2) with the analytical approximation (48). The plots are indistinguishable over a wide range of values of the effective length T.

A similar procedure is applied for the calculation of the nonadiabatic part of the integral (9), which is reduced following the above arguments to

$$E_{pN}(z,t)=\frac{E_{p0}}{r\sqrt{\pi}}\int_0^{+\infty}\exp[-f(\nu)]\cos\varphi(\nu)d\nu, \tag{B6}$$

where

$$\varphi(\nu)=\nu t+\frac{T\Gamma\nu}{\nu^2+\Gamma^2}, \tag{B7}$$

and t stands for $t-z/c$. To calculate this integral by the saddle-point method, one has to find the saddle points from the condition $f'(\nu)+i\varphi'(\nu)=0$. However, we use the previous condition, $f'(\nu)=0$, to find these points. Such an approach simplifies the calculation and gives a nice approximate expression for the integral although with less accuracy. This is because $f(\nu)$ has extrema at ν_s, which are real, and $\varphi(\nu)$ is almost linearly dependent on ν in the vicinity of the saddle points ν_s. The contribution to the integral (B6) near the saddle points is expressed as

$$E_{pN}(z,t) = \frac{E_{p0}}{r\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-B(x)} \cos F(x,t) dx, \tag{B8}$$

where we retain only the x^2 term in the expansion of $f(\nu)$ near the saddle point ν_s ($\nu = \nu_s + x$) and only the linear terms x in the expansion of $\varphi(\nu)$. The functions $B(x)$ and $F(x,t)$ are $B(x) = T_{wide} + x^2(1-\eta^{-1})/r^2$, $F(x,t) = (t-t_{dN})x + \omega_{wide}(t+t_{ph})$, where $\omega_{wide} = \Gamma\sqrt{\eta - 1}$, $T_{wide} = (2\eta - 1)(\Gamma/2r)^2$, $t_{dN} = (1-2\eta^{-1})\sqrt{T}/2r$, and $t_{ph} = \sqrt{T}/2r$. The calculation of the integral (B8) gives

$$E_{pN}(z,t) = \frac{E_{p0} \cos[\omega_{wide}(t+t_{ph})]}{\sqrt{1-\eta^{-1}}} \exp\left[-T_{wide} - \frac{r^2(t-t_{dN})^2}{4(1-\eta^{-1})} \right], \tag{B9}$$

which is Eq. (40), but defined with η, rather than with M.

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