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Antihydrogen atom in external electric and magnetic fields

L. Labzowsky[1,2] and V. Sharipov[1]
[1]*Institute of Physics, St. Petersburg State University, 198904, Uljanovskaya 1, Petrodvorets, St. Petersburg, Russia*
[2]*Petersburg Nuclear Physics Institute, 188350, Gatchina, St. Petersburg, Russia*

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A theoretical comparison of the behavior of the antihydrogen ($\overline{\mathrm{H}}$) and hydrogen (H) atoms in external electric and magnetic fields is made. It is shown that observable differences arise in the spectroscopic properties of H and $\overline{\mathrm{H}}$ atoms in parallel electric and magnetic fields of the order of 475 V/cm and 0.12 T, respectively.

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Recent experimental success in the production of antihydrogen atoms [1–3] makes comparative spectroscopical studies of the hydrogen (H) and antihydrogen ($\overline{\mathrm{H}}$) atoms realistic in the near future. This comparison may lead to the most accurate limits for possible CPT-violating effects [4]. It is interesting also to investigate all possible differences in the spectra of H and $\overline{\mathrm{H}}$ atoms in the absence of CPT violation. These differences occur in external electric and magnetic fields. In [5,6] a frequency shift of about 10^{-4} Hz between the maxima of the frequency distributions for the Lyman-α transition in H and $\overline{\mathrm{H}}$ atoms in the external electric field 475 V/cm was predicted. This tiny effect is approximately ten orders of magnitude smaller than the recent accuracy of the Lyman-α frequency measurements [7].

Another effect, namely, quantum beats due to the coherent excitation of $2S_{1/2}$ and $2P_{1/2}$ states of H and $\overline{\mathrm{H}}$ atoms in an external electric field, was examined in [9]. It was predicted that the beat signal should contain a part which is of the opposite sign for H and $\overline{\mathrm{H}}$ atoms. However, a more accurate investigation shows that, although a difference in the fluorescence signal for H and $\overline{\mathrm{H}}$ atoms does exist at the level $(\Gamma/\Delta_L)(W^0_{2S}/W^0_{2P})^{1/2}\approx 10^{-8}$ (Γ is the width of the $2p$ state, Δ_L is the Lamb shift, and W^0_{2S}, W^0_{2P} are the one-photon transition probabilities $2S$-$1S$ and $2P$-$1S$, respectively), a beat structure of the signal is absent.

In the present paper we propose another possibility to observe the difference in H and $\overline{\mathrm{H}}$ spectra in parallel electric and magnetic fields. The Hamiltonian of the H ($\overline{\mathrm{H}}$) atom in homogeneous electric and magnetic external fields looks like

$$\hat{H}=\hat{H}_0-\hat{\vec{d}}\cdot\vec{\varepsilon}-\hat{\vec{\mu}}\cdot\vec{\mathcal{H}}, \qquad (1)$$

where $\hat{H}_0$ is the Hamiltonian in the absence of the external fields, $\hat{\vec{d}}=\pm|e|\vec{r}$, $\hat{\vec{d}}$ is the electric dipole moment operator for the positron (electron) in an atom, $\vec{r}$ is the radius vector of the electron (positron), $|e|$ is the absolute value of the electron (positron) charge, $\hat{\vec{\mu}}=-\mu_0(\hat{\vec{L}}+2\hat{\vec{S}})$, $\mu_0=\pm|e|\hbar/2m_{lep}c$ is the Bohr magneton, m_{lep} is the electron (positron) mass, $\hat{\vec{L}}$ and $\hat{\vec{S}}$ are the orbital and spin angular momentum operators for the electron (positron), and $\vec{\varepsilon}$ and $\vec{\mathcal{H}}$ are the field strengths for the external electric and magnetic fields, respectively. In Eq. (1) we neglected the terms quantic in the magnetic field.

The Hamiltonian $\hat{H}_0$ is charge invariant; the total Hamiltonian (1) is evidently charge noninvariant. This can lead to various differences in the spectra of the H and $\overline{\mathrm{H}}$ atoms in the external fields. Below we will investigate these differences.

Differences in the energy levels of H and $\overline{\mathrm{H}}$ atoms in an external electric field are absent due to the absence of the linear Stark effect in atoms. The latter is prohibited by the combined parity (*CP*) conservation law. The known "linear" Stark effect in H atoms is actually proportional to $|e|$ and thus is the same for H and $\overline{\mathrm{H}}$ atoms. This "linear" effect arises due to the neglect of the Lamb shift Δ_L compared to the Stark matrix element $S=|\langle 2P|\vec{d}\cdot\vec{\varepsilon}|2S\rangle|$.

Another situation occurs in an external magnetic field, where Zeeman splitting proportional to μ_B does exist. The splitting picture is the same for H and $\overline{\mathrm{H}}$ atoms but the upper and lower Zeeman components correspond to different values of the electron angular momentum projections as shown in Fig. 1 (e.g., $m_s=+1/2$ for H and $m_s=-1/2$ for $\overline{\mathrm{H}}$ atoms in the case of the ground state). If the atoms are polarized and the polarization is the same for H and $\overline{\mathrm{H}}$ atoms, only the upper Zeeman sublevel will be populated in H and the lower sublevel will be populated in $\overline{\mathrm{H}}$, or vice versa. Then the ionization potentials will be different for H and $\overline{\mathrm{H}}$ atoms with

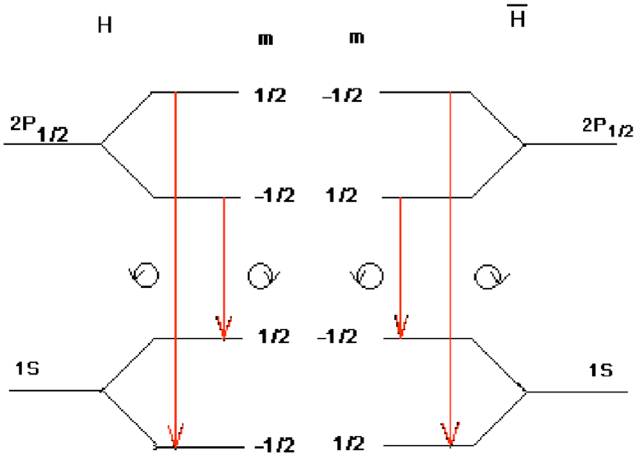

FIG. 1. The scheme of the $2P_{1/2}$-$1S$ transitions for H and $\overline{\mathrm{H}}$ atoms in an external magnetic field. It is assumed that the direction of the photon emission coincides with the direction of the magnetic field. The hyperfine structure of the levels is neglected. The circles with the right (left) arrows denote the right (left) circular polarization of the emitted photons.

the same polarization in the same external magnetic field. This difference even in the largest achievable field ~1 T is of relative order 10^{-5} and, probably, hardly observable. The ionization of the $\bar{\mathrm{H}}$ atom in a strong magnetic field was considered in [8] but the possibility to distinguish between H and $\bar{\mathrm{H}}$ atoms was not discussed in this work.

Consider now the transition probability for the Lyman-α $2P_{1/2}$-$1S$ transition in an external magnetic field. In what follows we will assume that only the circularly polarized photons are registered in the direction $\vec{n}$ parallel to the direction of the magnetic field. Then the transition $2P_{1/2}(m=1/2)\to 1S_{1/2}(m=-1/2)$ in the H atom provides right-handed photons and the transition $2P_{1/2}(m=-1/2)\to 1S_{1/2}(m=1/2)$ provides left-handed photons (see Fig. 1). The opposite situation will occur for the $\bar{\mathrm{H}}$ atom. Under these conditions an observer will see two peaks which would have only one difference: the low-frequency component of the doublet in the case of the H atom will be right handed and the high-frequency component will be left handed. An opposite picture will occur for the $\bar{\mathrm{H}}$ atom. This is the simplest spectroscopical difference between the H and $\bar{\mathrm{H}}$ atoms. The possibility to observe this effect depends only on the number of $\bar{\mathrm{H}}$ atoms available and the possibility to resolve the Zeeman structure. A more complicated situation arises when we consider the transitions $2P'_{1/2}\to 1S$ and $2S'\to 1S$ in external electric and magnetic fields. For simplicity we will assume these fields to be parallel. This situation was investigated thoroughly in [10] for the H atom. An expression for the probability W_{2S} in parallel electric and magnetic fields which follows from [10] looks like

$$W^{\pm}_{2S}=W^{0}_{2S}+S^{2}W^{0}_{2P}[A^{\pm}+B^{\pm}(\vec{s}\cdot\vec{h})]\pm S(W^{0}_{2S}W^{0}_{2P})^{1/2}\times[L^{\pm}+K^{\pm}(\vec{s}\cdot\vec{h})]. \tag{2}$$

In Eq. (2) $\vec{s}=i(\vec{e}\times\vec{e}^{*})$ is the photon spin, and $\vec{h}=\vec{\mathcal{H}}/\mathcal{H}$. Note that since $\vec{\mathcal{H}}$ is parallel to $\vec{n}$, the scalar product $(\vec{s}\cdot\vec{h})$ defines actually $(\vec{s}\cdot\vec{n})$, i.e., the helicity number $(\vec{s}\cdot\vec{n})=\pm 1$. The helicity values ±1 correspond to the right (left) circular polarizations. Thus Eq. (2) with $(\vec{s}\cdot\vec{n})=\pm 1$ describes the transition rates for two different transitions, with different frequencies and different kinds of emitted photons. The indices ± refer to the H and $\bar{\mathrm{H}}$ atoms. The coefficients $A^{\pm}$, $B^{\pm}$, $K^{\pm}$, $L^{\pm}$ are

$$A^{\pm}=\frac{1}{2}\left(\frac{1}{|\varepsilon^{\pm}(\mathcal{H})|^{2}}+\frac{1}{|\varepsilon^{\pm}(-\mathcal{H})|^{2}}\right), \tag{3}$$

$$B^{\pm}=\frac{1}{2}\left(\frac{1}{|\varepsilon^{\pm}(\mathcal{H})|^{2}}-\frac{1}{|\varepsilon^{\pm}(-\mathcal{H})|^{2}}\right), \tag{4}$$

$$K^{\pm}=-Im\left(\frac{1}{\varepsilon^{\pm}(\mathcal{H})}-\frac{1}{\varepsilon^{\pm}(-\mathcal{H})}\right), \tag{5}$$

$$L^{\pm}=-Im\left(\frac{1}{\varepsilon^{\pm}(\mathcal{H})}+\frac{1}{\varepsilon^{\pm}(-\mathcal{H})}\right), \tag{6}$$

where

$$\varepsilon^{\pm}(\mathcal{H})=\Delta_{L}+\frac{i}{2}\Gamma\pm\frac{2}{3}\mu_{0}\mathcal{H}. \tag{7}$$

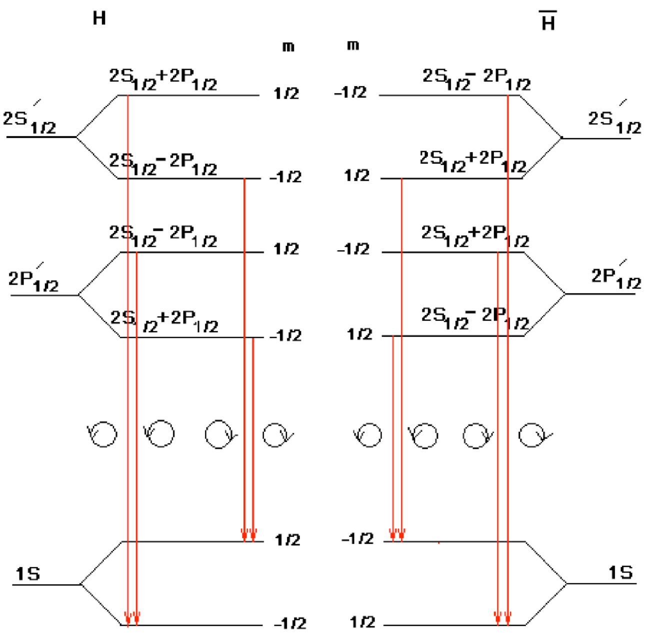

FIG. 2. The scheme of the $2P'_{1/2}$-$1S$ and $2S'_{1/2}$-$1S$ transitions for H and $\bar{\mathrm{H}}$ atoms in external parallel electric and magnetic fields. The direction of the photon emission is also parallel to both fields. For the field strength $\varepsilon=475$ V/cm the $2S'_{1/2}$-$1S$ transition is as strong as the $2P'_{1/2}$-$1S$ transition. The notation is the same as in Fig. 1.

In Eq. (7) compared to [10] we neglect for simplicity the contribution quadratic in the magnetic field and the hyperfine structure contribution. The most interesting situation occurs when the Zeeman sublevels $2S_{1/2}(m=-1/2)$ and $2P_{1/2}(m=-1/2)$ cross in the magnetic field 0.12 T [11]. This crossing occurs due to the difference in g factors of $2S$ and $2P$ electrons.

From Eqs. (3)–(6) it follows that $A^{+}=A^{-}$, $B^{+}=-B^{-}$, $L^{+}=L^{-}$, $K^{+}=K^{-}$. In the electric field 475 V/cm the coefficients A,B in Eq. (2) result in terms of the order of W^{0}_{2P}, while the other coefficients lead to small corrections of the relative order $\sim 10^{-8}$ (see the estimate given above).

The scheme of the levels is shown in Fig. 2. We use the notations $2P'_{1/2},2S'_{1/2}$ for the states in the external electric field, which contain actually a 50% mixture of the pure $2P_{1/2},2S_{1/2}$ states and can be distinguished only by their origin from pure $2P_{1/2},2S_{1/2}$ states. The Lyman-α transition $2P'_{1/2}\to 1S$ is very weakly affected by the electric field and in external electric and magnetic fields exhibits the standard Zeeman structure. For the $2S'_{1/2}\to 1S$ transition the high-frequency components [i.e., $2S'_{1/2}(m=1/2)\to 1S_{1/2}(m=-1/2)$ for the H atom and $2S'_{1/2}(m=-1/2)\to 1S_{1/2}(m=1/2)$ for the $\bar{\mathrm{H}}$ atom] are enhanced compared to the Lyman-α components due to the cancellation of terms $\Delta_{L}-2/3\mu_{0}\mathcal{H}$ in the energy denominators $\varepsilon^{\pm}(\mathcal{H})$. This enhancement occurs in the magnetic field $\mathcal{H}=0.12$ T and is proportional to $(\Delta_{L}/\Gamma)^{2}$. The low-frequency components are suppressed by a factor of 4. As in the case of pure magnetic field, the H and $\bar{\mathrm{H}}$ atoms in external magnetic and electric fields differ spectroscopically only if a light polarization analysis is performed. The transition rates and the radiation intensity distributions are the same. This holds, however, only when we neglect the coefficients $K^{\pm}, L^{\pm}$ in Eq. (2).

The inclusion of these terms leads to differences in transition rates for H and $\overline{\text{H}}$ atoms, though the maximum relative order of these differences achieved at the field values $\varepsilon = 475$ V/cm and $\mathcal{H} = 0.12$ T is $(W^0_{2S}/W^0_{2P})^{1/2} \approx 10^{-7}$. In the absence of the magnetic field Eq. (2) takes the form [10]

$$W^{\pm}_{2S} = W^0_{2S} \pm S(W^0_{2S}W^0_{2P})^{1/2} \frac{\Gamma}{\Delta_L^2 + \frac{1}{4}\Gamma^2} (\vec{n} \cdot \vec{f}) + S^2 W^0_{2P} \frac{1}{\Delta_L^2 + \frac{1}{4}\Gamma^2}, \tag{8}$$

where $\vec{f} = \vec{\varepsilon}/\varepsilon$. Similar formulas were obtained in [12,13]. Due to the second term in Eq. (8) the transition rates $2S$-$1S$ are different for H and $\overline{\text{H}}$ atoms. However, in the field 475 V/cm this term is small compared to the third term in Eq. (8). This smallness is again of the order $(\Gamma/\Delta_L)(W^0_{2S}/W^0_{2P})^{1/2} \approx 10^{-8}$. An expression analogous to Eq. (8) can be written also for $W^{\pm}_{2P}$ transitions in an electric field [5,6]. In this expression the term different for H and $\overline{\text{H}}$ atoms has the same smallness as in Eq. (8). The relative shift of the maxima of the radiation intensity distributions in H and $\overline{\text{H}}$ atoms, considered in [5,6], also originates from the second term in Eq. (8).

Summarizing, we have shown that the H and $\overline{\text{H}}$ atoms can be distinguished spectroscopically in external magnetic and electric fields. In the case of a magnetic field, provided that a radiation polarization analysis for $\overline{\text{H}}$ becomes available, the observation of the differences looks quite feasible: it requires only the resolution of Zeeman components for the Lyman-α transition and the polarization analysis of these components. In parallel magnetic and electric fields the difference is also detectable. In addition to the Zeeman components of the $2P'_{1/2}$-$1S$ transition different polarization occurs for the Zeeman components of the $2S'$-$1S$ transition.

Transition probabilities for the H and $\overline{\text{H}}$ atoms differ also in a pure electric field. In this case the transition rates are different, but the observation of these differences seems unrealistic at the moment. The total width of the $2P_{1/2}$ state is about 100 MHz and it is weakly affected by an external electric field. The natural line width of the Lyman-α $2P_{1/2}$-$1S_{1/2}$ transition was measured in [7]. The accuracy of this measurement does not exceed ± 6 MHz, i.e., the relative accuracy is about 5×10^{-2}. This should be compared with a relative difference of 10^{-7} in the transition rates for the H and $\overline{\text{H}}$ atoms. The same concerns the observation of the differences in the ionization potentials of polarized H and $\overline{\text{H}}$ atoms: these differences also look undetectable with the existing techniques.

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