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Achieving induced transparency with one- and three-photon destructive interference in a two-mode, three-level, double-Λ system

L. Deng and M. G. Payne
Electron and Optical Physics Division, NIST, Gaithersburg, Maryland 20899, USA

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We show that in a two-mode, three-level, double-Λ system an efficient multiphoton destructive interference involving one- and three-photon pumping pathways occurs, leading to a unique type of induced transparency. Unlike the conventional electromagnetically induced transparency achieved with a three-state Λ system, the induced transparency is critically dependent upon two distinctive relaxation processes involving the production and propagation of an internally generated field. When a two-mode probe field is injected under suitable conditions, we show that the two probe pulses, after a characteristic propagation distance, evolve into a pair of temporal, amplitude, and group velocity matched pulse, traveling loss free in a highly dispersive medium.

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Electromagnetically induced transparency (EIT) [1] achieved with a three-level Λ system has been demonstrated to be able to significantly reduce the absorption of a probe field tuned to a strong one-photon resonance. In two seminal studies [2,3], Harris has described how a pair of short and intense probe and pump pulses can evolve, in a three-state Λ-type EIT configuration, into a temporally matched pair that propagates losslessly in the medium after a characteristic initial propagation distance.

In this Rapid Communication we show that with a two-mode, three-level, double-Λ system it is possible to produce a pair of temporally, amplitude, and group velocity (TAG) matched ultraslow probe pulse [4,5]. The key difference, in comparison with the conventional EIT Λ scheme [1–3], is the suppression of a dark state population by an efficient multiphoton destructive interference [6], leading to a unique type of efficient induced transparency. Specifically, we show (1) under suitable conditions both temporal profiles, amplitudes, and group velocities of two probe pulses can be well matched, (2) substantial suppression of a dark state due to a robust destructive interference between a one- and a three-photon excitation channel, resulting in a unique type of induced transparency and remarkable suppression of probe pulse absorption, and (3) no requirement on having maximum atomic coherence in order to achieve 100% photon flux conversion efficiency.

Before presenting our work, we first cite several works on Raman double-Λ system. These works include lasing without inversion [7], cavity QED [8], optical phase conjugation [9], efficient parametric frequency conversion [10], and efficient Raman scattering [11,12]. We point out that all these studies (except Ref. [12]) require a four-level system and many rely on steady-state solutions to atomic responses (especially in the case of efficient frequency conversion [10]). Finally, we also point out several recent studies on double-Λ systems [13].

In the present study we consider a lifetime broadened three-state atomic medium interacting with a pulsed two-mode probe field and a two-mode continuous wave (cw) pump field (Fig. 1). The probe fields (pulse length τ at the entrance of the medium and angular frequencies $\omega_{pn}, n=1,2$) and pump fields (angular frequencies ω_{cn}) couple the transition $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$, respectively. We assume that the probe lasers are weak so that ground-state depletion can be neglected. The equations of motion for the atomic response and the probe fields can be written as (throughout the present work $n=1,2$ unless specified)

$$\frac{\partial A_2^{(n)}}{\partial t} = id_{pn}A_2^{(n)} + i\Omega_{cn}^* A_3 + i\Omega_{pn}^*, \tag{1a}$$

$$\frac{\partial A_3}{\partial t} = id_3 A_3 + i\Omega_{c1}A_2^{(1)} + i\Omega_{c2}A_2^{(2)}, \tag{1b}$$

$$\frac{\partial \Omega_{pn}^*}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{pn}^*}{\partial t} = i\kappa_{12}A_2^{(n)}. \tag{1c}$$

Here, $A_2^{(n)}$ is the part of the state $|2\rangle$ amplitude that carries the polarization at frequency $\omega_{pn}, d_{pn}=\delta_{pn}+i\gamma_2/2$, where δ_{pn} is the detuning of the probe laser (ω_{pn}) from the $|1\rangle \rightarrow |2\rangle$ resonance and γ_2 is the decay rate of the state $|2\rangle$. In addition, A_3 is the amplitude of state $|3\rangle$ and $d_3=\delta_3+i\gamma_3/2$, where δ_3

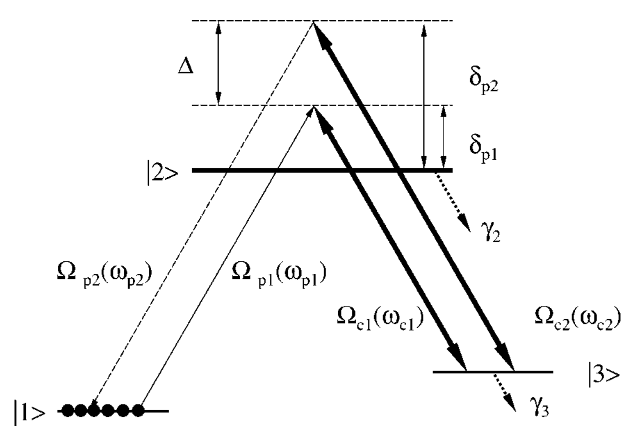

FIG. 1. Energy-level diagram for a three-state double-Λ system interacting with a two-mode probe and two-mode control fields. In reference to ultracold ^{87}Rb atomic vapor, we choose $|1\rangle=5S_{1/2}(F=1,M_F=-1)$, $|2\rangle=5P_{1/2}(F=2,M_F=0)$, $|3\rangle=5S_{1/2}(F=2,M_F=1)$.

$=\omega_{p1}-\omega_{c1}=\omega_{p2}-\omega_{c2}$ is the two-photon detuning between states $|1\rangle$ and $|3\rangle$ and γ_3 is the decay rate of state $|3\rangle$. We assume that two-photon resonances are always maintained so that $\delta_3=0$ and take $\gamma_3\simeq 0$. Finally, $2\Omega_{pn}(2\Omega_{cn})$ is the Rabi frequency of the probe (pump) field for the relevant frequency mode, $\kappa_{12}=2\pi N\omega_{pn}|D_{12}|^2/(\hbar c)$ with D_{12} and N being the dipole moment for transition $|1\rangle\to|2\rangle$ and concentration, respectively. We note that the major approximations used in deriving Eqs. (1a)–(1c) are the undepleted ground state ($A_1\simeq 1$) and the neglect of far off-resonant terms such as cross-mode stimulated emission with nonvanishing two-photon detunings (these approximations should always be accurate if the fields at ω_{pn} are sufficiently weak). No other approximations have been made in our semiclassical theory, Eqs. (1a)–(1c).

To solve Eqs. (1a)–(1c) we begin by assuming that $|\delta_{pn}|\gg\gamma_2$, $|\delta_{pn}\tau|\gg 1$, $|\delta_{p2}|\gg|\Omega_{pn}|$, $\Sigma_{n=1}^{2}|\Omega_{cn}\tau|^2/|d_{pn}\tau|\gg 1$, and $|\Omega_{pn}|\ll|\Omega_{cn}|$. These conditions ensure that the ground state remains undepleted and adiabatic processes remain effective. Let $\alpha_2^{(n)}$, α_3, and Λ_{pn}^* be the time Fourier transforms of $A_2^{(n)}$, A_3, and Ω_{pn}^*, respectively; we obtain $(n,m=1,2;n\neq m)$

$$\alpha_2^{(n)}=\frac{[|\Omega_{cm}|^2-(d_3+\omega)(d_{pm}+\omega)]\Lambda_{pn}^*-\Omega_{cn}^*\Omega_{cm}\Lambda_{pm}^*}{D}, \tag{2a}$$

$$\alpha_3=\frac{\Omega_{c1}(d_{p2}+\omega)\Lambda_{p1}^*+\Omega_{c2}(d_{p1}+\omega)\Lambda_{p2}^*}{D}, \tag{2b}$$

$$\frac{\partial\Lambda_{pn}^*}{\partial z}-i\frac{\omega}{c}\Lambda_{pn}^*=i\kappa_{12}\alpha_2^{(n)},\quad (n=1,2), \tag{2c}$$

where ω is Fourier transform variable,

$$D=(d_{p1}+\omega)(d_{p2}+\omega)(d_3+\omega)-|\Omega_{c1}|^2(d_{p2}+\omega)-|\Omega_{c2}|^2(d_{p1}+\omega). \tag{2d}$$

Equations (2a)–(2d) can be easily solved, yielding

$$\Lambda_{pn}^*=e^{\alpha_p z}(W_+^{(n)}e^{Lz}+W_-^{(n)}e^{-Lz}), \tag{3}$$

where $\alpha_p=L_0\Sigma_{n=1}^{2}[|\Omega_{cn}|^2-(d_{pn}+\omega)(d_3+\omega)]/2$, $L_0=i\kappa_{12}/[(d_{p1}+\omega)(d_{p2}+\omega)(d_3+\omega)-J]$, $J=(d_{p1}+\omega)|\Omega_{c2}|^2+(d_{p2}+\omega)|\Omega_{c1}|^2$, $\alpha_{12}=-L_0\beta_{12}$, $\beta_{12}=\Omega_{c1}^*\Omega_{c2}$, $L=L_0\sqrt{\beta_q^2+|\beta_{12}|^2}$, $W_\pm^{(n)}=[(L\pm\alpha_q)\Lambda_{pn}^*(0,\omega)\pm\alpha_{nm}\Lambda_{pm}^*(0,\omega)]/(2L)$, $\alpha_q=L_0\beta_q$, and $\beta_q=[|\Omega_{c1}|^2-|\Omega_{c2}|^2-(d_{p1}-d_{p2})(d_3+\omega)]/2$.

Although in general detailed solutions of the probe fields require numerical evaluation of the inverse transform of Eq. (3) using the complex quantities defined above, much physical insight can be gained if the exponents (i.e., $\alpha_p\pm L$) can be approximated as linear or quadratic functions of ω. The linear dependence on ω will correctly predict the propagation velocities and temporal profiles of the weak probe fields, whereas the inclusion of the quadratic terms in ω provides further corrections to temporal profiles, amplitudes, and group velocities due to pulse spreading and additional pulse attenuation. We note that the quadratic approximation can be quite accurate even when $\gamma_2\tau$ is relatively large, as with the lowest $S\to P$ transitions in alkali elements. Typically, when the linear or quadratic approximation to $\alpha_p\pm L$ is accurate it is sufficiently accurate to simply evaluate the coefficients in $W_\pm^{(n)}$ at $\omega=0$. With these approximations, we obtain

$$W_+^{(n)}=\frac{|\Omega_{c2}|^2}{|\Omega|^2}[\Lambda_{pn}^*(0,\omega)-S_1\Lambda_{pm}^*(0,\omega)], \tag{4a}$$

$$W_-^{(n)}=\frac{|\Omega_{c1}|^2}{|\Omega|^2}[\Lambda_{pn}^*(0,\omega)+S_2\Lambda_{pm}^*(0,\omega)], \tag{4b}$$

where $S_1=(\Omega_{c1}^*/\Omega_{c2}^*)\delta_{n,1}+(\Omega_{c1}/\Omega_{c2})\delta_{n,2}$, $S_2=(\Omega_{c2}/\Omega_{c1})\delta_{n,1}+(\Omega_{c2}^*/\Omega_{c1}^*)\delta_{n,2}$ ($\delta_{n,1}$ is the Kronecker δ function), and $|\Omega|^2=|\Omega_{c1}|^2+|\Omega_{c2}|^2$. In arriving at Eqs. (4a) and (4b) we have assumed that $|\Omega_{cn}|^2\gg|(d_{pn}+\omega)(d_3+\omega)|$ and $|J|\gg|(d_{p1}+\omega)\times(d_{p2}+\omega)(d_3+\omega)|$.

We now consider the adiabatic limit where we use Eqs. (4a) and (4b) and retain only the constant and linear terms in ω in the exponents in Eq. (3). Defining $K_0=\kappa_{12}|\Omega|^2/J$ and $\Omega_{pn(\pm)}^*=\Omega_{pn}^*[0,t-z/V_g^{(\pm)}]$, we obtain

$$\Omega_{pn}^*(z,t)=\frac{|\Omega_{c2}|^2e^{-iK_0z}}{|\Omega|^2}(\Omega_{pn+}^*-S_1\Omega_{pm+}^*)+\frac{|\Omega_{c1}|^2}{|\Omega|^2}(\Omega_{pn-}^*+S_2\Omega_{pm-}^*), \tag{5}$$

where $1/V_g^{(-)}=1/c+(\kappa_{12}/|\Omega|^2)$ and $1/V_g^{(+)}=1/c+(\kappa_{12}/|\Omega|^2)[(d_{p1}-d_{p2})^2|\Omega_{c1}|^2|\Omega_{c2}|^2/J^2]$. Equation (5) indicates that in general each probe mode breaks up into two pulses, each traveling at a different group velocity. It is, however, possible to match these group velocities with appropriately chosen operation parameters. To achieve this we require [14] $\mathrm{Re}[(d_{p1}-d_{p2})^2|\Omega_{c1}|^2|\Omega_{c2}|^2/J^2]\simeq 1$, and obtain $V_g^{(+)}\simeq V_g^{(-)}$. If we assume the initial condition of $\Omega_{p2}(0,t)=0$, then by maximizing Ω_{p2} keeping group velocity matched we have

$$\Omega_{pn}^*(z,t)=\frac{1}{2}[1+(-1)^{n-1}e^{-i(2\kappa_{12}z/\delta_2+i\gamma_2)}]\Omega_{p1-}^*. \tag{6}$$

We note that when $\exp(-2\kappa_{12}\gamma_2z/\delta_2^2)$ is close to unity then if $2\kappa_{12}z/\delta_2=m\pi$ and m is an odd integer, we get $\Omega_{p1}^*(z,t)=0$ and $\Omega_{p2}^*(z,t)=\Omega_{p1-}^*$. Therefore, with an appropriate medium thickness, the field that exits the resonant medium will be Ω_{p2} only, with an amplitude nearly equal to that of Ω_{p1} at the entrance of the medium. Since the frequencies of the two modes are nearly the same, this represents a near 100% photon flux conversion from Ω_{p1} to Ω_{p2}. On the other hand, if m is an even integer, we have $\Omega_{p2}(z,t)=0$ $\Omega_{p1}^*(z,t)=\Omega_{p1-}^*$. Thus, as the two interacting fields propagate through the medium the state of the probe field oscillates between the two frequency modes as a function of propagation distance. Note that there are no restrictions on having maximum atomic coherence between states $|1\rangle$ and $|3\rangle$ in order to achieve near unity conversion. Indeed, there is a very small excited state population in this problem.

The second unique feature of the present work is a robust multiphoton destructive interference. We note that both Ω_{p1}^* and Ω_{p2}^* have a velocity component [the first terms in Eq. (5)] that decays in exactly the same way. In the case where

$\Omega_{p2}(0,t)=0$ this feature has very interesting implications. At large z, where the fast decaying part is negligible, we get

$$\Omega_{pn}^*(z,t) = \frac{\Omega_{c1}\Omega_{cn}^*}{|\Omega|^2}\Omega_{p1-}^*, \tag{7}$$

which lead to $\Omega_{p1}^*(z,t)/\Omega_{p2}^*(z,t)=\Omega_{c1}^*/\Omega_{c2}^*\rightarrow\Lambda_{p1}^*=(\Omega_{c1}^*/\Omega_{c2}^*)\Lambda_{p2}^*$. Using this result in Eqs. (2a) and (2b) and assuming $|\Omega_{cn}|^2\gg|\omega d_{pn}|$ we immediately obtain $\alpha_2^{(n)}\simeq 0$. This implies that at a sufficient depth into the medium the amplitude of A_2 is strongly suppressed by a destructive interference between a one- (Ω_{p1}^*) and a three-photon $(\Omega_{c1}^*,\Omega_{c2}^*,\Omega_{p2}^*)$ pathway to drive the $|1\rangle\rightarrow|2\rangle$ transition transparent. This multiphoton destructive interference-based induced transparency is very different from the conventional three-state Λ EIT system where the transparency is the result of the destructive interference of two single-photon pathways with two externally supplied fields. It has been shown that such a multiphoton-based interference is a common feature of other highly efficient four-wave mixing (FWM) processes [15]. As a consequence, if two matched pulses satisfying the above relation are injected into the medium, they will propagate with identical velocities and with very little distortion or attenuation.

It is instructive to examine the formation of the dark state before and after the onset of effective three-photon destructive interference. This is another important difference between the system studied here and the conventional three-state Λ scheme widely used in EIT related works. In the conventional EIT picture, a dark state is established immediately after the probe field enters the medium (assuming that the cw control field is already present). This is, however, *not* the case of the system studied here. In fact, there is no dark state in the medium prior to the establishment of the three-photon destructive interference deep inside the medium. A characteristic propagation distance is required so that the generated wave is strong enough to open the back coupling channel, leading to the three-photon destructive interference. In other words, unlike the conventional EIT scheme, the dark state has a root in the nonlinear wave mixing process. In fact, it is established by the interplay of the one-photon and three-photon coupling after a characteristic propagation distance. This characteristic propagation distance can be short or long depending on the concentration and operation conditions. As a comparison, the commonly known dark state picture in the conventional EIT does not rely on any propagation effect. Before the characteristic propagation distance at which the fast damping terms become negligible, one does not have a dark state or the multiphoton induced transparency. Indeed, the view of changing from state $|1\rangle$ to state $|3\rangle$ by gradually extinguishing the driving field, as indicated in the dark state picture of the conventional EIT scheme, cannot be simply applied in the present problem because of the role of the internally generated field. If one extinguishes the control field Ω_{c1}, a photon of frequency $\omega_{SR}\simeq\omega_{c2}$ will be generated by stimulated Raman process. The characteristic of the process changes from conventional FWM to a parametric FWM. Consequently, both two-photon $(\Omega_{p1}^*+\Omega_{SR}$ excitation balances $\Omega_{c2}^*+\Omega_{PFWM}$ excitation) and three-photon $(\Omega_{p1}^*+\Omega_{SR}+\Omega_{c2}^*$ excitation balances Ω_{PFWM} excitation) destructive interferences can occur after characteristic propagation distances under suitable driving conditions. If, on the other hand, one extinguishes the control field Ω_{c2}, the generated field and the three-photon destructive interference disappear, and one loses the TAG pulse pair. These features of the dark state in the present study do not have a counterpart in the conventional EIT process. We emphasize that in contrast to the conventional EIT scheme the unique type of induced transparency is critically dependent upon the production and propagation of an internally generated field and upon two distinctive relaxation processes for each velocity component. There is no equivalent mechanism in the conventional Λ-type EIT system.

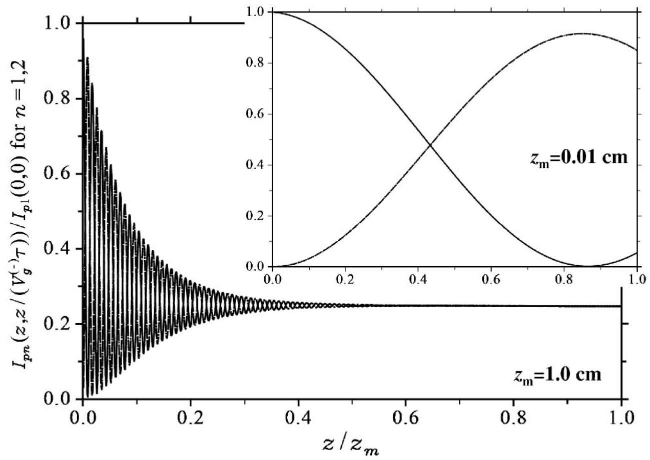

FIG. 2. Normalized peak intensities of the ω_{p1} mode (solid line) and ω_{p2} mode (dashed line) probe waves as a function of z/z_m for $z_m=1.0$ cm and $z_m=0.01$ cm (the inset). Parameters used: $\tau=10^{-4}$ s, $N=10^{14}\text{cm}^{-3}$, $|\Omega_{c1}\tau|=|\Omega_{c2}\tau|=2136$, $\delta_{p1}\tau=0$, $\delta_{p2}\tau=1.256\times10^5$, $|\Omega_{p1}(0,0)\tau|=1$, $|\Omega_{p2}(0,0)\tau|=0$, $\gamma_2\tau=3610$, $\delta_3\tau=0$, $\gamma_3\tau=0.05$, $\kappa_{12}\tau=2.28\times10^7$ cm s^{-1}, and $V_g^{(-)}\tau=0.4$ cm. Each curve contains two traces obtained from the numerical solutions of Eqs. (2a)–(2c) and the analytical solutions of Eq. (7), respectively. The agreement between the two equations is excellent and the two traces for each mode cannot be distinguished.

The above results are the consequence of linearization of the coefficients and exponents permitted by the assumption of good adiabatic behavior in the atomic response. Corrections to this adiabatic theory can be derived analytically to account for probe pulse spreading and additional attenuation. Due to space limitation, solutions to Eq. (3) using quadratic approximation with different pulse lengths and delays will be presented elsewhere. In the following we give numerical examples to demonstrate the validity of our analytical solutions.

For numerical simulations we consider cold ^{87}Rb atomic vapor. In the first case we take $\Omega_{p1}(0,0)\tau=1$ and $\Omega_{p2}(0,0)\tau=0$. From Eq. (6) we find that the first destructive interference occurs at $z_1=0.008\,65$ cm. We thus take the medium thickness to be $z_m=z_1=0.01$ cm. In Fig. 2 we plot the normalized probe field intensities as a function of z/z_m for the case of $z_m=0.01$ cm (the inset) and $z_m=1.0$ cm. The numerical solutions are obtained by solving Eqs. (1a)–(1c) without any approximations. The results from the two methods are so that they cannot be distinguished on the graph. Note that the peak of $|\Omega_{p2}(z,t_r/\tau)|^2/|\Omega_{p1}(0,0)|^2$ (the inset) indicates a conversion of 91% is near $z=0.008\,65$ cm, as predicted.

It is instructive to consider a case where $\Lambda_{p1}^* = (\Omega_{c1}^*/\Omega_{c2}^*)\Lambda_{p2}^*$ is satisfied and $\Omega_{p1}(0,0)\tau = \Omega_{p2}(0,0)\tau = 1$. In such a matched injection case Eq. (8) predicts that two fields Ω_{p1} and Ω_{p2} form a perfectly matched pair, propagate with identical group velocity, and experience very little attenuation and pulse distortion. Extensive numerical simulations for such injection-matched pairs agree well with the analytical solution Eq. (7), and typically less than 15% attenuation of the probe fields can be achieved for an extended propagation distance of 1 cm. This is a remarkably high transparency for such a highly resonant and optically thick medium. In a separate study we will systematically investigate the advantages of such matched injection conditions and explore applications of this effect in other wave propagation problems.

We have investigated a unique type of induced transparency resulted from multiphoton destructive interference. We have shown the formation of two TAG matched ultraslow pulse pairs. In addition, we have discussed the key differences between the present scheme and the conventional EIT scheme, and we have shown that the unique effect and the underlying physics do not have equivalent counterparts in the conventional EIT system. We emphasize that the unique type of induced transparency is the result of two distinctive relaxation processes and is critically dependent on the three-photon destructive interference involving the internally generated field. The robust three-photon destructive interference and the efficient multiphoton-based induced transparency predicted here are also expected to occur in Doppler broadened media under modified driving conditions due to the the broad linewidths. Of course, the requirement for near adiabatic behaviors of the system response will also be subject to appropriate modifications.

The TAG matched propagation of a pair of ultraslow probe pulses in a highly transparent medium discussed here may be applied to other multiwavelength experiments in the ultraslow propagation regime. The unique type of highly efficient induced transparency enabled by one- and three-photon destructive interference may provide yet another way to achieve lossless propagation in a highly dispersive resonant medium. This could lead to intriguing applications in the field of optoelectronics.

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[14] For instance, with $|\delta_{pn}| \gg \gamma_2$, we require $(\delta_{p1} - \delta_{p2})^2|\Omega_{c1}|^2|\Omega_{c2}|^2 \simeq (\delta_{p1}|\Omega_{c2}|^2 + \delta_{p2}|\Omega_{c1}|^2)^2$. Thus, if one chooses $\delta_{p1} = -\delta_{p2}$, then $|\Omega_{c2}|^2 \simeq (3+\sqrt{8})|\Omega_{c1}|^2$ gives matched group velocity. We also require that Re[…]≫Im[…] which can be easily satisfied because of the condition $|\delta_{pn}| \gg \gamma_2$.

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