Preparation of Greenberger-Horne-Zeilinger entangled states with multiple superconducting quantum-interference device qubits or atoms in cavity QED

Chui-Ping Yang and Siyuan Han
Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045, USA

(Received 7 June 2004; published 29 December 2004; publisher error corrected 30 December 2004)

A scheme is proposed for generating Greenberger-Horne-Zeilinger (GHZ) entangled states of multiple superconducting quantum-interference device (SQUID) qubits by the use of a microwave cavity. The scheme operates essentially by creating a single photon through an auxiliary SQUID built in the cavity and performing a joint multiqubit phase shift with assistance of the cavity photon. It is shown that entanglement can be generated using this method, deterministic and independent of the number of SQUID qubits. In addition, we show that the present method can be applied to preparing many atoms in a GHZ entangled state, with tolerance to energy relaxation during the operation.

DOI: 10.1103/PhysRevA.70.062323 PACS number(s): 03.67.Lx, 85.25.Dq, 42.50.Dv

I. INTRODUCTION

Entanglement is the cornerstone of quantum computation and communication. During the past few years, rapid progress has been made in generation and engineering of quantum entanglement. Experimental realizations of entangled states with four photons [1], four ions [2], two atoms in microwave cavity QED [3], and two excitons in a single quantum dot [4] have been reported. On the other hand, based on cavity QED, a large number of theoretical methods for creating multiqubit entanglement with atoms, ions, quantum dots, and charge qubits have been presented [5–11].

In this paper, our goal is to present a way to prepare many (superconducting quantum-interference device (SQUID)) qubits in the Greenberger-Horne-Zeilinger (GHZ) type of entangled states [12]. The scheme is based on the cavity QED technique. However, unlike previous proposals [5–7] where photon detection outside the cavity or passing qubits (e.g., atoms) through the cavity is required, the present scheme operates essentially by building an auxiliary SQUID in a cavity to serve as a microwave photon generator and by performing a multiqubit joint phase shift with the aid of the cavity photon. As shown below, the present scheme has the following distinct features: (i) The entanglement preparation is deterministic and independent of the number of SQUID qubits in the cavity. (ii) Because no tunneling between the SQUID qubit levels $|0\rangle$ and $|1\rangle$ is required, the decay from the level $|1\rangle$ can be made negligibly small during the operation, via adjusting the potential barrier between the qubit levels [13]. (iii) The created entanglement is a superposition of different combination of two lowest levels $|0\rangle$ and $|1\rangle$; thus decoherence of the generated entanglement, caused by the cavity loss and spontaneous emission, is greatly suppressed. In addition, it is interesting to note that the method can be extended to prepare many Λ-type three-level atoms in a GHZ entangled state, without real excitation of higher energy levels for each atom during the entire operation.

The motivation of this work is threefold. (i) SQUIDs have recently attracted much attention in the quantum-information community. Because they are relatively easy to scale up and have been demonstrated to have long decoherence times [14–16], they have been considered as promising candidates for building up superconducting quantum computers [17]. (ii) For superconducting flux qubits, Amin *et al.* have proposed a scheme for realizing an *arbitrary single-qubit rotation* without tunneling in a three-level SQUID qubit, by the use of a small detuning [18]. Later Yang and Han showed that large detuning of the driving fields from the upper state can be applied to implement single-qubit rotation without energy relaxation in the three-level SQUID qubit [19] (see also recent work by Kis and Paspalakis [20]). In addition, schemes for entangling two flux qubits within cavity QED and proposals for creating entanglement between a flux qubit and an electromagnetic field have also been proposed [20–23]. However, how to prepare multiqubit entangled states with flux qubits based on cavity QED has not been thoroughly investigated. (iii) Multiqubit GHZ states are of great interest in the foundations of quantum mechanics and measurement theory, and may prove to be useful in quantum-information processing [24], communication [25], error-correction protocols [26], and high-precision spectroscopy [27].

This paper is outlined as follows. In Sec. II, we show a way to prepare GHZ entangled states of multiple SQUID qubits in a cavity and then give a brief discussion of the experimental issues. In Sec. III, we extend the method to create multiqubit GHZ states with three-level atoms via cavity QED. A concluding summary is given in Sec. IV.

II. GENERATION OF MULTI-SQUID-QUBIT GHZ STATES

Let us consider N SQUIDs $(1,2,\ldots,N)$ and an auxiliary SQUID a in a single-mode microwave cavity [Fig. 1(a)]. In the following, the auxiliary SQUID a serves to create cavity photons. The SQUIDs considered in this paper are rf SQUIDs each consisting of a Josephson tunnel junction in a superconducting loop (typical size of a rf SQUID is on the order of 10–100 μm). The Hamiltonian of a rf SQUID (with junction capacitance C and loop inductance L) can be written in the usual form [28]

$$H_s = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_x)^2}{2L} - E_J \cos\left(2\pi\frac{\Phi}{\Phi_0}\right), \qquad (1)$$

where Φ, the magnetic flux threading the ring, and Q, the total charge on the capacitor, are the conjugate variables of

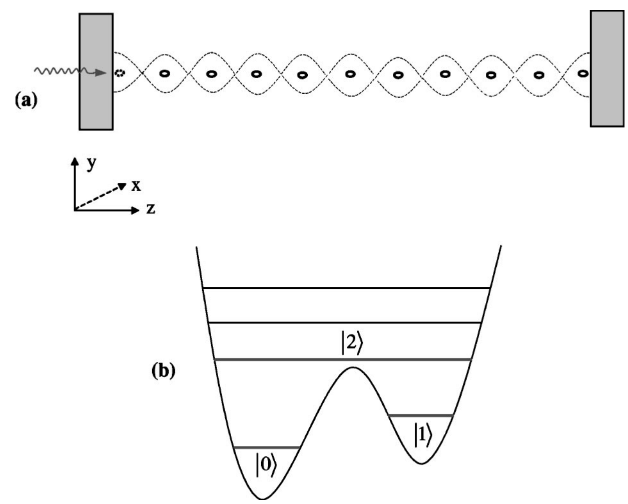

FIG. 1. (a) The schematic setup of SQUIDs $(1,2,\ldots,N)$ and an auxiliary SQUID a (dotted-line circle) in a microwave cavity. The magnetic components of the pulses and the cavity mode are in the Y direction. Each SQUID is placed in the X-Z plane. The auxiliary SQUID is used as a microwave photon generator. (b) Level diagram of a SQUID with the Λ-type three lowest levels $|0\rangle$, $|1\rangle$, and $|2\rangle$.

the system (with the commutation relation $[\Phi,Q]=i\hbar$), Φ_x is the static (or quasistatic) external flux applied to the ring, and $E_J \equiv I_c\Phi_0/2\pi$ is the Josephson coupling energy (I_c is the critical current of the junction and $\Phi_0=h/2e$ is the flux quantum).

Consider a Λ-type configuration formed by the three lowest levels of each SQUID, denoted by $|0\rangle$, $|1\rangle$, and $|2\rangle$ with energy eigenvalues E_0, E_1, and E_2, respectively [Fig. 1(b)]. To prepare the SQUIDs $(1,2,\ldots,N)$ in a GHZ state, we will need two types of interactions—the cavity mode interacting resonantly with the transition between the levels $|0\rangle$ and $|2\rangle$ of SQUID a, and the microwave pulse interacting resonantly with the transition between the levels $|0\rangle$ and $|2\rangle$ of SQUID a. For the first type of interaction, when the cavity mode is initially in the photon-number state $|n\rangle$, one can get the following state evolution in the interaction picture:

$$|0\rangle_a|n\rangle_c \rightarrow \cos\sqrt{n}gt|0\rangle_a|n\rangle_c - i\sin\sqrt{n}gt|2\rangle_a|n-1\rangle_c,$$

$$|2\rangle_a|n\rangle_c \rightarrow -i\sin\sqrt{n+1}gt|0\rangle_a|n+1\rangle_c + \cos\sqrt{n+1}gt|2\rangle_a|n\rangle_c, \tag{2}$$

where the subscripts a and c represent SQUID a and the cavity mode, respectively; and g is the coupling constant between the cavity mode and the $|0\rangle\leftrightarrow|2\rangle$ transition of SQUID a. The expression of g is given by [21]

$$g = \frac{1}{L}\sqrt{\frac{\omega_c}{2\mu_0\hbar}}\langle 0|\Phi|2\rangle_a \int_S \mathbf{B}_c(\mathbf{r})\cdot d\mathbf{S}, \tag{3}$$

where S is any surface bounded by the SQUID ring, $\mathbf{r}$ is the position vector on S, and $\mathbf{B}_c(\mathbf{r})$ is the magnetic component of the normal mode of the cavity with frequency ω_c. For a standing-wave cavity, one has $\mathbf{B}_c(z)=\mu_0\sqrt{2/V}\cos kz$ with wave number k, cavity volume V, and cavity axis z. In addition, after an interaction time t, the second type of interaction leads to the following state rotation for SQUID a (in the interaction picture):

$$|0\rangle_a \rightarrow \cos\Omega_{02}t|0\rangle_a - i\sin\Omega_{02}t|2\rangle_a,$$

$$|2\rangle_a \rightarrow -i\sin\Omega_{02}t|0\rangle_a + \cos\Omega_{02}t|2\rangle_a, \tag{4}$$

where Ω_{02} is the Rabi frequency of the pulse, which takes the following form [21]:

$$\Omega_{02} = \frac{1}{2L\hbar}\langle 0|\Phi|2\rangle_a \int_S \mathbf{B}_{\mu w}(\mathbf{r})\cdot d\mathbf{S} \tag{5}$$

for a microwave pulse with magnetic component $\mathbf{B}_{\mu w}(\mathbf{r},t) = \mathbf{B}_{\mu w}(\mathbf{r})\cos\omega_{\mu w}t$ Here, $\omega_{\mu w}$ is the carrier frequency of the microwave pulse and $\mathbf{B}_{\mu w}(\mathbf{r})$ is the amplitude of the magnetic component of the pulse.

Now let us turn to how to prepare SQUIDs $(1,2,\ldots,N)$ in a GHZ state. SQUID a and SQUIDs $(1,2,\ldots,N)$ have the Λ-type level configuration as depicted in Fig. 2(a), for which the transition between any two levels is far off resonant with the cavity mode (e.g., via prior adjustment of the level spacings). Suppose that the cavity mode, SQUID a, and each of SQUIDs $(1,2,\ldots,N)$ are initially in $|0\rangle_c$ (the vacuum state), $(|0\rangle - i|2\rangle)/\sqrt{2}$, and $(|0\rangle+|1\rangle)/\sqrt{2}$, respectively. The initial states for the SQUIDs can be prepared using many different techniques such as (i) applying a microwave pulse resonant with the transition between the levels $|0\rangle$ and $|1\rangle$; (ii) applying two microwave pulses separately, which are resonant with the $|0\rangle\leftrightarrow|2\rangle$ transition and the $|1\rangle\leftrightarrow|2\rangle$ transition, respectively [21]; and (iii) applying two microwave pulses simultaneously, with a small detuning or a large detuning from the upper level $|2\rangle$ [18,19]. The whole procedure for preparing SQUIDs $(1,2,\ldots,N)$ in the GHZ type of entangled state is shown as follows.

Step (i). Leave the level structure of SQUIDs $(1,2,\ldots,N)$ unchanged while adjusting the level spacings of SQUID a such that the $|0\rangle\leftrightarrow|2\rangle$ transition of SQUID a is resonant with the cavity mode [Fig. 2(b)]. After an interaction time $\tau_1 = \pi/(2g)$, the transformation $|0\rangle_a|0\rangle_c \rightarrow |0\rangle_a|0\rangle_c$ and $|2\rangle_a|0\rangle_c \rightarrow -i|0\rangle_a|1\rangle_c$ is obtained for the cavity mode and SQUID a. The initial state of the whole system $\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)(|0\rangle_a - i|2\rangle_a)|0\rangle_c$ thus becomes

$$\prod_{l=1}^{N}(|0\rangle_l + |1\rangle_l)(|0\rangle_c - |1\rangle_c)|0\rangle_a, \tag{6}$$

which can be written as

$$\left[\prod_{l=1}^{N}(|0\rangle_l + |1\rangle_l)|0\rangle_c - \prod_{l=1}^{N}(|0\rangle_l + |1\rangle_l)|1\rangle_c\right]|0\rangle_a, \tag{7}$$

where the subscript l represents SQUID $l(l=1,2,\ldots,N)$.

Step (ii). Adjust the level structure of SQUID a back to the previous situation [Fig. 2(c)] such that the cavity mode does not couple to this SQUID. In the meanwhile, adjust the level structure of SQUIDs $(1,2,\ldots,N)$ so that the detuning $\delta=\omega_{20}-\omega_c$ meets the condition $\delta \gg g'\sqrt{n+1}$ [Fig. 2(c)]. Here, ω_{20} is the transition frequency between the levels $|0\rangle$

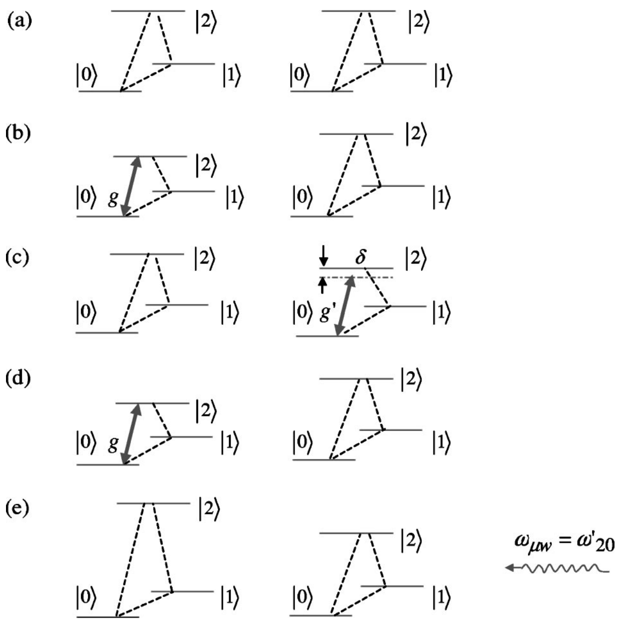

FIG. 2. The relevant level structures (reduced) of the SQUIDs $(1,2,\ldots,N)$ and the auxiliary SQUID a during GHZ-state preparation. In (a), (b), (c), (d), and (e), figures on the left side represent the level structures for SQUID a, while figures on the right size represent the level structures of SQUIDs $(1,2,\ldots,N)$. g is the resonant coupling constant between the cavity mode and the $|0\rangle\leftrightarrow|2\rangle$ transition of SQUID a, while g' is the non-resonant coupling strength between the cavity mode and the $|0\rangle\leftrightarrow|2\rangle$ transition of SQUIDs $(1,2,\ldots,N)$. The $|0\rangle\leftrightarrow|2\rangle$ and $|1\rangle\leftrightarrow|2\rangle$ transition frequencies for SQUIDs $(1,2,\ldots,N)$ are, respectively, denoted as ω_{20} and ω_{21} (not shown), while the $|0\rangle\leftrightarrow|2\rangle$ and $|1\rangle\leftrightarrow|2\rangle$ transition frequencies for SQUID a are, respectively, indicated by ω'_{20} and ω'_{21} (not shown). $\omega_{\mu w}$ is the carried frequency of the applied microwave pulse. In (e), the level spacings for SQUID a is set to be much different from that for each of SQUIDs $(1,2,\ldots,N)$, such that SQUIDs $(1,2,\ldots,N)$ are decoupled from the applied pulse. In addition, the transition between any two levels linked by a dashed line is far off resonant with the cavity mode.

and $|2\rangle$ for SQUIDs $(1,2,\ldots,N)$, $\overline{n}$ is the mean photon number of the cavity mode, and g' is the nonresonant coupling strength between the cavity mode and the $|0\rangle\leftrightarrow|2\rangle$ transition of SQUIDs $(1,2,\ldots,N)$. The effective Hamiltonian of SQUIDs $(1,2,\ldots,N)$ and the cavity mode in the interaction picture is thus given by (setting $\hbar=1$) [7,29]

$$H=\lambda\left[\sum_{l=1}^{N}(|2\rangle_l\langle 2|cc^\dagger-|0\rangle_l\langle 0|c^\dagger c)+\sum_{l,k=1}^{N}|2\rangle_l\langle 0|\otimes|0\rangle_k\langle 2|\right],\quad l\neq k, \tag{8}$$

where $\lambda=(g')^2/\delta$. The first and second terms of Eq. (8) describe the photon-number-dependent Stark shifts, while the third term describes the "dipole" coupling between the lth and the kth SQUIDs mediated by the cavity mode. Note that both the first term and the third term in Eq. (8) acting on the state (7) result in zero, i.e., only the term $-\lambda\Sigma_{l=1}^{N}|0\rangle_l\langle 0|c^\dagger c$ has a contribution to the time evolution of the state (7). Hence, it is easy to see that the state (7), under the Hamiltonian (8), evolves into

$$\left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)|0\rangle_c-\prod_{l=1}^{N}(e^{i\lambda t}|0\rangle_l+|1\rangle_l)|1\rangle_c\right]|0\rangle_a. \tag{9}$$

In the case of $t=\tau_2=\pi/\lambda$, we have from Eq. (9)

$$\left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)|0\rangle_c-(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l)|1\rangle_c\right]|0\rangle_a, \tag{10}$$

which implies that after the interaction time τ_2, a conditional phase shift $|0\rangle_l|0\rangle_c\to|0\rangle_l|0\rangle_c$, $|1\rangle_l|0\rangle_c\to|1\rangle_l|0\rangle_c$, $|0\rangle_l|1\rangle_c\to-|0\rangle_l|1\rangle_c$, and $|1\rangle_l|1\rangle_c\to|1\rangle_l|1\rangle_c$ $(l=1,2,\ldots,N)$ has been realized for each of SQUIDs $(1,2,\ldots,N)$ simultaneously, with the control of the cavity photon.

The state (10) shows that SQUIDs $(1,2,\ldots,N)$ have been entangled with each other after the above operations. Note that they are also entangled with the cavity mode. Thus, all one needs to do now is to disentangle SQUIDs $(1,2,\ldots,N)$ from the cavity mode by the remaining two steps.

Step (iii). Adjust the level structure of SQUIDs $(1,2,\ldots,N)$ back to the previous configuration while bringing the $|0\rangle\leftrightarrow|2\rangle$ transition of SQUID a to resonance with the cavity mode [Fig. 2(d)]. After an interaction time $\tau_3=\pi/(2g)$, the cavity mode and SQUID a undergo the transformation $|0\rangle_a|0\rangle_c\to|0\rangle_a|0\rangle_c$ and $|0\rangle_a|1\rangle_c\to-i|2\rangle_a|0\rangle_c$. The state of the whole system is then given by

$$\left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)|0\rangle_a+i(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l)|2\rangle_a\right]|0\rangle_c, \tag{11}$$

i.e., the original entanglement between SQUIDs $(1,2,\ldots,N)$ and the cavity mode has been transferred to the entanglement between SQUIDs $(1,2,\ldots,N)$ and SQUID a.

Step (iv). The level structures of SQUIDs $(1,2,\ldots,N)$ are kept unchanged while the $|0\rangle\leftrightarrow|2\rangle$ transition of SQUID a is brought to far off resonance with the cavity mode [Fig. 2(e)]. Then, apply a microwave $\pi/2$ pulse $(2\Omega_{02}t=\pi/2$, where t is the pulse duration) with frequency $\omega_{\mu w}=\omega'_{20}$ (ω'_{20} is the $|0\rangle\leftrightarrow|2\rangle$ transition frequency of SQUID a). This pulse creates a rotation $|0\rangle\to|0\rangle-i|2\rangle$ and $|2\rangle\to-i|0\rangle+|2\rangle$ for SQUID a while leaving the state of SQUIDs $(1,2,\ldots,N)$ and the state of the cavity mode unchanged. The state (11) is then transformed as follows:

$$\left\{\left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)+(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l)\right]|0\rangle_a-i\left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)-(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l)\right]|2\rangle_a\right\}|0\rangle_c. \tag{12}$$

The state (12) demonstrates that if a measurement shows the SQUID a is in the state $|0\rangle$, then the remaining SQUIDs

$(1,2,\ldots,N)$ are projected onto the following state:

$$\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)+(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l). \qquad (13)$$

On the other hand, the state (12) demonstrates that if a measurement shows the SQUID a is in the state $|2\rangle$, then SQUIDs $(1,2,\ldots,N)$ are prepared in the state

$$\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)-(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l). \qquad (14)$$

The states (13) and (14) can be rewritten as

$$|+\rangle_1|+\rangle_2\cdots|+\rangle_N+(-1)^N|-\rangle_1|-\rangle_2\cdots|-\rangle_N, \qquad (15)$$

$$|+\rangle_1|+\rangle_2\cdots|+\rangle_N-(-1)^N|-\rangle_1|-\rangle_2\cdots|-\rangle_N, \qquad (16)$$

where $|+\rangle_l=|0\rangle_l+|1\rangle_l$, and $|-\rangle_l=|0\rangle_l-|1\rangle_l$ $(l=1,2,\ldots,N)$. Since $|+\rangle_l$ is orthogonal to $|-\rangle_l$; the states (15) and (16) are both GHZ entangled states [13] of N SQUID qubits.

The irrelevant SQUIDs in each step described above need to be decoupled from the cavity field or pulse. Yet, this requirement can be achieved by the adjustment of the level spacings of the SQUIDs. Note that in a SQUID system, the level spacings can be easily changed by adjusting the external flux Φ_x or the critical current I_c (e.g., for variable barrier rf SQUIDs) [13].

One sees from the above description the following points.

(a) The level spacings of SQUIDs $(1,2,\ldots,N)$ are only adjusted in step (ii).

(b) Because the same detuning δ is set for each of SQUIDs $(1,2,\ldots,N)$, the level spacings for SQUIDs $(1,2,\ldots,N)$ can be synchronously adjusted, e.g., via changing the common external flux Φ_x.

(c) During the operation, the level $|2\rangle$ for each of SQUIDs $(1,2,\ldots,N)$ is unpopulated. Thus, decoherence due to spontaneous emission from this level is greatly suppressed.

(d) As addressed above, when SQUID a is detected in the state $|0\rangle$ $(|2\rangle)$, SQUIDs $(1,2,\ldots,N)$ are prepared in the GHZ state (15) [(16)] with certainty. Hence, the entanglement generation is deterministic.

(e) Since the operation time τ_2 in step (ii) is independent of the number of SQUIDs N, the present method can in principle be applied to create GHZ entangled states with a large number of SQUIDs.

It is necessary to give a discussion on experimental matters. First, the pulse-SQUID interaction time should be much shorter than the energy relaxation time γ^{-1} of the level $|2\rangle$. Second, the typical "cavity–auxiliary SQUID" interaction time τ_1, $\tau_3=\pi/(2g)$ should be much smaller than γ^{-1}. However, the constraint $\tau_2=\pi/\lambda<\gamma^{-1}$ can be greatly relaxed because the level $|2\rangle$ for each of SQUIDs $(1,2,\ldots,N)$ is unpopulated during the whole operation. Third, the total cavity-SQUID interaction time $\tau=\tau_1+\tau_2+\tau_3$ should be much smaller than the lifetime of the cavity mode, which is given by $\kappa^{-1}=Q/2\pi\nu_c$. Here, Q is the (loaded) quality factor of the cavity and ν_c is the cavity field frequency. Last, direct coupling between SQUIDs needs to be negligible since this interaction is not intended. In principle, these requirements can be realized, because (i) the SQUIDs can be made to have a sufficiently long energy relaxation time and thus spontaneous decay of the SQUIDs is negligible during the operation; (ii) the pulse-SQUID interaction time can be shorten by increasing the intensity of the microwave pulses, such that the decay of the level $|2\rangle$ is negligible during the pulse duration; (iii) the typical time τ_1 and τ_3 can be reduced by increasing the "cavity–auxiliary SQUID" coupling constant g (e.g., by varying the energy level structure of the auxiliary SQUID), thus leading to τ_1, $\tau_3\ll\gamma^{-1}$; (iv) κ^{-1} can be increased by employing a high-Q cavity; and (v) direct interaction between SQUIDs can be made negligibly small as long as $D\gg d$ (where D is the distance between the two nearest SQUIDs and d is the linear dimension of each SQUID).

For the sake of definitiveness, let us consider SQUIDs with $C=36$ fF, $L=36$ pH, $\Phi_x=0.4993\Phi_0$, $\beta_L=1.10$, and $R\sim10^8\ \Omega$ which are demonstrated in a recent experiment [30]. Here, β_L is the SQUID's potential shape parameter and R is the junction's damping resistance. With a choice of these parameters, the SQUIDs have the desired three lowest levels as shown in Fig. 1(b). Note that SQUIDs with these parameters are available at the present time [14–16]. For the parameters presented here, the energy relaxation time γ^{-1} is on the order of 1.6×10^{-5} s, the coupling matrix element $\phi_{20}\equiv\langle0|\Phi|2\rangle/\Phi_0$ between the levels $|0\rangle$ and $|2\rangle$ is $\sim2.9\times10^{-2}$, and the $|0\rangle\leftrightarrow|2\rangle$ transition frequency is ~49.2 GHz. Hence, we choose $\nu_c=49.2$ GHz as the cavity mode frequency. For a superconducting standing-wave cavity with a volume $30\times1\times1\ \mathrm{mm}^3$ and a SQUID with a $45\times45\ \mu\mathrm{m}^2$ loop located at one of the antinodes of the cavity-mode magnetic field, a simple calculation gives the coupling constant $g\sim1.2\times10^9\ \mathrm{s}^{-1}$, which results in τ_1, $\tau_3\sim1.3\times10^{-9}$ s, much smaller than γ^{-1}. On the other hand, as a rough estimate, we assume $g'\sim0.5g$ and $\delta\sim10g'$, which can be readily achieved by adjusting the level spacings. As a result, we have $\tau\sim5.5\times10^{-8}$ s, much shorter than $\kappa^{-1}\sim6.4\times10^{-7}$ s for a cavity with $Q=2\times10^5$, which might be available within the present technologies or soon, since a quasi-one-dimensional on-chip superconducting microwave cavity (resonator) with a $Q>10^6$, patterned into a thin superconducting film deposited on the surface of a silicon chip, has been experimentally demonstrated [31] (also see Refs. [32–35] regarding its application for loaded superconducting qubits or semiconductor qubits).

For a cavity with $\nu_c=49.2$ GHz, the wavelength of the cavity mode is $\lambda\sim6$ mm, i.e., 0.2 times of the cavity length. When each SQUID is placed at an antinode of the $\mathbf{B}_c$ field [Fig. 1(a)], the ratio D/d would be ~70 for $d=45\ \mu$m and $N=11$ (where N is the number of SQUIDs in the cavity). Note that the dipole field generated by the current in each SQUID ring at a distance $r\gg d$ decreases as r^{-3}. Thus, the condition of negligible direct coupling between SQUIDs is very well satisfied. The result presented here shows that entangling ten SQUIDs is possible by the use of the cavity described above. We remark that the number of SQUIDs to be entangled can in principle be increased with increment of the cavity length.

In the above, we considered a single-mode microwave cavity. As a matter of fact, this is not necessary, since for a multimode cavity one can choose one mode to interact with the SQUIDs while having all other cavity modes well decoupled from the three lowest levels of the SQUIDs during the operation (e.g., by properly designing the device parameters). In addition, a three-dimensional (3D) cavity described above is not necessarily required, since the method presented here is applicable to 1D or 2D microwave resonators as long as the conditions described above can be met.

III. GHZ-STATE GENERATION WITH MANY ATOMS

In recent years, much attention has been paid to the generation of highly entangled states with atomic systems. Two-atom entangled states have been experimentally demonstrated in microwave cavity QED [3]. In addition, based on cavity QED, theoretical proposals have been presented for creating multiatom entangled states. We may cite, as especially relevant to this work, the very recent work by Zheng [7], where a method was proposed for creating GHZ states with many two-level atoms, by simultaneously sending all atoms through a nonresonant cavity. The major advantage of Zheng's scheme is that the cavity decay is greatly suppressed due to the virtual excitation of the cavity mode during the atoms crossing the cavity. However, it is noted that the excited level $|e\rangle$ of each atom is always populated during the operation. Thus, decoherence, due to spontaneous emission from the excited level $|e\rangle$, may become a severe problem. In the following, we will present an alternative way to implement the same task, which, as shown below, does not involve real excitation of the excited level and therefore decoherence caused by the energy relaxation from the excited level is greatly reduced. The scheme presented here is actually a generalization of the method described in Sec. II to GHZ-state generation in atomic systems.

We point out that it is not our intention to cast aspersions on existing approaches to entanglement; rather we simply wish to add one further element to the discussion in this work. Our discussion focuses on creating entanglement with immunity to decoherence caused by energy relaxation.

Consider N identical atoms $(1,2,\ldots,N)$ each having a Λ-type level configuration formed by two ground states and an excited state. In accordance with the previous section, we use $|0\rangle$ and $|1\rangle$ to represent the two ground states and $|2\rangle$ to indicate the excited state. The GHZ state of N atoms $(1,2,\ldots,N)$ can be prepared using the following prescription.

First, send a two-level atom a (with ground level $|\widetilde{0}\rangle$ and excited level $|\widetilde{1}\rangle$) through a single-mode cavity (Fig. 3). Assume that the atom a is initially in the state $|\widetilde{0}\rangle_a-|\widetilde{1}\rangle_a$ and that the cavity mode is initially in the vacuum state $|0\rangle_c$. Suppose that the atomic transition is resonant with the cavity mode. Choose the atomic velocity appropriately such that the time for the atom a crossing the cavity equals $\pi/(2g)$, where g is the cavity-atom coupling constant. Thus, after the atom a exits the cavity, the cavity is left in the state $|0\rangle_c+i|1\rangle_c$ while the atom a is in the state $|\widetilde{0}\rangle_a$.

Second, send the N atoms $(1,2,\ldots,N)$ initially in the state $\Pi_{l=1}^{N}(|0\rangle_l+|1\rangle_l)$ through the cavity simultaneously (Fig. 3). Assume that the dipole transition between $|0\rangle$ and $|1\rangle$ is

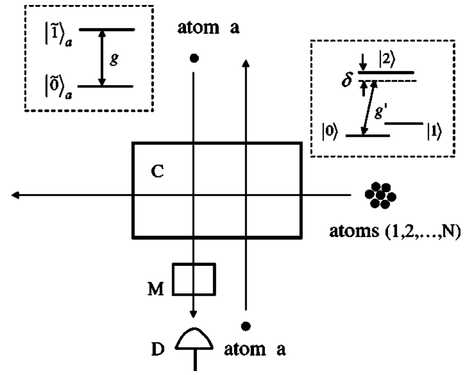

FIG. 3. Sketch of the setup for the preparation of multiatom GHZ states. C, M, and D represent the cavity, the classical field, and the detector, respectively. The two-level structure inside the left dashed-line box is for atom a while the three-level structure inside the right dashed-line box is for atoms $(1,2,\ldots,N)$.

forbidden due to the definite parity of the wave function and the $|1\rangle\leftrightarrow|2\rangle$ transition is decoupled or highly detuned from the cavity mode, while the detuning $\delta=\omega_{20}-\omega_c$ between the $|0\rangle\leftrightarrow|2\rangle$ transition and the cavity mode satisfies the condition $\delta\gg g'\sqrt{\overline{n}+1}$. Here, ω_{20} is the transition frequency between the levels $|0\rangle$ and $|2\rangle$, ω_c is the cavity mode frequency, $\overline{n}$ is the mean photon number of the cavity mode, and g' is the cavity-atom coupling strength for the $|0\rangle\leftrightarrow|2\rangle$ transition. Under this consideration, there is no energy exchange between the cavity mode and the atoms $(1,2,\ldots,N)$, and the effective Hamiltonian for the atoms $(1,2,\ldots,N)$ and the cavity mode has the same form as (8). If the passage time of the atoms $(1,2,\ldots,N)$ through the cavity equals to $\pi/\lambda=\pi\delta/(g')^2$ (e.g., by choosing the atomic velocity properly), then the system will be in the following state:

$$\left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)|0\rangle_c+i(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l)|1\rangle_c\right]|\widetilde{0}\rangle_a. \tag{17}$$

Third, send the atom a back through the cavity at the same velocity (Fig. 3). Thus, after this atom exits the cavity, the state of the system is given by

$$\left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)|\widetilde{0}\rangle_a+(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l)|\widetilde{1}\rangle_a\right]|0\rangle_c. \tag{18}$$

Fourth, let the atom a enter a classical field which performs a Hadamard transformation $|\widetilde{0}\rangle\rightarrow|\widetilde{0}\rangle+|\widetilde{1}\rangle$ and $|\widetilde{1}\rangle\rightarrow|\widetilde{0}\rangle-|\widetilde{1}\rangle$ on the atom a (Fig. 3). After that, the state (18) changes into

$$\Bigg\{\left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)+(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l)\right]|\widetilde{0}\rangle_a + \left[\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)-(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l)\right]|\widetilde{1}\rangle_a\Bigg\}|0\rangle_c. \tag{19}$$

Last, after the atom a exits from the classical field, one can measure its state using a detector (Fig. 3). In the case when the atom a leaves the classical field in the excited state $|\tilde{1}\rangle$, the detector gives a signal by ionizing the excited atom a and detecting the free electron. In contrast, if the atom a leaves the classical field in the ground state $|\tilde{0}\rangle$, the detector gives no signal. Now we assume that the detector gives a signal. In this case, one can see from Eq. (19) that the N atoms $(1,2,\ldots,N)$ collapse into the following entangled GHZ state:

$$\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)+(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l). \tag{20}$$

On the other hand, if the detector gives no signal, it follows from Eq. (19) that the N atoms $(1,2,\ldots,N)$ are projected onto the following entangled GHZ state:

$$\prod_{l=1}^{N}(|0\rangle_l+|1\rangle_l)-(-1)^N\prod_{l=1}^{N}(|0\rangle_l-|1\rangle_l). \tag{21}$$

From the above description, it is concluded that the present scheme has the following advantages.

(i) No adjustment of the level spacings for each atom is required during the operation.

(ii) Similar to Ref. [7], all atoms $(1,2,\ldots,N)$ are initially in the same state. Therefore the initial state of the atoms $(1,2,\ldots,N)$ can be easy to prepare in an experiment.

(iii) The present proposal also requires only one cavity.

(iv) The excited level $|2\rangle$ for each atom is unpopulated during the operation. Hence, decoherence due to spontaneous emission from level $|2\rangle$ is greatly suppressed.

For our method to work, the total cavity-atom interaction time $\pi/g+\pi\delta/(g')^2$ should be much smaller than the cavity decay time κ^{-1}, so that the cavity dissipation is negligible. As a rough estimate, consider $g\sim g'$ and $\delta=10g'$. With this choice, one can easily find that the error probability caused due to the population of the level $|2\rangle$ would be on the order of 0.04 and the required relationship between the coupling constant and the cavity decay rate would be g, $g'\gg 11\pi\kappa$.

IV. CONCLUSION

Before our conclusion, several points related to this work need to be addressed. First, the idea of coupling multiple qubits globally with a resonant structure and tuning the individual qubits to couple and decouple them from the resonator has been previously presented for charge-based qubits [36]. Rather, our scheme is for a different system and it differs in the details of both the qubits and the coupling structure. In our case, we consider a system consisting of flux-based qubits (SQUIDs) coupled via a single-mode microwave cavity field, while the system described in [36] comprises charge qubits and a LC-oscillator mode in the circuit. Second, the type of effective Hamiltonian (8) was proposed previously for a trapped-ion-based quantum processor [29] or atom-cavity- based quantum processor [7]. However, we first note that using this Hamiltonian, a joint phase shift can be performed simultaneously on many qubits with assistance of the cavity photon, which not only simplifies the entanglement preparation but also reduces the energy relaxation significantly. Last, our method is different from that in Ref. [8] where the authors showed how multi-ion GHZ entangled states can be realized; that method is, however, based on performing a controlled-NOT simultaneously on all ions through a dispersive interaction. We note that a joint controlled-NOT operation is rather difficult to implement in the present system due to different physical mechanisms.

In summary, we have proposed a scheme for creating multi-SQUID-qubit GHZ entangled states with the use of a microwave cavity. The present scheme is based primarily on inside-cavity photon creation and absorption via an auxiliary SQUID in the cavity. As a result, rapid adjustments of level spacings of SQUIDs are needed to ensure that the cavity mode can be decoupled from the SQUIDs during pulsed microwave manipulation, or to control the cavity-SQUID interaction. Nevertheless, we believe that the present proposal is of great interest because of its advantages: (i) all operations are performed within the cavity, e.g., no photon detection outside the cavity is required; (ii) the preparation of entanglement is deterministic and independent of the number of SQUIDs in the cavity; (iii) as tunneling between the qubit levels $|0\rangle$ and $|1\rangle$ is not required during the operation, the decay from level $|1\rangle$ can be made negligibly small during the operation, via adjusting the potential barrier between the qubit levels $|0\rangle$ and $|1\rangle$ [13]; and (iv) the entangled state (15) and (16) is a superposition of different combination of two lowest levels $|0\rangle$ and $|1\rangle$; thus decoherence of the generated entanglement, caused by the cavity loss and spontaneous emission, is greatly suppressed. To the best of our knowledge, our scheme is the first to demonstrate the possibility of multiqubit entanglement in SQUIDs within cavity QED. Finally, it is interesting to note that the present method can be applied to implement multiatom GHZ entangled states with tolerance to energy relaxation. The method presented here is quite general; it is applicable to the generation of multiqubit entanglement in other types of solid-state system with the three-level configuration described above, such as quantum dots [37].

ACKNOWLEDGMENTS

This research was partially supported by the National Science Foundation QuBIC program (ECS-0201995), and AFOSR (F49620-01-1-0439), funded under the Department of Defense University Research Initiative on Nanotechnology (DURINT) Program and by the ARDA.

[1] J. W. Pan, M. Daniell, S. Gasparoni, G. Weihs, and A. Zeilinger, Phys. Rev. Lett. **86**, 4435 (2001).

[2] C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland, and C. Monroe, Nature (London) **404**, 256 (2000).

[3] E. Hagley, X. Maitre, G. Nogues, C. Wunderlich, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **79**, 1 (1997); S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, *ibid.* **87**, 037902 (2001).

[4] G. Chen, N. H. Bonadeo, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, and L. J. Sham, Science **289**, 1906 (2000).

[5] L. M. Duan and H. J. Kimble, Phys. Rev. Lett. **90**, 253601 (2003).

[6] J. I. Cirac and P. Zoller, Phys. Rev. A **50**, R2799 (1994); C. C. Gerry, *ibid.* **53**, 2857 (1996).

[7] S. B. Zheng, Phys. Rev. Lett. **87**, 230404 (2001).

[8] J. Steinbach and C. C. Gerry, Phys. Rev. Lett. **81**, 5528 (1998).

[9] K. Mølmer and A. Sørensen, Phys. Rev. Lett. **82**, 1835 (1999).

[10] X. Wang, M. Feng, and B. C. Sanders, Phys. Rev. A **67**, 022302 (2003).

[11] S. L. Zhu, Z. D. Wang, and P. Zanardi, e-print, quant-ph/0403004.

[12] D. M. Greenberger, M. Horne, and A. Zeilinger, Am. J. Phys. **58**, 1131 (1990); D. Bouwmeester *et al.*, Phys. Rev. Lett. **82**, 1345 (1999).

[13] S. Han, J. Lapointe, and J. E. Lukens, *Single-Electron Tunneling and Mesoscopic Devices* (Springer-Verlag, Berlin, 1991), Vol. 31, pp. 219–222.

[14] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296** 886 (2002).

[15] Y. Yu, S. Han, X. Chu, S.-I. Chu, and Z. Wang, Science **296**, 889 (2002).

[16] I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Science **299**, 1869 (2003).

[17] M. F. Bocko, A. M. Herr, and M. J. Feldman, IEEE Trans. Appl. Supercond. **7**, 3638 (1997).

[18] M. H. S. Amin, A. Y. Smirnov, and Alec Maassen van den Brink, Phys. Rev. B **67**, 100508 (2003).

[19] C. P. Yang and S. Han, Phys. Lett. A **321**, 273 (2004).

[20] Z. Kis and E. Paspalakis, Phys. Rev. B **69**, 024510 (2004).

[21] C. P. Yang, S. I. Chu, and S. Han, Phys. Rev. A **67**, 042311 (2003).

[22] A. Yu. Smirnov and A. M. Zagoskin, e-print, cond-mat/0207214.

[23] M. J. Everitt, T. D. Clark, P. B. Stiffell, H. Prance, R. J. Prance, and J. F. Ralph, e-print quant-ph/0307181.

[24] M. Hillery, V. Buzek, and A. Berthiaume, Phys. Rev. A **59**, 1829 (1999).

[25] See, for many references, S. Bose, V. Vedral, and P. L. Knight, Phys. Rev. A **57**, 822 (1998).

[26] D. P. DiVincenzo and P. W. Shor, Phys. Rev. Lett. **77**, 3260 (1996); J. Preskill, Proc. R. Soc. London, Ser. A **454**, 385 (1998).

[27] J. J. Bollinger, W. M. Itano, D. Wineland, and D. Heinzen, Phys. Rev. A **54**, R4649 (1996); S. F. Huelga, C. Macchiavello, T. Pellizzari, A. K. Ekert, M. B. Plenio, and J. I. Cirac, Phys. Rev. Lett. **79**, 3865 (1997).

[28] S. Han, R. Rouse, and J. E. Lukens, Phys. Rev. Lett. **76**, 3404 (1996).

[29] A. Sørensen and K. Mølmer, Phys. Rev. Lett. **82**, 1971 (1999).

[30] Y. Yu, D. Nakada, J. C. Lee, B. Singh, D. S. Crankshaw, T. P. Orlando, K. K. Berggren, and W. D. Oliver, Phys. Rev. Lett. **92**, 117904 (2004).

[31] P. K. Day, H. G. LeDuc, B. Mazin, A. Vayonakis, and J. Zmuidzinas, Nature (London) **425**, 817 (2003).

[32] S. M. Girvin, Ren-Shou Huang, Alexandre Blais, Andreas Wallraff, and R. J. Schoelkopf, e-print cond-mat/0310670.

[33] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, e-print cond-mat/0407325.

[34] P. Zhang, Z. D. Wang, J. D. Sun, and C. P. Sun, e-print quant-ph/0407069.

[35] L. I. Childress, A. S. Sorensen, and M. D. Lukin, e-print quant-ph/0309106.

[36] Y. Makhlin, G. Schoen, and A. Shnirman, Nature (London) **398**, 305 (1999).

[37] For quantum dots, the level spacings can be changed via adjusting the external electronical field. For the details, see P. Pradhan, M. P. Anantram and Kang L. Wang, e-print quant-ph/0002006.