

Stabilizing quantum metastable states in a time-periodic potential

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Metastability of a particle trapped in a well with a time-periodically oscillating barrier is studied in the Floquet formalism. It is shown that the oscillating barrier causes the system to decay faster in general. However, avoided crossings of metastable states can occur with the less stable states crossing over to the more stable ones. If in the static well there exists a bound state, then it is possible to stabilize a metastable state by adiabatically increasing the oscillating frequency of the barrier so that the unstable state will eventually cross over to the stable bound state. It is also found that increasing the amplitude of the oscillating field may change a direct crossing of states into an avoided one.

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I. INTRODUCTION

Ever since the advent of quantum mechanics, quantum tunneling has been an important and fascinating subject. This phenomenon arises frequently in physics. In fact, one of the first successful applications of quantum mechanics has been the explanation of the α decay of atoms as a quantum tunneling process [1]. Recent examples include tunneling phenomena in semiconductors and superconductors [2], in Josephson junction systems [3], resonant tunneling in heterojunction nanostructures [4], tunneling ionization of atoms [5], photon-assisted tunneling in superconducting junctions and semiconductor superlattices [6], etc.

In cosmology, quantum metastable states play an essential role in some versions of the inflationary models of the early universe [7]. In these models inflation of the early universe is governed by a Higgs field trapped in a metastable state. Inflation ends when the metastable state decays to the true ground state of the universe. During inflation the universe expands exponentially. It is thus obvious that the metastable state of the Higgs field is trapped in a rapidly varying potential. The problem is therefore a truly time-dependent one. Unfortunately, owing to the inherent difficulties of the problem, more often than not one has to consider the decay of the Higgs field in a quasistationary approximation, in which the decay is studied assuming a static potential [8]. Surely this approximation is hard to justify, but for the present one has to be content with it. Ultimately one hopes to be able to tackle the nonstationary case. To this end, it is desirable to gain some insights first by studying metastability in time-dependent potential in simple quantum-mechanical models.

An early attempt at studying the effects of time-varying forces on quantum metastability appears in Fisher's work [9], which was motivated by an experiment on quantum tunneling of the phase in a current-biased Josephson junction with a weak microwave perturbation [10]. In this work Fisher considered the general problem of quantum tunneling in a metastable well with a weak oscillatory force. There he reformulated the standard WKB approach to quantum decay in order to include a weak time-dependent perturbation. For a class of metastable potentials which interpolates between the cubic potential and a truncated harmonic-oscillator potential, he showed that the decay rate is generally enhanced

by the weak oscillatory force. The potential considered by Fisher has a number of oscillatorlike levels near its minimum. The opposite situation where only two levels are present was considered by Sokolovski [11], who studied the effect of a small ac field mixing two levels in the well on the tunneling rate in a semiclassical framework.

The results in [9,11] are quite general for a class of weak oscillatory forces. However, it is desirable to consider other possibilities, e.g., exact solutions and/or nonperturbative results. In this respect, we had considered previously [12] an exactly solvable quantum metastable system with a moving potential which has height and width scaled in a specific way introduced by Berry and Klein [13]. We found that in this model a small but finite nondecay probability could persist at large time limit for an expanding potential.

In this paper we consider another simple driven quantum metastable model in which a particle is trapped in a well with a periodically driven rectangular barrier. In order to do away with any restriction of amplitude or frequency of the driving force, and of the number of states in the potential, we treat the problem in the framework of the Floquet formalism [14–17]. An exact expression determining the Floquet quasienergies of stable or metastable states in the well is derived. From the solution of this equation we find that while the oscillating barrier makes the system decay faster in general, there is the possibility that avoided crossings of metastable states can occur with the less stable states crossing over to the more stable ones.

That an oscillating potential can affect the tunneling property of a system has also been noticed before, e.g., in quantum transport process [16–19]. In [17,18] it was found that a particle can be localized in one side of a time-dependent double well if the amplitude and the frequency of the oscillating field were chosen properly. In [16] it was demonstrated that a propagating particle at appropriate incident energy can be trapped into a bound state by an oscillating square well. Our example shows how a time-periodic field can modify the metastability of a decaying state.

II. THE MODEL

The model we consider consists of a particle of mass m trapped in a square well with a harmonically oscillating barrier,

$$V(x,t) = \begin{cases} \infty, & x < 0, \\ 0, & 0 \leq x < a, \\ V_0 + V_1 \cos(\omega t), & a \leq x \leq b, \\ V_0', & x > b. \end{cases} \quad (1)$$

Here V_0 , V_1 , V_0' and ω are positive parameters, with $V_0' < V_0$ and $V_1 < V_0 - V_0'$. According to the Floquet theorem, the wave function of a time-periodic system has the form $\Psi_\varepsilon(x,t) = e^{-i\varepsilon t/\hbar} \Phi_\varepsilon(x,t)$, where $\Phi_\varepsilon(x,t)$ is a periodic function with the period $T = 2\pi/\omega$, i.e., $\Phi_\varepsilon(x,t+T) = \Phi_\varepsilon(x,t)$, and ε is the Floquet quasienergy, which we will call Floquet energy

for brevity. It should be noted that the Floquet energy is determined only modulo $\hbar\omega$. For if $\{\varepsilon, \Phi_\varepsilon\}$ is a solution of the Schrödinger equation, then $\{\varepsilon' = \varepsilon + n\hbar\omega, \Phi_{\varepsilon'} = \Phi_\varepsilon \exp(in\omega t)\}$ is also a solution for any integer n . But they are physically equivalent as the total wave function Ψ_ε is the same [15]. All physically inequivalent states can be characterized by their reduced Floquet energies in a zone with a width $\hbar\omega$. We therefore consider solutions of ε only in the first Floquet zone, i.e., $\varepsilon \in [0, \hbar\omega)$.

Following the procedures described in [16] (see also [19]), we get the wave function as follows:

$$\Psi(x,t) = e^{-i\varepsilon t/\hbar} \Phi_\varepsilon(x,t) = e^{-i\varepsilon t/\hbar} \begin{cases} \sum_{n=-\infty}^{\infty} A_n \sin(k_n x) e^{-in\omega t}, & 0 \leq x < a, \\ \sum_{n=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} (a_l e^{q_l x} + b_l e^{-q_l x}) J_{n-l}(V_1/\hbar\omega) e^{-in\omega t}, & a \leq x \leq b, \\ \sum_{n=-\infty}^{\infty} t_n e^{ik'_n x} e^{-in\omega t}, & x > b, \end{cases} \quad (2)$$

where

$$\begin{aligned} k_n &= \sqrt{2m(\varepsilon + n\hbar\omega)}/\hbar, \\ q_l &= \sqrt{2m(V_0 - \varepsilon - l\hbar\omega)}/\hbar, \\ k'_n &= \sqrt{2m(\varepsilon + n\hbar\omega - V_0')}/\hbar, \end{aligned} \quad (3)$$

and J_n is the Bessel function. In the region $x > b$, we have adopted Gamow's outgoing boundary condition, namely, there is no particle approaching the barrier from the right [1]. Matching the wave function and its first derivative at the boundaries $x=a$ and $x=b$, we obtain the relations among the coefficients A_n , a_n , b_n and t_n :

$$\begin{aligned} A_n \sin(k_n a) &= \sum_l (a_l e^{q_l a} + b_l e^{-q_l a}) J_{n-l}(\alpha), \\ k_n A_n \cos(k_n a) &= \sum_l q_l (a_l e^{q_l a} - b_l e^{-q_l a}) J_{n-l}(\alpha), \\ t_n e^{ik'_n b} &= \sum_l (a_l e^{q_l b} + b_l e^{-q_l b}) J_{n-l}(\alpha), \\ ik'_n t_n e^{ik'_n b} &= \sum_l q_l (a_l e^{q_l b} - b_l e^{-q_l b}) J_{n-l}(\alpha), \end{aligned} \quad (4)$$

where $\alpha \equiv V_1/\hbar\omega$. The Floquet energy is determined from these relations by demanding nontrivial solutions of the coefficients. In practice, however, we must truncate the above equations to a finite number of terms, or sidebands as they are usually called in the literature, e.g., $n=0, \pm 1, \dots, \pm N$.

The number N is determined by the frequency and the strength of the oscillation as $N > V_1/\hbar\omega$ [16].

We proceed to determine the Floquet energy as follows. We first separate the boundary conditions for the central band ($n=0$) from those for the subbands ($n \neq 0$) in Eq. (4). From the boundary conditions for the subbands ($n \neq 0$), one can relate the coefficients a_l and b_l ($l \neq 0$) with the coefficients a_0 and b_0 through the following two relations:

$$a_l = f_{la}(k_0, k'_0, \omega, V_1) a_0 + f_{lb}(k_0, k'_0, \omega, V_1) b_0, \quad (5)$$

$$b_l = g_{la}(k_0, k'_0, \omega, V_1) a_0 + g_{lb}(k_0, k'_0, \omega, V_1) b_0, \quad (6)$$

where f 's and g 's are functions determined as follows. Eliminating the A_n 's and t_n 's in Eq. (4), we can obtain

$$\begin{aligned} A_{n,n}^- e^{q_n a} J_0 a_n + A_{n,n}^+ e^{-q_n a} J_0 b_n + \sum_{l \neq n, 0} A_{n,l}^- e^{q_l a} J_{n-l} a_l \\ + \sum_{l \neq n, 0} A_{n,l}^+ e^{-q_l a} J_{n-l} b_l = -A_{n,0}^- e^{q_0 a} J_n a_0 - A_{n,0}^+ e^{-q_0 a} J_n b_0 \end{aligned} \quad (7)$$

and

$$\begin{aligned} B_{n,n}^+ e^{q_n b} J_0 a_n + B_{n,n}^- e^{-q_n b} J_0 b_n + \sum_{l \neq n, 0} B_{n,l}^+ e^{q_l b} J_{n-l} a_l \\ + \sum_{l \neq n, 0} B_{n,l}^- e^{-q_l b} J_{n-l} b_l = -B_{n,0}^+ e^{q_0 b} J_n a_0 - B_{n,0}^- e^{-q_0 b} J_n b_0, \end{aligned} \quad (8)$$

with

$$A_{n,l}^{\pm} \equiv \cos k_n a \pm \frac{q_l}{k_n} \sin k_n a, \quad B_{n,l}^{\pm} \equiv 1 \pm i \frac{q_l}{k_n}. \quad (9)$$

Equations (7) and (8) allow us to solve for a_l and b_l in terms of a_0 and b_0 in the forms of Eqs. (5) and (6) by means of the Cramer's rule in matrix algebra. As mentioned before, in practice a truncated version of Eqs. (7) and (8) has to be used.

Using Eqs. (5) and (6) we can rewrite the boundary conditions for the central band $n=0$ as

$$A_0 \sin(k_0 a) = F_1(k_0, k'_0; \omega, V_1) e^{q_0 a} a_0 + F_2(k_0, k'_0; \omega, V_1) e^{-q_0 a} b_0,$$

$$\begin{aligned} k_0 A_0 \cos(k_0 a) &= F_3(k_0, k'_0; \omega, V_1) q_0 e^{q_0 a} a_0 \\ &\quad - F_4(k_0, k'_0; \omega, V_1) q_0 e^{-q_0 a} b_0, \end{aligned} \quad (10)$$

$$t_0 e^{ik'_0 b} = F_5(k_0, k'_0; \omega, V_1) e^{q_0 b} a_0 + F_6(k_0, k'_0; \omega, V_1) e^{-q_0 b} b_0,$$

$$\begin{aligned} ik'_0 t_0 e^{ik'_0 b} &= F_7(k_0, k'_0; \omega, V_1) q_0 e^{q_0 b} a_0 \\ &\quad - F_8(k_0, k'_0; \omega, V_1) q_0 e^{-q_0 b} b_0, \end{aligned}$$

where the coefficients $F_i(k_0; \omega, V_1)$ ($i=1, \dots, 8$) are

$$F_1(k_0, k'_0; \omega, V_1) = J_0(\alpha) + e^{-q_0 a} \sum_{l \neq 0} (f_{1a} e^{q_l a} + g_{1a} e^{-q_l a}) J_{-l}(\alpha),$$

$$F_2(k_0, k'_0; \omega, V_1) = J_0(\alpha) + e^{q_0 a} \sum_{l \neq 0} (f_{1b} e^{q_l a} + g_{1b} e^{-q_l a}) J_{-l}(\alpha),$$

$$F_3(k_0, k'_0; \omega, V_1) = J_0(\alpha) + e^{-q_0 a} \sum_{l \neq 0} \frac{q_l}{q_0} (f_{1a} e^{q_l a} - g_{1a} e^{-q_l a}) J_{-l}(\alpha),$$

$$F_4(k_0, k'_0; \omega, V_1) = J_0(\alpha) - e^{q_0 a} \sum_{l \neq 0} \frac{q_l}{q_0} (f_{1b} e^{q_l a} - g_{1b} e^{-q_l a}) J_{-l}(\alpha),$$

$$F_5(k_0, k'_0; \omega, V_1) = J_0(\alpha) + e^{-q_0 b} \sum_{l \neq 0} (f_{1a} e^{q_l b} + g_{1a} e^{-q_l b}) J_{-l}(\alpha),$$

$$F_6(k_0, k'_0; \omega, V_1) = J_0(\alpha) + e^{q_0 b} \sum_{l \neq 0} (f_{1b} e^{q_l b} + g_{1b} e^{-q_l b}) J_{-l}(\alpha),$$

$$F_7(k_0, k'_0; \omega, V_1) = J_0(\alpha) + e^{-q_0 b} \sum_{l \neq 0} \frac{q_l}{q_0} (f_{1a} e^{q_l b} - g_{1a} e^{-q_l b}) J_{-l}(\alpha),$$

$$F_8(k_0, k'_0; \omega, V_1) = J_0(\alpha) - e^{q_0 b} \sum_{l \neq 0} \frac{q_l}{q_0} (f_{1b} e^{q_l b} - g_{1b} e^{-q_l b}) J_{-l}(\alpha).$$

By demanding nontrivial solutions of the coefficients a_0 , b_0 , A_0 , and t_0 in Eq. (10), we obtain an equation which determines the Floquet energy ε :

$$\begin{aligned} F_4 \frac{q_0}{k_0} \tan k_0 a + F_2 &= \frac{F_8 q_0 + i F_6 k'_0}{F_7 q_0 - i F_5 k'_0} \left(F_3 \frac{q_0}{k_0} \tan k_0 a - F_1 \right) \\ &\quad \times e^{-2q_0(b-a)}. \end{aligned} \quad (11)$$

We recall here that k_0 , q_0 , k'_0 are functions of the Floquet

energy ε [cf. Eq. (3)]. If the solutions ε of Eq. (11) are complex (real) numbers, the corresponding Floquet states are metastable (stable) states. The nondecay probability $P(t)$, which is the probability of the particle still being trapped by the potential barrier at time $t > 0$, is given by

$$\begin{aligned} P(t) &= \frac{\int_0^b |\Psi(x, t)|^2 dx}{\int_0^b |\Psi(x, 0)|^2 dx} = e^{2 \operatorname{Im}(\varepsilon)t/\hbar} \frac{\int_0^b |\Phi_\varepsilon(x, t)|^2 dx}{\int_0^b |\Phi_\varepsilon(x, 0)|^2 dx} \\ &\equiv e^{2 \operatorname{Im}(\varepsilon)t/\hbar} h(t), \end{aligned} \quad (12)$$

with $P(0)=1$. The imaginary part of the Floquet energy, which enters $P(t)$ via the factor $\exp[2 \operatorname{Im}(\varepsilon)t/\hbar]$, gives a measure of the stability of the system. Unlike the static case, however, here $P(t)$ is not a monotonic function of time, owing to the time-dependent function $h(t)$ after the exponential factor in Eq. (12). But since $h(t)$ is only a periodic function oscillating between two values which are of order one, the essential behavior of $P(t)$ at large times is still mainly governed by the exponential factor. Hence, as a useful measure of the nondecay rate of the particle in the well, we propose a coarse-grained nondecay probability $\bar{P}(t)$ defined as

$$\bar{P}(t) \equiv e^{2 \operatorname{Im}(\varepsilon)t/\hbar} \langle h(t) \rangle, \quad (13)$$

where $\langle h(t) \rangle$ is the time average of $h(t)$ over one period of oscillation. The graphs of $P(t)$ and $\bar{P}(t)$ will be given in the next section.

It is easily seen that the coefficients $F_i(k_0; \omega, V_1)$ all approach one in the limit $\alpha = V_1/\hbar\omega \rightarrow 0$,

$$\lim_{V_1/\omega \rightarrow 0} F_i(k_0; \omega, V_1) \rightarrow 1, \quad i = 1, \dots, 8. \quad (14)$$

Hence in the limit $V_1 \rightarrow 0$ or $\omega \rightarrow \infty$, Eq. (11) reduces to the corresponding equation for the case of static potential with potential V_0 in the region $a \leq x \leq b$, and the Floquet energy in this limit is just the (real or complex) eigenenergy of the static case. This is understandable, since in the limit $V_1 \rightarrow 0$ the potential becomes static, and at high frequencies the particle in the well will only see a time-averaged barrier of effective height V_0 [19].

III. NUMERICAL RESULTS

We now study numerical solutions of Eq. (11) with a specific potential. We take $a=1$, $b=2$, $V_0=15$, and $V'_0=V_0/2$ in the atomic units (a.u.) ($e=m_e=\hbar=1$). In the static case this potential supports one bound state, with energy $E_0/V_0=0.232123$, and one metastable state, with complex energy $E_1/V_0=0.864945-0.00255261i$. For the oscillating potential, we solve Eq. (11) in 2-sideband approximation, i.e., we take $N=2$. This is accurate enough for oscillating frequency $\omega \geq V_1/2$.

Figures 1 and 2 present the graphs of the real and imaginary parts of the Floquet energy (ε/V_0) as a function of $\omega/V_0 \geq 0.2$ with $V_1=0.1V_0$ and $0.2V_0$, respectively. We find that the solutions of Eq. (11) have the form $\varepsilon = \varepsilon_0 + n\omega$ (n

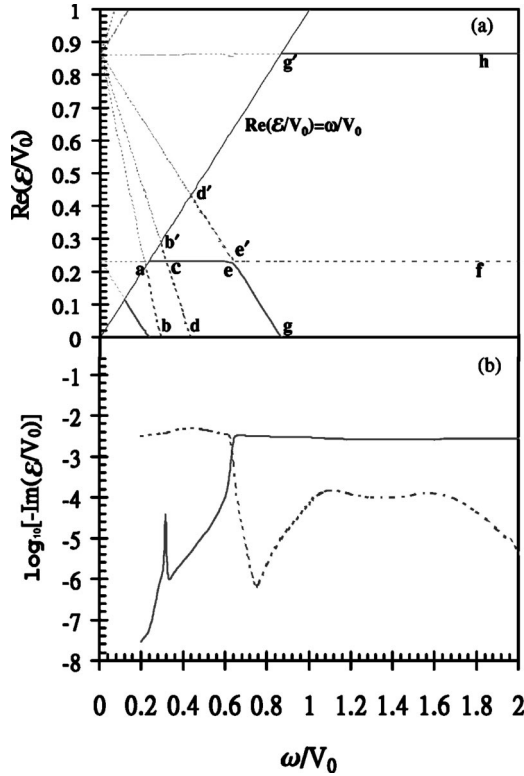


FIG. 1. The Floquet energies (ϵ/V_0) of the two metastable states versus the barrier oscillating frequency (ω/V_0) for $V_0=15$ a.u., $V'_0=V_0/2$, and $V_1/V_0=0.1$ in the atomic units (a.u.) ($e=m_e=\hbar=1$). In (a) the real parts of the Floquet energies are shown in the first Floquet zone under the line $\text{Re}(\epsilon)=\omega$ (the straight line). The light dotted lines show how the different branches emanate from the two states in the static case [with $E_0/V_0=0.232123$ and $\text{Re}(E_1)/V_0=0.864945$]. In (b) the corresponding imaginary parts of the Floquet energies of the two states are plotted. The dotted curve corresponds to the state with real parts given along the path $abb'cdd'e'f$, and the solid curve corresponds to the state with real parts given along $cegg'h$.

$=0, \pm 1, \pm 2, \dots$), with $\text{Re}(\epsilon_0)$ (the horizontal branch) laying close to the energies E_0 and $\text{Re}(E_1)$ in the static potential. That is, these branches of $\text{Re}(\epsilon)$ emanate from either E_0 or $\text{Re}(E_1)$ at $\omega=0$. Branches emerging from the same point have the same imaginary part. Numerical results show that, with the barrier oscillating, the stable state (E_0) in the static case becomes unstable, and the unstable state (E_1) will decay even faster. For simplicity, in Figs. 1(a) and 2(a) we show only six branches ($n=0, \pm 1, \pm 2$ and -3) emerging from $\text{Re}(E_1)$, and only the central branch ($n=0$) and a subband ($n=-1$) from E_0 . As mentioned before, we only take solutions in the first Floquet zone, $\text{Re}(\epsilon)$ (modulo ω), which are points under the line $\text{Re}(\epsilon)=\omega$.

In Fig. 3 we give the graphs of the probability density $|\Psi|^2$ in the well and the barrier with the same parameters as in Fig. 2 for the two metastable states at frequency $\omega/V_0=0.62$. The Floquet energies of the less stable and the more stable state are $\epsilon/V_0=0.251714-0.004995i$ and $0.227343-0.001456i$, respectively. Four time frames, namely, $\tau \equiv t \times V_0=0, 100, 200$ and 300 a.u., are shown, with the prob-

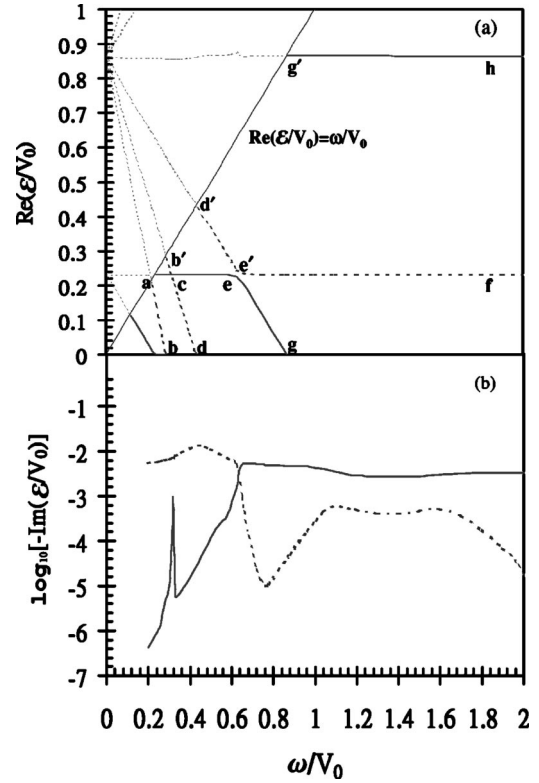


FIG. 2. Same plot as Fig. 1 for $V_0=15$ a.u., $V'_0=V_0/2$, and $V_1/V_0=0.2$.

ability density normalized to unity within $0 \leq x \leq 2$ at $\tau=0$. One can clearly see that the less stable state (dashed curve) decays much faster than the more stable state (solid curve). The nondecay probability (oscillatory curve) $P(t)$, Eq. (12), and the coarse-grained nondecay probability (monotonic curve) $\bar{P}(t)$, Eq. (13), of these two states are shown in Fig. 4. It is clear that the coarse-grained function $\bar{P}(t)$ is a mono-

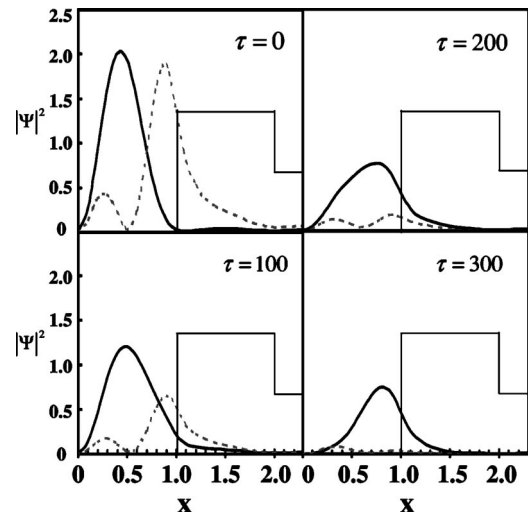


FIG. 3. Probability density $|\Psi|^2$ in the well and the barrier with the same parameters as in Fig. 2 for the two metastable states at frequency $\omega/V_0=0.62$. Probability density normalized to unity within $0 \leq x \leq 2$ at $\tau \equiv tV_0=0$ a.u.. The more (less) stable state is indicated by solid (dashed) curve.

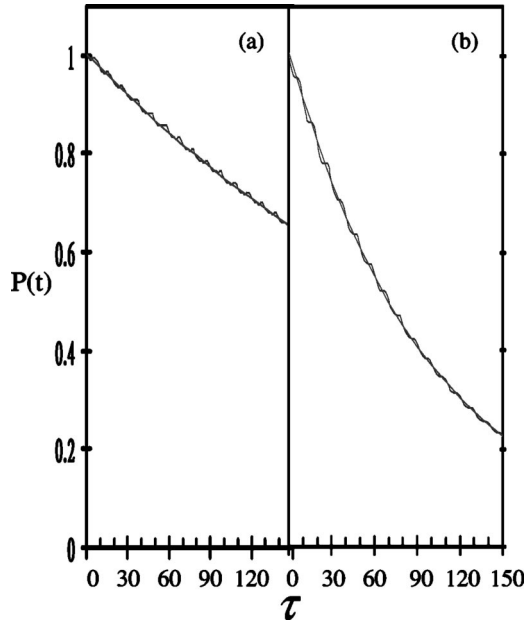


FIG. 4. Nondecay probability (oscillatory curve) $P(t)$ and the coarse-grained nondecay probability (monotonic curve) $\bar{P}(t)$ as a function of time for the more stable (a) and the less stable (b) state in Fig. 3.

tonic function, and does give a smooth measure of the stability of the system.

From Figs. 1 and 2 we also see that a direct crossing occurs at frequency $\omega \approx \text{Re}(E_1 - E_0)/2$ (point c). However, as ω approaches the frequency $\omega \approx \text{Re}(E_1 - E_0) = 0.632822V_0$, an avoided crossing (e, e') between the real parts of the Floquet energies occurs. Figure 2 indicates that larger values of V_1 only enhance the instability of the system and the repulsion between the two levels at avoided crossing. Thus as the frequency ω is increased, the state emanating from E_1 has Floquet energy with real part given by values along the path $abb'cdd'e'f$ (the dark dotted curve), while the real part of Floquet energy of the state emerging from E_0 lies along the path $cegg'h$ (the solid curve). The imaginary parts of these two paths are depicted in Figs. 1(b) and 2(b). One sees that an exchange of the imaginary parts takes place at the avoided crossing ee' . In Figs. 5 and 6, we show the probability density of the two states within the potential barrier just before and just after the avoided crossing. Together with Fig. 3, these plots demonstrate clearly the switching of the states. Beyond the avoided crossing, the upper state becomes more stable than the lower state. This gives the possibility of stabilizing an unstable state by an oscillating field. We recall that as $\omega \rightarrow \infty$, Eq. (11) reduces to the one for the static potential. In the example considered here, the lower state supported by the static well is a stable bound state, and hence the unstable upper Floquet state can be made stable in the high frequency limit. Even more simply, the same aim can be achieved by adiabatically tuning down the amplitude V_1 just after the avoided crossing, as in this limit the potential becomes the static one.

Finally, it is interesting to note here the role of amplitude V_1 of the oscillating barrier in the model. As we have seen,

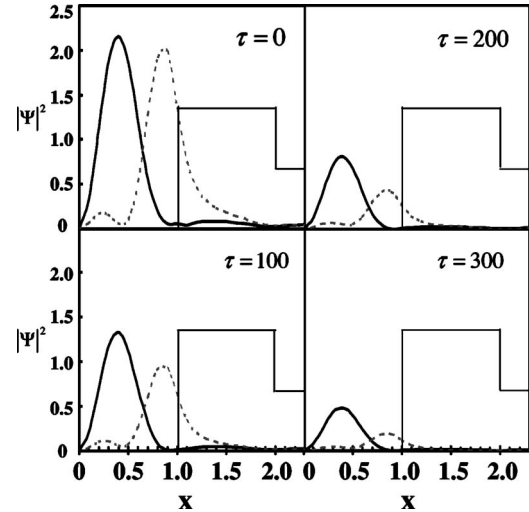


FIG. 5. Same plot as Fig. 3 at frequency $\omega/V_0 = 0.63$, just before the avoided crossing. The lifetimes of these two metastable states are comparable.

the presence of a nonvanishing V_1 always makes the system less stable. However, if V_1 is reduced, an avoided crossing may turn into a direct one. In the present case, the avoided crossing ee' changes into a direct crossing for $V_1/V_0 < 0.03$. Conversely, increasing V_1 could change a direct crossing into an avoided crossing. At an avoided crossing, the imaginary parts of the Floquet energies cross, while the real parts do not. At a direct crossing, it is the real parts, not the imaginary parts, that cross. But the more (less) stable state has the tendency to become less (more) stable. This is evident from the Floquet energy at the direct crossing point c in Figs. 1 and 2. These observations are consistent with the semiclassical results obtained in [11] by perturbative methods. Hence, by a combination of adiabatic changes of the frequency and the amplitude of the oscillating barrier, one can manipulate the stability of different states in a quantum potential: tune up V_1

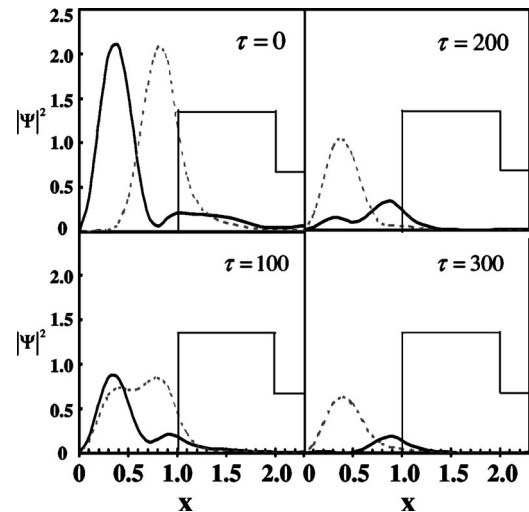


FIG. 6. Same plot as Fig. 3 at frequency $\omega/V_0 = 0.64$, just after the avoided crossing. The dashed (solid) curve represents the originally less (more) stable state, which now becomes more (less) stable.

until a direct crossing becomes an avoided one, increase ω so that the avoided crossing is passed, then reduce V_1 to make the potential static. In the process, two states in the well are interchanged.

IV. SUMMARY

To summarize, our results show that an oscillating potential barrier generally makes a metastable system decay faster. However, the existence of avoided crossings of metastable states can switch a less stable state to a more stable one. If in the static well there exists a bound state, then it is possible to stabilize a metastable state by adiabatically changing the oscillating frequency and amplitude of the barrier so that the unstable state will eventually cross over to the stable bound state. Thus a time-dependent potential can be used to control the stability of a particle trapped in a well.

Finally, we would like to comment on the differences between the stabilization of the decaying state discussed in this work and the interesting phenomenon of the suppression of ionization of atom (also called stabilization of atom) in superintense, high frequency laser pulses [20]. In this later phenomenon, it was found that while an atom is generally ionized by absorption of photons, the ionization rate of the atom

can be dramatically suppressed as the intensity (amplitude) of the laser field exceeds a certain threshold. Thus stabilization of the atom is attained by increasing the intensity. On the contrary, stabilization of the decaying state described here is achieved by increasing the frequency of the oscillating barrier to the threshold at which an avoided crossing in the Floquet energies takes place, regardless of the amplitude of the field. Also, in our case stabilization is against quantum tunneling through a potential barrier, while in the case of atomic stabilization it is against ionization by photon absorption. Another difference of the two phenomena is that an atom in the ground state is stable when not being irradiated by the laser field, but the decaying state to be stabilized in our system is already unstable even in the absence of the oscillating field owing to quantum tunneling effect. Furthermore, suppression of ionization can be studied using the methods of classical nonlinear dynamics and chaos [21], but tunneling through a barrier considered here is a characteristic and fundamental quantum phenomenon.

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