Theory of a compound large-angle atom beam splitter. II. Initial state deflection

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The theory of a compound, large-angle atom beam splitter [A. Zh. Muradyan, A. A. Poghosyan, and P. R. Berman, Phys. Rev. A **68**, 033604 (2003)] is generalized to allow for initial-state deflection. Atoms are prepared in an initial state by an off-resonant standing-wave field and then subject to two-standing-wave fields that couple the initial state to a final state. By a proper choice of parameters, atoms in the initial state can be deflected or split as a result of the interactions with the fields. The role of relaxation is considered.

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In Ref. [1] a theory of a compound, atom beam splitter was developed, based on the coupling of standing wave fields to a Λ-type three-level atom (see Fig. 1). Atoms are prepared initially in state 1 and are subjected to an intense off-resonant standing-wave field acting on the 1-2 transition, preferably in the Raman-Nath regime. This preliminary interaction creates widely spread momentum states in state 1 (see Fig. 2). Following this interaction, a pair of $\pi/2$-phase-shifted standing-wave fields act on the coupled 1-2 and 2-3 transitions. As a result, the outlying momentum states are transferred to internal state 3, while the intermediate ones are left in internal state 1. In this manner, one generates a large-angle beam splitter, provided one can isolate the atoms exiting the interaction zone in state 3. For the field parameters chosen in Ref. [1], it was not possible to produce a beam splitter on the initially populated internal state.

Our beam splitter differs somewhat from conventional beam splitters in that the atoms emerge in a coherent superposition of two internal states, but the desired final-state momentum components are associated with only one of the internal states. This is in contrast to beam splitters (i) involving atoms in a *single* internal state whose momentum distribution is modified by an atom-field interaction or passage through a material grating [2,3] and (ii) involving atoms for which an atom-field interaction leads to different momentum components associated with each of two different internal states of the atoms [4]. In Ref. [1], it is possible to maintain the coherence of atoms in state 3 while eliminating the contribution to the wave function from atoms in state 1. This could be achieved, for example, by selectively ionizing atoms in state 1 since the ionizing fields would leave the atoms in state 3 untouched. Note that the compound beam splitter discussed in this paper is based on techniques that are similar in spirit to those proposed by Cohen *et al.* [5] and Rohwedder [3] who use multiple-atom optics elements to enhance a desired output.

From an experimental point of view it may be desirable to construct a beam splitter in the initially occupied state. In this Brief Report, we show that, within the context of the model represented schematically in Fig. 1, it is possible to choose the fields to achieve this goal. Moreover, the same scheme may be used as a beam reflector, rather than a beam splitter, for appropriate input fields.

Assuming the same time envelope for both laser pulses, $\Omega_p(z,t)=\Omega_p(z)\mathrm{sech}(t/T)$, $\Omega_s(z,t)=\Omega_s(z)\mathrm{sech}(t/T)$, one can show that, provided the initial-state amplitudes are $(C_1(z,-\infty)\neq 1, C_3(z,-\infty)=0)$, the lower-state probability amplitudes following the interaction are given by [1]

$$C_1(z,+\infty)=[B\sin^2\theta+\cos^2\theta]C_1(z,-\infty), \tag{1a}$$

$$C_3(z,+\infty)=(B-1)\sin\theta\cos\theta C_1(z,-\infty), \tag{1b}$$

where

$$\sin\theta=\Omega_p(z)/\sqrt{\Omega_p^2(z)+\Omega_s^2(z)},$$

$$\cos\theta=\Omega_s(z)/\sqrt{\Omega_p^2(z)+\Omega_s^2(z)}, \tag{2}$$

$$B=\frac{\Gamma\left(\frac{1}{2}+\gamma+i\delta\right)\Gamma\left(\frac{1}{2}+\gamma+i\delta\right)}{\Gamma\left(\frac{1}{2}+\gamma+i\delta-\alpha(z)\right)\Gamma\left(\frac{1}{2}+\gamma+i\delta+\alpha(z)\right)}, \tag{3}$$

$\Gamma(x)$ is the gamma function,

$$\alpha(z)=\sqrt{\Omega_p^2(z)+\Omega_s^2(z)}T \tag{4}$$

is a dimensionless pulse area,

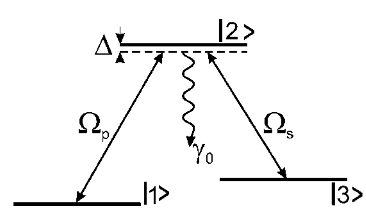

FIG. 1. Atom-field geometry. Standing-wave pulses $\Omega_p(z,t)$ and $\Omega_s(z,t)$ drive coupled atomic transitions $|1\rangle$-$|2\rangle$ and $|2\rangle$-$|3\rangle$, respectively. Relaxation from state $|2\rangle$ is out of the Λ system.

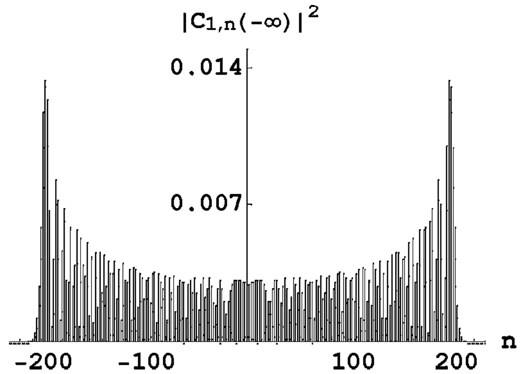

FIG. 2. Momentum distribution, generated by an intense off-resonant standing wave. The chosen value $U=200$ is in an intermediate range of experimentally accessible values.

$$\delta=\frac{1}{2}\Delta T, \quad \gamma=\frac{1}{2}\gamma_0 T \tag{5}$$

are dimensionless detuning and decay parameters, and $\Delta=\omega_p-\omega_{21}=\omega_s-\omega_{31}$ is an atom-field detuning. A frequency chirp, considered in Ref. [1], has been set equal to zero.

To get the momentum amplitude distribution after the interaction, one should expand Eqs. (1a) and (1b) into Fourier series. The parameter B and the trigonometric functions are rather complicated functions of the coordinate z and an analytical expression relating the final-state Fourier components to the initial ones cannot be obtained, in general. Stated in another way, the interference between different initial-state Fourier components is rather complicated. To simplify matters, we choose

$$\Omega_p(z)=\Omega_p \sin kz, \quad \Omega_s(z)=\Omega_s \cos kz, \tag{6}$$

$\Omega_p=\Omega_s=\alpha T$. The pulse area α and the parameter B are independent of z and

$$\sin\theta=\sin kz, \quad \cos\theta=\cos kz. \tag{7}$$

Then, if one expands the amplitudes appearing in Eqs. (1a) and (1b) in Fourier series as $C_1(z,-\infty)=\sum_{n=-\infty}^{\infty}C_{1,n}(-\infty)e^{2inkz}$, $C_3(z,\infty)=\sum_{n=-\infty}^{\infty}C_{3,n}(\infty)e^{2inkz}$ [assuming the initial state amplitude contains only even powers of (kz)], one finds

$$C_{1,n}(\infty)=\frac{1+B(\alpha,\beta,\gamma)}{2}C_{1,n}(-\infty) + \frac{1-B(\alpha,\beta,\gamma)}{4}[C_{1,n+1}(-\infty)+C_{1,n-1}(-\infty)], \tag{8a}$$

$$C_{3,n}(\infty)=\frac{1-B(\alpha,\beta,\gamma)}{4i}[C_{1,n+1}(-\infty)-C_{1,n-1}(-\infty)], \tag{8b}$$

a relatively simple relationship between the initial- and final-state momentum amplitudes.

In Ref. [1], an initial, off-resonant standing-wave field pulse acted on atoms in state 1 to produce state amplitudes that satisfied $C_{1,n+1}(-\infty)\simeq C_{1,n-1}(-\infty)$ for low- and intermediate-momentum states; these values were then used as *initial* conditions for the pair of standing-wave pulses having amplitudes given in Eq. (6). As can be deduced from Eqs. (8), such initial conditions would suppress the intermediate-momentum states associated with level 3, but not those with level 1. To suppress the intermediate-momentum states in level 1, the initial momentum-state amplitudes should satisfy the condition $C_{1,n+1}(-\infty)\simeq -C_{1,n-1}(-\infty)$ for small and intermediate values of n. To this end, one can choose the initial, preparatory field to be an off-resonant field having spatial amplitude

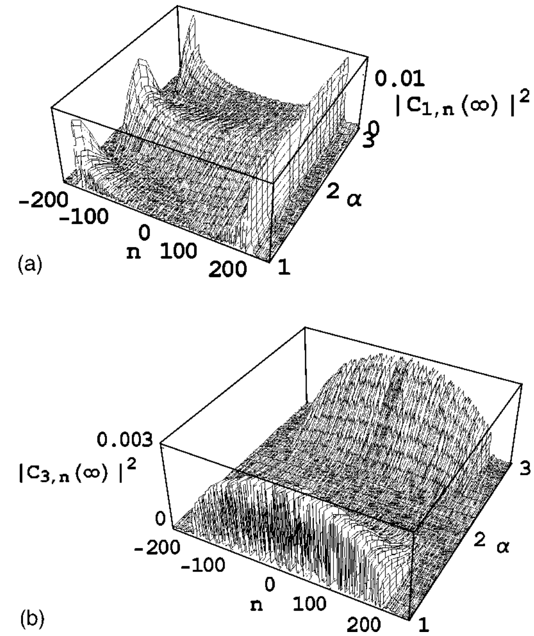

FIG. 3. (a) Momentum distribution in state 1 at exact resonance with decay ($\beta=0,\gamma=0.01$), which is a periodic function of pulse area α. The original ($\alpha=0$) distribution is repeated at values $\alpha=2,4,\ldots$. For values $\alpha=1,3,\ldots$, the scheme acts as a beam splitter (the frontal plane represents one of them), while at intermediate values $\alpha=1/2,3/2,\ldots$ it acts as a beam reflector. (b) Momentum distribution in state 3 for the same parameters. As we can see from both figures, during the interaction part of population has transferred to state 2 and than decayed out of the Λ system.

$$\Omega_{SW}(z)=\Omega_{SW}\sin(kz\pm\pi/4). \tag{9}$$

This field produces an initial-state amplitude for the second pair of fields given by

$$C_1(z,-\infty)=e^{\pm iU\sin 2kz}=\sum_{n=-\infty}^{\infty}(\pm 1)^n J_n(U)e^{2inkz}, \tag{10}$$

having Fourier coefficients

$$C_{1,n}(-\infty)=(\pm 1)^n J_n(U), \tag{11}$$

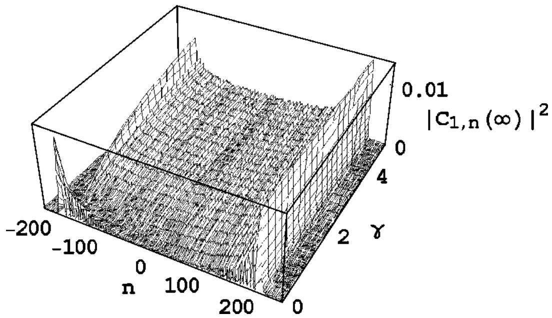

FIG. 4. Momentum distribution in level 1 as a function of relaxation rate γ. The frontal plane corresponds to that in Fig. 3 with $\alpha=1$, when the the suppression of intermediate momentum states is most effective. As is seen the increase of γ first depresses the left-hand side of distribution, transforming it into a reflector type, and then, as can be expected, gradually restores the initial distribution.

where U is the pulse area of the preparatory field and $J_n(U)$ is a Bessel function of order n. This initial momentum distribution is illustrated in Fig. 2. For $n \leqslant U, U \gg 1$, the asymptotic formula [6]

$$J_n(U) \approx \sqrt{\frac{2}{\pi U}} \cos\left(U - \frac{n\pi}{2} - \frac{\pi}{4}\right) \tag{12}$$

guarantees that $C_{1,n+1}(-\infty) \simeq -C_{1,n-1}(-\infty)$ for small and intermediate values of n. As such, the combination of both field pulses results in a suppression of low- and intermediate-momentum components in level 1 with a corresponding transfer of these components to level 3. Note that the preparatory field (9) is $\pi/4$ shifted relative to the fields (6) coupling the adjacent quantum transitions.

As is seen from Eqs. (8), in order to maximize the efficiency of the process—i.e. the transmission of intermediate momentum states from level 1 into level 3—the parameter B should be real and close to -1. By choosing $\delta=0$, one ensures that B is real. If, simultaneously, $\gamma=0$, then

$$B = \cos(\pi\alpha)$$

and optimal transfer occurs for odd values of pulse area α. For even values of α, $B=1$ and the final momentum spectrum reverts to the initial one. Such behavior is analogous to that encountered with π and 2π pulses. The final momentum distributions for levels 1 and 3 as a function of α are illustrated in Fig. 3.

Another interesting regularity is seen in Fig. 3(a). For $\alpha = 1/2, 3/2, \ldots$, atoms in state 1 are reflected mainly to the right (n positive). This can be explained as follows: When $\alpha = n+1/2, B=0$. Using the recursion relation $J_{n+1}(U) + J_{n-1}(U) = (2n/U) J_n(U)$, one finds

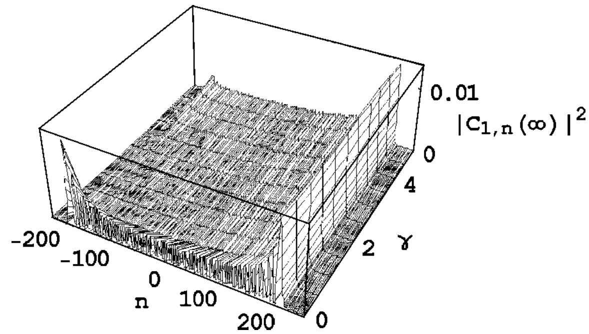

FIG. 5. A graph, analogous to that in Fig. 4 for $\alpha=2$, when momentum distribution equals the initial distribution when $\gamma=0$.

$$C_{1,n}^2(\infty) = \frac{1}{4}\left(1 + \frac{n}{U}\right)^2 J_n^2(U). \tag{13}$$

If $U \gg 1$, the appreciable values of $J_n(U)$ as a function of n are spread between $n=-U$ and $n=U$ (Fig. 2). Therefore, the momentum amplitudes are small for negative n and increase with increasing n. The probability $C_{1,n}^2(\infty)$ achieves its maximum value for $n \approx U$.

In general, relaxation decreases the efficiency of the beam splitter. We consider only integral α, but the results are fairly general. For odd α, the beam splitter is optimized with $\gamma=0$ ($B=-1$). As γ increases from 0 to $1/2$, B rises from -1 to 0; B stays approximately equal to zero for $1/2 < \gamma < \alpha$ and then rises to unity for $\gamma \gg \alpha$. As such, the beam splitter distribution is first converted into a reflector distribution with increasing γ before reverting to the initial distribution for $\gamma \gg \alpha$. For even α, the momentum distribution is unchanged from that produced by the preparatory pulse if $\gamma=0$ ($B=1$). As γ increases from 0 to $1/2$, B falls from 1 to 0; B stays approximately equal to zero for $1/2 < \gamma < \alpha$ and then rises to unity for $\gamma \gg \alpha$. As such, the initial distribution is first converted into a reflector distribution with increasing γ before reverting to the initial distribution for $\gamma \gg \alpha$. These features are illustrated in Figs. 4 and 5, respectively.

As was noted above, the compound beam splitter is a totally coherent device. Atoms are prepared in a linear superposition of low-momentum states for atoms in state 3 and high-momentum states for atoms in state 1. The atoms in state 3 can be selectively removed from the beam without seriously affecting the coherence of the atoms in state 1. As such the compound beam splitter can serve as the first element of an atom interferometer. To complete the interferometer the beams would have to be recombined in a coherent manner. It is possible that compound atom deflectors could be used to achieve this goal, but we have not yet investigated this possibility in detail.

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