Conditioned homodyne detection at the single-photon level: Intensity-field correlations for a two-level atom in an optical parametric oscillator

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We consider intensity-field correlation functions for a two-level atom in a degenerate optical parametric oscillator (OPO), which would result from a conditioned homodyne measurement. Analytic results are obtained in the limit of weak driving fields using quantum trajectory methods for both the transmitted and fluorescent fields. This system is unique in that after detection of a photon, it is known that one excitation is in the system, in either the atom or cavity mode. We find large violations of inequalities satisfied by classical fields, for both transmitted and fluorescent fields. This is in contrast to the usual cavity QED system of an atom in a driven cavity where we do not find nonclassical behavior in the intensity-field correlation function in fluorescence. This is understood in terms of a relationship between the intensity-field correlation function and the second-order intensity correlation function, as well as the different amount of field-atom entanglement in the two systems. We show that for weak-field cavity QED one must have photon bunching to have nonclassical behavior in the intensity-field correlation function. We compare our results to those of an ordinary OPO. Finally, we also consider cross correlations, where we examine the transmitted (fluorescent) field conditioned by detection of a fluorescent (transmitted) photon.

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I. INTRODUCTION

Carmichael and co-workers [1] have recently introduced an intensity-field correlation function that has proven to be of great interest. Typically quantum opticians have dealt with the field-field correlation function $g^{(1)}(\tau) = \langle E^*(0)E(\tau)\rangle / \langle E\rangle^2$ or the intensity-intensity correlation function $g^{(2)}(\tau) = \langle I(0)I(\tau)\rangle / \langle I\rangle^2$. Carmichael *et al.* introduced the correlation function

$$h_\theta(\tau) = \frac{\langle I(0)E_\theta(t)\rangle}{\langle I\rangle\langle E_\theta\rangle}, \tag{1}$$

where E_θ is an electric-field quadrature. This is in essence a correlation between particlelike aspects of light (intensity measurement as photon detection) and wavelike aspects (the interferometric nature of a field quadrature measurement). For an optical parametric oscillator (OPO) or a driven cavity QED system, this correlation function has been predicted to exhibit large violations of a Schwartz inequality that would be satisfied by classical fields [1]. It has also been proposed as a measurement of squeezing that is independent of detector efficiency [1]. Recent experiments on the cavity QED system have verified the large violations for the transmitted field [2], as well as the relation of $h_\theta(\tau)$ to the spectrum of squeezing [3]. This experimental program has also examined quantum fluctuations at the subphoton level [4]. Denisov *et al.* [5] have shown that for stronger fields $h_\theta(\tau)$ is not time symmetric, and this has been traced to a violation of detailed balance.

Here we investigate the behavior of $h_\theta(\tau)$ for a two-level atom inside an OPO, in the weak-driving-field limit, and look at both transmitted and fluorescent fields.

Jin and Xiao [6,7] considered phase and intensity bistability for this system. Further they considered the spectrum of squeezing and incoherent spectra for that system. Agarwal [8] has previously considered the two-level atom in an OPO, with a strong driving field incident directly on the atoms, from the side of the cavity. He considered the strong driving limit where the external field dressed the atoms and found modifications of the Mollow triplet in that case. Clemens *et al.* [9] considered the incoherent spectrum in this system in the weak-field limit and found a variety of nonclassical effects. In the strong-coupling regime, the incoherent spectrum consisted of a vacuum-Rabi doublet with holes in each sideband. Outside the strong-coupling regime, spectral holes and narrowing were reported. These were attributed to quantum interference between various emission pathways, which vanishes when the number of intracavity photons increases and the number of pathways increases. Our results here illustrate, again, the nonclassical nature of this system with weak driving fields.

In this paper we find that the nonclassical behavior of $h_\theta(\tau)$ for the transmitted field is on the same order as that of an OPO with *no* atom in it. The new feature here is that the fluorescent field exhibits nonclassical behavior of the same order as the transmitted field. This does not occur in the fluorescent field of the driven cavity with a two-level atom in it; in the ordinary OPO, there is of course no fluorescence. This is explained in terms of the strong entanglement induced between the atom and cavity mode upon detection. Also we understand this in terms of a relation between $h_\theta(0)$ and $g^{(2)}(0)$ for weakly driven cavity QED systems.

Further we present results for the transmitted field conditioned on the direction of a fluorescent photon and vice versa. We find that the conditioned field is not strongly dependent on whether it was conditioned on detection of a

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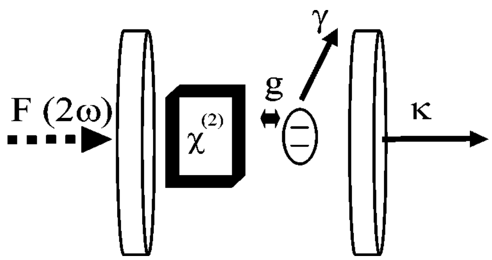

FIG. 1. Two-level atom inside a driven optical parametric oscillator. F^2 is the input photon flux at frequency 2ω, g is the atom-field coupling, γ is the spontaneous emission rate out the sides of the cavity, and 2κ is the rate of intracavity intensity decay.

transmitted *or* fluorescent field. Again this is a consequence of the entanglement in the system. In Sec. II we examine the physical system under consideration. The intensity-field correlation function for transmitted light is calculated in Sec. III. The intensity-field correlation of the fluorescent light is considered in Sec. IV. Section V consists of an examination of cross correlations, where we examine the transmitted (fluorescent) field conditioned by detection of a fluorescent (transmitted) photon. We then conclude in Sec. VI.

II. PHYSICAL SYSTEM

We first consider a single two-level atom inside an optical cavity, which also contains a material with a $\chi^{(2)}$ nonlinearity. The atom and cavity are assumed to be resonant at ω and the system is driven by light at 2ω. The system is shown in Fig. 1.

The interaction of this driving field with the nonlinear material produces light at the subharmonic ω. This light consists of correlated pairs of photons or (very weakly) quadrature squeezed light. In the limit of weak driving fields, these correlated pairs are created in the cavity and eventually two photons leave the cavity either through the end mirror or as fluorescence out the side before the next pair is generated. Hence we may view the system as an atom-cavity system driven by the occasional pair of correlated photons. This is in contrast to shining weakly squeezed light onto the cavity, as it is not certain in that case that both entangled photons get into the cavity.

In the language of squeezed light, we are interested in the limit $N \to 0$, where $N = \sinh^2 r$ is the average photon number of the squeezed field, with r the usual squeezing parameter. As N is increased the effects we consider here vanish. We wish to understand these effects in terms of photon correlations rather than the usual effects of quadrature squeezed light, where typically the largest nonclassical effects are seen in the large-N limit. The system is described by a master equation in Lindblad form

$$\dot{\rho} = -i\hbar[H,\rho] + \mathcal{L}_{diss}\rho \equiv \mathcal{L}\rho, \tag{2}$$

where the system Hamiltonian is

$$H = i\hbar F(a^{\dagger^2} - a^2) + i\hbar g(a^\dagger \sigma_- - a\sigma_+) + \hbar\omega\left(a^\dagger a + \frac{1}{2}\sigma_z\right). \tag{3}$$

Here, $g = \mu(\omega_0/\hbar\epsilon_0 V)^{1/2}$ is the usual Jaynes-Cummings atom-field coupling in the rotating-wave and dipole approximations. The cavity-mode volume is V, and the atomic dipole matrix element connecting ground and excited states is μ. The effective two-photon driving field F is proportional to the intensity $I_{in}(2\omega_0)$ of a driving field at twice the resonant frequency of the atom (and resonant cavity) and the $\chi^{(2)}$ of the nonlinear crystal in the cavity, as

$$F = -i\kappa_{in}\left(\frac{\mathcal{F}}{\pi}\right)\sqrt{\frac{\varepsilon_0 VT}{\hbar\omega}}e^{i\phi}\chi^{(2)}I_{in}(2\omega). \tag{4}$$

The cavity finesse is $\mathcal{F}$, and T and ϕ are the intensity transmission coefficient and phase change at the input mirror. We also have $\kappa_{in} = cT'/L$ as the cavity-field loss rate through the input mirror. The transmission T' of the input mirror is taken to be vanishingly small, with a large $I_{in}(2\omega_0)$, so that F is finite. Hence we effectively consider a single-ended cavity. The dissipative Liouvillian describing loss due to the leaky end mirror and spontaneous emission out the side of the cavity is

$$L_{diss}\rho = \frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a). \tag{5}$$

Here γ is the spontaneous emission rate to all modes other than the privileged cavity mode, hereafter referred to as the vacuum modes. The field decay rate of the cavity at the output mirror is κ. As we are working in the weak-driving-field limit, we only consider states of the system with up to two quanta—i.e.,

$$|0-\rangle, |0+\rangle, |1-\rangle, |1+\rangle, |2-\rangle. \tag{6}$$

Here, the first index corresponds to the excitation of the field (n=number of quanta) and the second index denotes the number of energy quanta in the atoms (+ for ground state and − for excited state). The use of a truncated basis in the weak field is well known in cavity QED; the canonical system of an atom or atoms in a driven optical cavity [10]. It has also been used previously in work on this very system, validated by simulations including more photon states; see in particular Fig. 8 in Ref. [9].

We describe the system by a conditioned wave function, which evolves via a non-Hermitian Hamiltonian, and associated collapse processes [11]. These are given by

$$|\psi_c(t)\rangle = \sum_{n=0}^{\infty} C_{g,n}(t)e^{-iE_{g,n}t}|g,n\rangle + C_{e,n}(t)e^{-iE_{e,n}t}|e,n\rangle, \tag{7}$$

$$H_D = -i\kappa a^\dagger a + -i\frac{\gamma}{2}\sigma_+ + i\hbar F(a^{\dagger^2} - a^2) + i\hbar g(a^\dagger\sigma_- - a\sigma_+), \tag{8}$$

where we also have collapse operators

$$\mathcal{C} = \sqrt{\kappa}a, \tag{9}$$

$$\mathcal{A} = \sqrt{\frac{\gamma}{2}}\sigma_-, \tag{10}$$

representing cavity loss and spontaneous emission, respectively.

Let us address the feasibility of experiments on this system. It would be very difficult to place a second-harmonic crystal inside a small-volume microcavity; however, here we seek a very small nonlinearity. This is to ensure that we are in the weak-field limit; the driving field that the atoms and cavity mode see is the occasionaly pair of photons. One could envision a nonlinear material in the mirror itself or a second atomic species and isotope with external fields to create a nonlinearity. The problem that must be faced is the alteration of the Q of the cavity dueto this nonlinear material. Nonunit photodetection efficiency does not affect normalized correlation functions; it merely lowers the count rates and increases the time the experiment must run. This may pose practical problems in terms of how long lasers can be locked. The intensity-field correlation function has been measured though and has been shown to be an efficiency-independent measure of squeezing [2–4]. With a zero mean field, the addition of an offset field is a new complication in this setup. The atom must be known to be in the cavity, but this can be detected by various means; the atom must stay in the cavity for several atomic or cavity lifetimes, whichever is shorter. The averaging is over a set of correlations after a trigger event.

III. $h_\theta(\tau)$ FOR THE TRANSMITTED FIELD

We now turn to our calculations for $h_\theta(\tau)$. For a quantized field we have

$$h_\theta^{TT}(\tau) = \frac{\langle \mathcal{T}{:}\hat{a}^\dagger(0)\hat{a}(0)\hat{a}_\theta(\tau){:}\rangle}{\langle \hat{a}^\dagger\hat{a}\rangle\langle \hat{a}_\theta\rangle} = \frac{\langle {:}\hat{a}^\dagger(0)\hat{a}_\theta(\tau)\hat{a}(0){:}\rangle}{\langle \hat{a}^\dagger\hat{a}\rangle\langle \hat{a}_\theta\rangle}, \tag{11}$$

where we have utilized normal and time ordering and defined the electric-field quadrature operator (as measured by a balanced homodyne detector)

$$\hat{a}_\theta = \frac{1}{2}(\hat{a}e^{-\imath\theta} + \hat{a}^\dagger e^{\imath\theta}), \tag{12}$$

where θ is the phase of the local oscillator with respect to the average signal field. The superscript TT refers to the fact that the field and intensity of interest are the transmitted ones.

This normalized correlation function is not well defined for a field of zero average value (i.e., $\langle\hat{a}\rangle=0$). In that case, it is convenient to introduce an offset by combining the input field with an offset field at a beam splitter. The signal mode is then $\hat{b}=\hat{a}+\alpha e^{\imath\theta}$ where θ is adjusted to match the local oscillator phase. The choice of

$$\alpha = \sqrt{\langle \hat{a}^\dagger\hat{a}\rangle} \tag{13}$$

results in the maximum signal-to-noise ratio in an experiment.

As with other correlation functions, like the second-order intensity correlation function $g^{(2)}(\tau)$, restrictions can be placed on $h_\theta(\tau)$ *if* there is an underlying positive-definite probability distribution function for the amplitude and phase of the electric field—i.e., that the field is classical albeit stochastic. If one ignores third-order correlations that vanish in the weak-field limit, Carmichael *et al.* have shown that [1]

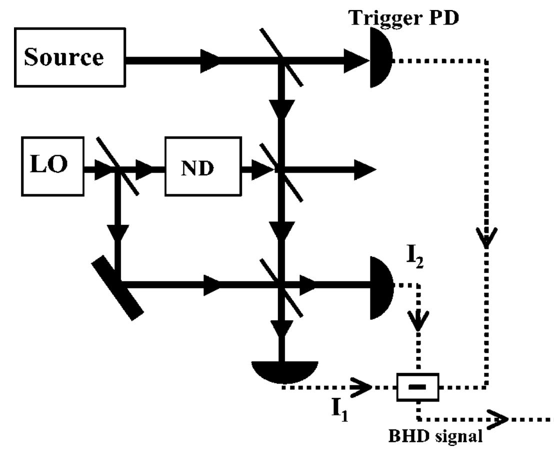

FIG. 2. A schematic of the measurement of $h_\theta(\tau)$. This is a balanced homodyne detection (BHD), conditioned on a trigger photodetection (PD). A part of the strong local oscillator (LO) is used as the offset field after adjusting the offset amplitude with a neutral density filter (ND)

$$h_\theta^{TT}(\tau) = 1 + 2\frac{\langle{:}\Delta\hat{a}_\theta(0)\Delta\hat{a}_\theta(\tau){:}\rangle}{\langle\Delta\hat{a}^\dagger\Delta\hat{a}\rangle}, \tag{14}$$

and we see that the intensity-field correlation function is connected to the spectrum of squeezing:

$$S_\theta(\omega) \propto \int_0^\infty d\tau \cos(\omega\tau)[h_\theta(\tau) - 1]. \tag{15}$$

From this, it has been shown that the Schwartz inequality would yield

$$0 \leqslant h_\theta(0) - 1 \leqslant 1 \tag{16}$$

or, generalizing to any τ,

$$0 \leqslant h_\theta(\tau) \leqslant 2 \tag{17}$$

for classical fields. Whenever there is squeezing, these inequalities do not hold for $h_\theta(\tau)$. Giant violations of these inequalities have been predicted for an optical parametric oscillator and a group of N atoms in a driven optical cavity [1] and have been recently observed in the cavity QED system [2–4].

How does one perform a measurement of $h_\theta(\tau)$? One first detects a photon, waits a time τ, and measures $\langle\hat{a}_\theta\rangle$. A practical way to do that is shown in Fig. 2. Recall that the operator $\hat{a}$ is a mode with both signal from the source and offset field, $\hat{a}=\alpha+\Delta\hat{a}$. To understand this we examine the structure of

$$h_\theta^{TT}(\tau) = \frac{\langle \hat{a}^\dagger(0)\hat{a}_\theta(\tau)\hat{a}(0)\rangle}{\langle \hat{a}^\dagger \hat{a}\rangle\langle \hat{a}_0\rangle}. \tag{18}$$

We see that with the $\hat{a}$ acting to the right and the $\hat{a}^\dagger$ acting to the left at $t=0$, a collapsed state is prepared, the collapse being that of the loss of a photon from the field corresponding to a detection event. Then at $t=\tau$ one measures $\langle \hat{a}_\theta\rangle$ *conditioned* on the previous detection. This differs from a direct measurement of $\langle \hat{a}_\theta\rangle$ with no conditioning. An ensemble average of the latter measurements (necessary to get a good signal-to-noise ratio) would yield zero due to phase fluctuations. The conditioned balanced homodyne (BHD) measurement essentially looks at members of the ensemble with the same phase, a phase that is set by the photodetection. The result is that

$$h_\theta^{TT}(\tau) = \frac{\langle n\rangle_{SS}\langle \hat{a}_\theta(\tau)\rangle_c}{\langle n\rangle_{SS}\langle \hat{a}_\theta(\tau)\rangle_{SS}} = \frac{\langle \hat{a}_\theta(\tau)\rangle_c}{\langle \hat{a}_0(\tau)\rangle_{SS}}. \tag{19}$$

We now construct an analytic solution using the quantum trajectory method and again look at weak driving fields. We find

$$\langle \hat{a}^\dagger(0)\hat{a}_\theta(\tau)\hat{a}(0)\rangle = \langle \psi_c^T|\hat{a}_\theta|\psi_c^T\rangle, \tag{20}$$

where $|\psi_c^T\rangle$ is the collapsed state produced by the photodetection event. We need only keep the states with two or less excitations (total in the cavity mode or internal energy) for weak driving fields.

The equations for the relevant probability amplitudes are

$$\dot{C}_{g,0} = -FC_{g,2},$$

$$\dot{C}_{g,1} = gC_{e,0} - \kappa C_{g,1},$$

$$\dot{C}_{e,0} = -gC_{g,1} - \gamma/2C_{e,0},$$

$$\dot{C}_{g,2} = g\sqrt{2}C_{e,1} + FC_{g,0} - 2\kappa C_{g,2},$$

$$\dot{C}_{e,1} = -\sqrt{2}gC_{g,2} - (\kappa + \gamma/2)C_{e,1}. \tag{21}$$

The steady-state solutions are easy to find (to order F):

$$C_{g,0}^{SS} = 1,$$

$$C_{g,1}^{SS} = 0,$$

$$C_{e,0}^{SS} = 0,$$

$$C_{g,2}^{SS} = \frac{F}{2}\frac{\kappa + \gamma/2}{g^2 + \kappa(\kappa + \gamma/2)},$$

$$C_{e,1}^{SS} = \frac{-1}{\sqrt{2}}\frac{gF}{g^2 + \kappa(\kappa + \gamma/2)}, \tag{22}$$

where we assume that the system starts in the ground state and that $C_{g,0}\sim 1$ for weak fields. After a collapse, the wave function will evolve from the collapsed state back to the steady state. The zero- and two-photon amplitudes scale as F. As the one-photon amplitudes are fed by decay from the two-photon states, we assume that they also scale as F. The expressions for $h_\theta(\tau)$ just depend on the one-photon amplitudes conditioned on a detection event in the fluorescent or transmitted field. The solution to these is

$$C_{g,1}(\tau) = \exp\left[-\left(\frac{\kappa}{2}+\frac{\gamma}{4}\right)\tau\right]\left[C_{g,1}(0)\cosh(\Omega\tau/2) + 2\frac{gC_{e,0}(0) - \left(\frac{\kappa}{2}-\frac{\gamma}{4}\right)C_{g,1}(0)}{\Omega}\sinh(\Omega\tau/2)\right], \tag{23}$$

$$C_{e,0}(\tau) = \exp\left[-\left(\frac{\kappa}{2}+\frac{\gamma}{4}\right)\tau\right]\left[C_{e,0}(0)\cosh(\Omega\tau/2) + 2\frac{\left(\frac{\kappa}{2}-\frac{\gamma}{4}\right)C_{e,0}(0) - gC_{g,1}(0)}{\Omega}\sinh(\Omega\tau/2)\right], \tag{24}$$

with

$$\Omega = \sqrt{(\kappa - \gamma/2)^2 - 4g^2}. \tag{25}$$

The steady-state photon number is given by

$$\langle \hat{a}^\dagger \hat{a}\rangle = 2|C_{g,2}^{SS}|^2 + |C_{e,1}^{SS}|^2. \tag{26}$$

For an initial trigger detection in the transmitted field, the appropriate collapsed state is given by

$$|\psi_c^T\rangle = \frac{\hat{a}|\psi_{SS}\rangle}{|\hat{a}|\psi_{SS}\rangle|}. \tag{27}$$

In the weak-field limit this becomes

$$|\psi_c^T\rangle = \frac{\sqrt{2}C_{g,2}^{SS}|g,1\rangle + C_{e,1}^{SS}|e,0\rangle}{\sqrt{2|C_{g,2}^{SS}|^2 + |C_{e,1}^{SS}|^2}}. \tag{28}$$

Note that there is no population in the ground state. Upon detection of a transmitted photon, as they are created in pairs, we find ourselves certain in the knowledge that one quantum is in the system, either in a cavity-mode excitation (photon) or an internal excitation of the atom. While this might be a difficult way to prepare such a state, by proper choice of g, κ, and γ, almost any superposition of $|e,0\rangle$ and $|g,1\rangle$ may be created. In the weak field the probability of more than two quanta in the system initially is negligible; this is not the case for higher excitations, where correlated pairs begin to overlap. As this certainty of the number of quanta is at the heart of all the nonclassical effects observed, these will vanish as the driving field increases. It is this driving of the system by the occasional pair of photons in an entangled state that creates most of the interesting effects. After the detection, the system evolves in time,

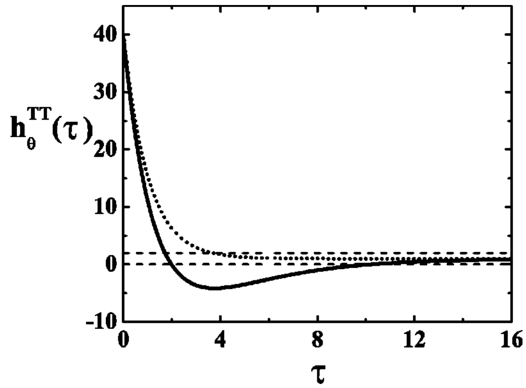

FIG. 3. Plot of $h_\theta^{TT}(\tau)$ vs $\tau=\kappa t$ for the two-level atom in an OPO (solid line) and for an ordinary OPO (dotted line). We use $\kappa/\gamma=5.0$, $g/\gamma=1.5$, and $F/\gamma=0.01$, with $\langle\hat{a}^\dagger\hat{a}\rangle=7.5\times10^{-4}$. The dashed lines indicate the range allowed for classical fields.

$$|\psi_c^T\rangle = C_{g,1}^{CT}(\tau)|g,1\rangle + C_{e,0}^{CT}(\tau)|e,0\rangle, \tag{29}$$

where the superscript CT indicates a collapse associated with a photon detection in transmission. The appropriate initial conditions are

$$C_{g,1}^{CT}(0) = \frac{\sqrt{2}C_{g,2}^{SS}}{\sqrt{2|C_{g,2}^{SS}|^2+|C_{e,1}^{SS}|^2}}, \tag{30}$$

$$C_{e,0}^{CT}(0) = \frac{C_{e,1}^{SS}}{\sqrt{2|C_{g,2}^{SS}|^2+|C_{e,1}^{SS}|^2}}. \tag{31}$$

In terms of the one-photon probability amplitudes, we find

$$h_\theta^{TT}(\tau) = 1 + \frac{\sqrt{2}C_{g,1}^{CT}(\tau)C_{g,2}^{SS} + C_{e,0}^{CT}(\tau)C_{e,1}^{SS}}{\sqrt{2|C_{g,2}^{SS}|^2+|C_{e,1}^{SS}|^2}} + \frac{C_{g,1}^{CT}(\tau)\cos\theta}{\sqrt{2|C_{g,2}^{SS}|^2+|C_{e,1}^{SS}|^2}}. \tag{32}$$

The first two terms are of order unity, while the third term is of order $1/F$. For weak fields, this term can be arbitrarily large, in violation of the inequality (16). In Fig. 3 we have a plot of $h_\theta^{TT}(\tau)$ for weak coupling ($g/\gamma=1.5$, $g/\kappa=0.375$), with cavity decay dominant over spontaneous emission ($\kappa/\gamma=5.0$). We find large violations of the inequality (16), both above ($h_\theta^{TT}(\tau)>2$) and below ($h_\theta^{TT}(\tau)<0$) the classically allowed region. For the ordinary OPO, only the former is true (the dotted line in Fig. 3). In this and all following figures, we have chosen a value of F that has been shown to place us in the weak-field limit in previous work [9]. The overall size of the violations of the inequality (16) is of the same order ($1/F$) as it is in the ordinary OPO [1]. For weak coupling (g/γ or $g/\kappa\ll1$), with spontaneous emission dominant over cavity decay ($\kappa/\gamma>1$) we find only violations above, as in the ordinary OPO, again of the same order. In this case there is no difference between the two-level atom in an OPO and the ordinary OPO. This is due to the fact that the probability of the atom to be in the ground state is quite high; after detection of a transmitted photon, the state is very close to that of a single-photon Fock state in the cavity as in the ordinary OPO. In Fig. 4 we have a plot of $h_\theta^{TT}(\tau)$ for strong coupling ($g/\gamma=5.0$, $g/\kappa=10.0$), and we find large violations of inequality (16), both above and below the classically allowed region, with the appearance of vacuum-Rabi oscillations. These oscillations are of course due to the interchange of energy between the cavity mode and atom, which does not occur in the ordinary OPO, as there is no atom there.

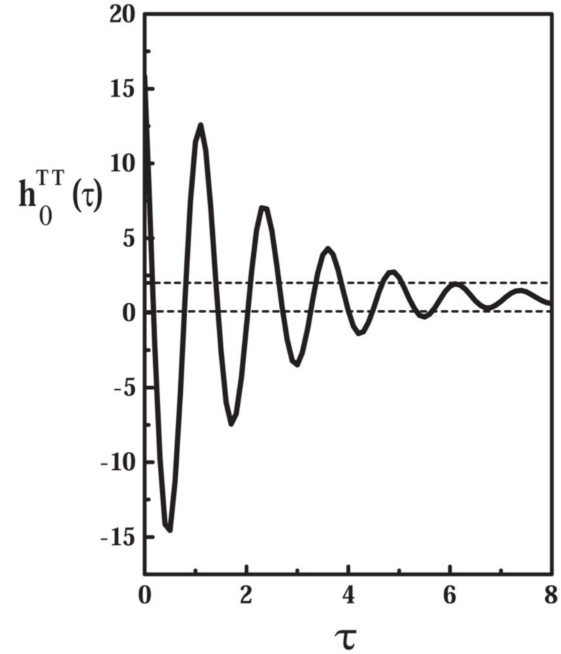

FIG. 4. Plot of $h_\theta^{TT}(\tau)$ vs $\tau=\gamma t$ for $\kappa/\gamma=0.5$, $g/\gamma=5.0$, and $F/\gamma=0.1$ with $\langle\hat{a}^\dagger\hat{a}\rangle=4.0\times10^{-4}$. The dashed lines indicate the range allowed for classical fields.

IV. $h_\theta(\tau)$ FOR THE FLUORESCENT FIELD

The fluorescent field is proportional to the dipole moment of the atom:

$$\hat{E}_{fl} \propto \hat{\sigma}_- e^{-i\omega t} + \hat{\sigma}_+ e^{i\omega t}. \tag{33}$$

The intensity of the fluorescent field is given by

$$I_{fl} \propto \langle\hat{\sigma}_+\sigma_-\rangle, \tag{34}$$

and we define the dipole quadrature operator

$$\sigma_\theta = \frac{1}{2}(\hat{\sigma}_- e^{-i\theta} + \hat{\sigma}_+ e^{i\theta}). \tag{35}$$

By considering photon detection in fluorescence followed by a balanced homodyne measurement of the fluorescent field, we obtain

$$h_\theta^{FF}(\tau) = \frac{\langle\mathcal{T}{:}\hat{\sigma}_+(0)\hat{\sigma}_-(0)\hat{\sigma}_\theta(\tau){:}\rangle}{\langle\hat{\sigma}_+\hat{\sigma}_-\rangle\langle\hat{\sigma}_0\rangle} = \frac{\langle{:}\hat{\sigma}_+(0)\hat{\sigma}_\theta(\tau)\hat{\sigma}_-(0){:}\rangle}{\langle\hat{\sigma}_+\hat{\sigma}\rangle\langle\hat{\sigma}_0\rangle}. \tag{36}$$

Here again we must add an offset as the average fluorescent field $\langle\hat{\sigma}_-\rangle=0$. The size of the offset is chosen again to maximize the signal-to-noise ratio in an experiment, $\alpha=\sqrt{\langle\sigma_+\hat{\sigma}_-\rangle}=\sqrt{\langle\Delta\hat{\sigma}_+\Delta\hat{\sigma}_-\rangle}$:

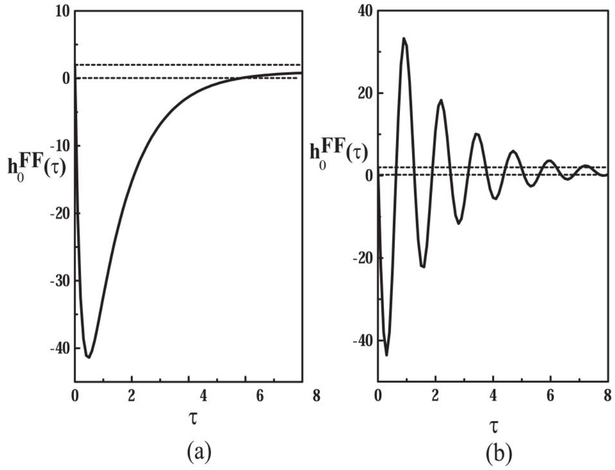

FIG. 5. $h_\theta^{FF}(\tau)$ for the fluorescent field vs $\tau=\gamma t$ for (a) $\kappa/\gamma=3.0$, $g/\gamma=1.0$, and $F/\gamma=0.1$ and (b) $\kappa/\gamma=0.5$, $g/\gamma=5.0$, and $F/\gamma=0.1$. The dashed lines indicate the range allowed for classical fields.

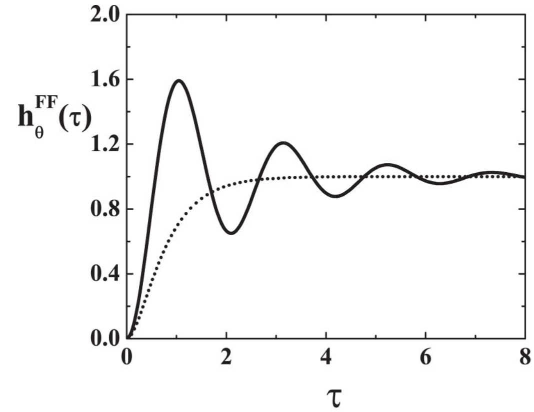

FIG. 6. $h_\theta^{FF}(\tau)$ for the fluorescent field of a two-level atom in a driven optical cavity vs $\tau=\gamma t$ for (a) $\kappa/\gamma=0.5$, $g/\gamma=3.0$, and $F/\gamma=0.1$ and (b) $\kappa/\gamma=5.0$, $g/\gamma=2.0$, and $F/\gamma=0.1$.

$$h_\theta^{FF}(\tau) = 1 + 2\frac{\langle:\Delta\hat{\sigma}_\theta(0)\Delta\hat{\sigma}_\theta(\tau):\rangle}{\langle\Delta\hat{\sigma}_+\Delta\hat{\sigma}_-\rangle}. \tag{37}$$

In terms of quantum trajectories, what is the state of the system after emission of a fluorescent photon out the side? The corresponding collapse operator is $(\sqrt{\gamma/2})\hat{\sigma}_-$. So the state of the system after the emission of a fluorescent photon is

$$|\psi_c^F\rangle = \frac{\hat{\sigma}_-|\psi_{SS}\rangle}{|\hat{\sigma}_-|\psi_{SS}\rangle|} = |g,0\rangle, \tag{38}$$

where the latter relation holds in the weak-field limit. Initially there is no entanglement between the atom and field; we have a product of atom in ground state and a one-photon Fock state for the cavity mode. However, due to the fact that there is no vacuum field contribution to this state, on a time scale of $1/g$ we find substantial entanglement. In the weak-field limit, we find that

$$h_\theta^{FF}(\tau) = 1 + C_{g,1}^{CF}(\tau) + \frac{C_{e,0}^{CF}(\tau)}{C_{e,1}^{SS}}\cos\theta \tag{39}$$

$$=2 + \frac{C_{e,0}^{CF}(\tau)}{C_{e,1}^{SS}}\cos\theta, \tag{40}$$

where *CF* means conditioned on detection in fluorescence. In Fig. 5 we have a plot of $h_\theta^{FF}(\tau)$ for weak coupling ($g/\gamma=0.1$, $g/\kappa=0.02$), with cavity decay dominant over spontaneous emission ($\kappa/\gamma=5.0$). At $\tau=0.0$, we find $h_\theta^{FF}(0)=2.0$, which is not nonclassical. It is not 0, as the offset field makes a contribution to the measured field here. Very quickly $h_\theta^{FF}(\tau)$ decreases below zero and we find a large violation of inequality (16), but only above the classically allowed region, not also below as was the case for the transmitted field. The same holds true when spontaneous emission is the dominant loss mechanism with weak coupling. Here we again find violations of the inequality (16) or order $1/F$. This does *not* occur in a driven atom-cavity system. There we have found no violations of the inequality (16). In Fig. 5(b) we have a plot of $h_\theta^{TT}(\tau)$ for strong coupling ($g/\gamma=5.0$, $g/\kappa=10.0$), with cavity decay and spontaneous emission loss rates equivalent ($\kappa/\gamma=0.5$). Again, at $\tau=0.0$, we find $h_\theta^{FF}(0)=2.0$, which is not nonclassical. At later times, we find large violations of inequality (16) from above and below. Recall that h_θ^{FF} is essentially a quadrature-field measurement of the fluorescent field given that a fluorescent photon was detected at $\tau=0$. The fluorescent field is π out of phase with the driving field, which is reflected in the initially decreasing behavior of h_θ^{FF}. Due to the presence of the offset (in phase with the local oscillator, 0° in our plots), h_θ^{FF} is really a quadrature-field measurement of $\hat{a}=\alpha+\Delta\hat{a}$, which is the sum of the offset (in phase with the LO) and the radiated dipole field (out of phase with the LO). Otherwise $h_\theta^{FF}(0)$ would be zero, as the envelope of the fluorescent field vanishes after a spontaneous emission event, as there is no net dipole.

In Fig. 6, we exhibit h_θ^{FF} for a two-level atom in a driven optical cavity in the weak field limit. Here as before [10] the driving field is resonant with the atom and cavity. Notice that there is no nonclassical behavior either for weak or strong coupling. A thorough examination of parameter space has found no nonclassical behavior in the fluorescent field conditioned on detection of a fluorescent photon for this system. The value of h_θ^{FF} at $\tau=0.0$ is 0.0 as there is no dipole to emit after emission of a fluorescent photon. So we find that in the case of h_θ^{FF} one does not necessarily have nonclassical behavior as opposed to the value of the second-order intensity correlation function $g^{(2)}(0)$ at zero delay time; this of course comes from the inability of a single atom to simultaneously fluoresce two photons. In the present case, the nonclassicality stems from the generation of strong entanglement between the atom and cavity field after the detection at $\tau=0.0$; in the ordinary cavity QED (CQED) system, after the first detection there is still a large vacuum component in the state after detection and hence there is not as much atom-cavity entanglement as in the OPO system.

One can also show that the inequality

$$h_\theta^2(\tau) \leqslant g^{(2)}(\tau) \tag{41}$$

must be satisfied if the underlying field is classical in nature. In both cases considered here (two-level atom inside an OPO) and the usual CQED system of atoms in a resonantly driven cavity, one has

$$g^{(2)}(0) = |C_{g,2}^{ss}|^2 = h_\theta^2(0). \tag{42}$$

To have nonclassical behavior in $h_\theta(0)$ one must have bunching; this makes sense as the first photon serves as a trigger, and if a second photon is not around, there is no signal. With this in hand, we can understand why one has nonclassical behavior for $h_\theta(0)$ for the transmitted field but not for the fluorescent field for the cavity QED system. In the case of the fluorescence, one *always* has perfect antibunching for one atom, and hence no nonclassical behavior in either of the two cases considered here at $\tau=0.0$. In the case of transmission, one can have bunching, and all reported nonclassical behavior in $h_\theta(0)$ to date has been in such a regime. For later times τ, we have

$$g_{TT}^{(2)}(\tau) = |C_{g,1}^{CT}(\tau)|^2 = |h_\theta^{TT}(\tau)|^2. \tag{43}$$

Recall that the second-order intensity correlation function $g^{(2)}(\tau)$ must satisfy certain inequalities if there is an underlying classical field. These are

$$g^{(2)}(0) \geqslant 1, \tag{44}$$

$$g^{(2)}(0+) \geqslant g^{(2)}(0), \tag{45}$$

$$|g^{(2)}(\tau) - 1| \leqslant |g^{(2)}(0) - 1|. \tag{46}$$

Violations of the last inequality (46) are referred to as overshoots and undershoots, respectively. In the fluorescence from the ordinary CQED system, this inequality is not violated; in particular, the second-order intensity correlation function is initially zero and rises monotonically to unity for weak fields or it can oscillate between 0 and 2 for strong coupling as shown in Fig. 6. Hence by Eq. (43) there will be no nonclassical behavior in $h_\theta^{FF}(\tau)$. On the other hand, overshoots and undershoots are common for the second-order intensity correlation function for the transmitted light in the ordinary CQED system. Hence there we can find nonclassical behavior in $h_\theta^{TT}(\tau)$. In the case of the two-level atom in an OPO, we have strong bunching in the transmitted field and not in fluorescence. But we have strong overshoots in both systems, leading to nonclassical behavior in $h_\theta^{FF}(\tau)$. The overshoots for the two-level atom in an OPO are noted by considering the square of $h_\theta^{FF}(\tau)$ as plotted in Fig. 5.

V. CROSS CORRELATIONS

In this section, we consider the measurement of a transmitted- (fluorescent-) field conditioned on the detection of a fluorescent (transmitted) photon. First we consider the fluorescent-field measurement triggered by a detection of a transmitted photon,

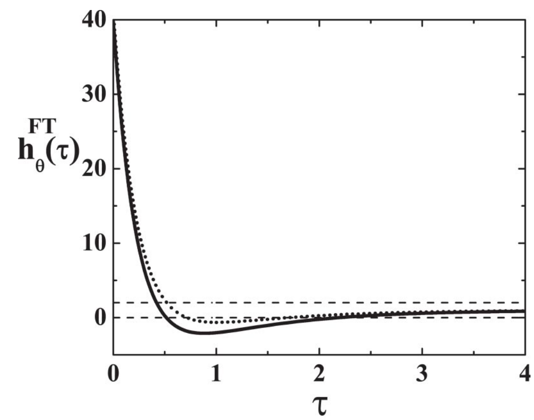

FIG. 7. $h_\theta^{FT}(\tau)$ for a two-level atom in an optical parametric oscillator vs $\tau=\gamma t$ for $\kappa/\gamma=5.0$, $g/\gamma=1.5$, and $F/\gamma=0.1$. The dashed line is $h_\theta^{TT}(\tau)$ for the same parameters.

$$h_\theta^{FT}(\tau) = 1 + C_{g,1}^{CF}(\tau) + \frac{C_{e,0}^{CF}(\tau)}{C_{e,1}^{SS}} \cos\theta, \tag{47}$$

where here the superscript *CF* refers to a conditioning on a fluorescent detection. This is easily obtained by using the solutions (23) and (24) with initial conditions given by Eq. (38). In Fig. 7, we plot $h_\theta^{TF}(\tau)$ for weak coupling. We see that the transmitted field measured by the homodyne detector is essentially the same, whether it is conditioned on a detection in fluorescence or transmission. This is due to the fact that after either type of detection, the excitation that is left in the system is either totally (fluorescent click conditioned) or mainly (transmitted click conditioned) in the cavity field. In Fig. 8, we look at the fluorescent field conditioned on either a transmitted or fluorescent click. Here we see a difference in $h_\theta(0)$; this is easily explained as the detection of a fluorescent photon places the atom in a ground state and the fluorescent field is then zero. For a transmitted photon triggering event, we do not necessarily have the atom in the ground state.

In the case of strong coupling, we do of course see vacuum-Rabi oscillations in the cross correlations, as well as a large phase shift in the fluorescent field depending on

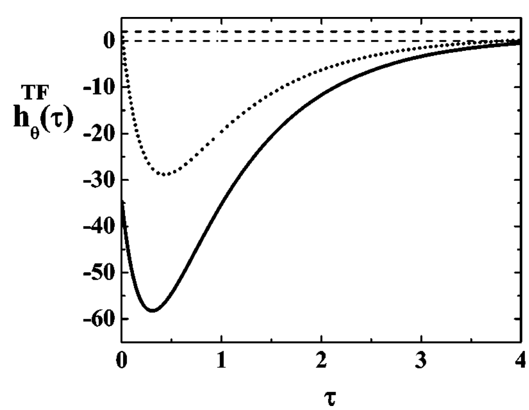

FIG. 8. $h_\theta^{TF}(\tau)$ for a two-level atom in an optical parametric oscillator vs $\tau=\gamma t$ for $\kappa/\gamma=5.0$, $g/\gamma=1.5$, and $F/\gamma=0.1$. The dashed line is $h_\theta^{FF}(\tau)$ for the same parameters.

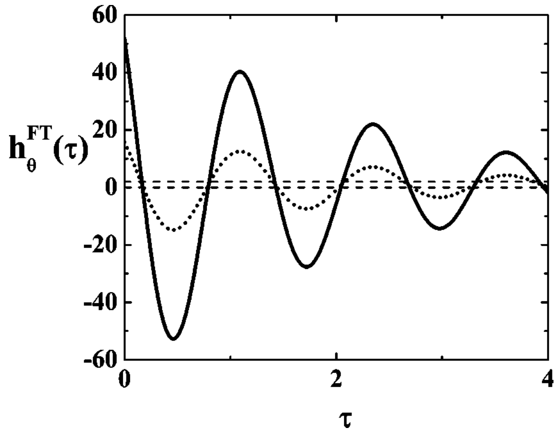

FIG. 9. $h_\theta^{FT}(\tau)$ for a two-level atom in an optical parametric oscillator vs $\tau=\gamma t$ for $\kappa/\gamma=0.5$, $g/\gamma=5.0$, and $F/\gamma=0.1$. The dashed line is $h_\theta^{TT}(\tau)$ for the same parameters.

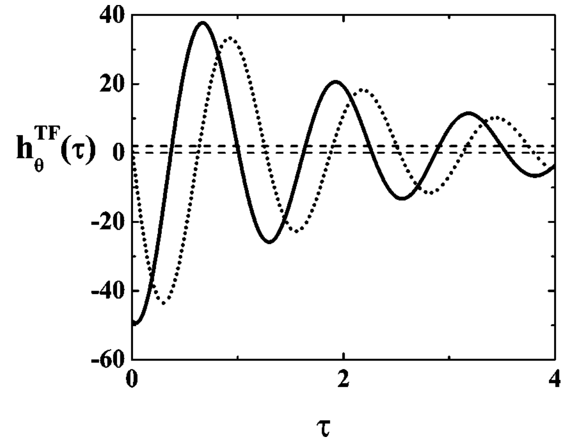

FIG. 10. $h_\theta^{TF}(\tau)$ for a two-level atom in an optical parametric oscillator vs $\tau=\gamma t$ for $\kappa/\gamma=0.5$, $g/\gamma=5.0$, and $F/\gamma=0.1$. The dashed line is $h_\theta^{FF}(\tau)$ for the same parameters.

which type of event triggered the field. The fluorescent field measured after a transmission event triggering is essentially the fluorescent field of a single atom driven by a single photon; this is because after a fluorescent click we know that there is a photon in the cavity. For the transmitted field, we see in Fig. 9 that if the trigger event is a fluorescent photon rather than a transmitted photon, we have a larger field (and larger nonclassicality). This is due to the fact that detection of a fluorescent event puts the atom in the ground state, with one photon in the cavity. For strong coupling, there is nearly an equal probability for the remaining excitation to be in the atom or field; hence, a fluorescent trigger will yield a larger field. For the fluorescent field, we see in Fig. 10 that on detection of a transmitted photon, there is a resultant dipole field; for a fluorescent trigger, that is not the case.

VI. CONCLUSIONS

We have investigated the intensity-field correlation functions for transmitted and fluorescent fields of a two-level atom in an optical parametric oscillator in the weak-field limit. In this limit we essentially have a cavity QED system where an occasional pair of photons appears in the cavity and interacts with the system. After detection of a transmitted or fluorescent photon, we know that there is one excitation left in the system. For the intensity-field correlation function, which is essentially a quadrature-field measurement conditioned on a photon detection, we have found violations of the classical inequality (16). Unlike the OPO without a two-level atom our system violates the upper *and* lower bounds over a wide range of parameters. Vacuum-Rabi oscillations appear for large Jaynes-Cummings couplings $(g>\kappa,\gamma)$. These inequalities are also violated for the fluorescent field, resulting from spontaneous emission from the atom. The inequality is violated from below only in the weak-coupling regimes and both above and below in the strong-coupling regime. We also find that for this system and the ordinary CQED system there is no nonclassical behavior in $h_\theta^{FF}(0)$. This is due to the fact that detection of a fluorescent photon puts the atom into the ground state and the system into a product of field and atomic states with no entanglement. We have also tied this to the fact that for weak fields there is a relation between nonclassical intensity-field correlations and bunching. We have further examined cross correlations—for example, the homodyned fluorescent field after detection of a fluorescent photon. This is essentially the electric field of an atom being driven from the ground state by a single photon field.

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