# Atom diode: A laser device for a unidirectional transmission of ground-state atoms 

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#### Abstract

An atom diode, i.e., a device that lets the ground-state atom pass in one direction but not in the opposite direction in a velocity range, is devised. It is based on the adiabatic transfer achieved with two lasers and a third laser potential that reflects the ground state.


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The detailed control of internal and/or translational atomic states is a major goal of quantum optics. Optical elements in which the roles of light and matter are reversed such as mirrors, gratings, interferometers, or beam splitters made of laser light or magnetic fields, allow to manipulate atomic waves. Atom chips [1,2] and atom-optic circuits [3] have been also realized recently. The aim of this paper is to propose simple models for an "atom diode," a laser device that lets the neutral atom in its ground state pass in one direction but not in the opposite direction for a range of incident velocities. A diode is a very basic control element in a circuit and many applications are possible for atomic trapping or quantum information processing.

More specifically, our goal is to model an atom-field interaction so that the ground-state atom is transmitted when traveling, say, from left to right, and it is reflected if coming from the right. We shall describe effective three-level and two-level atom models, for simplicity in one dimension, to achieve the desired behavior. The one-dimensional description is accurate if the atom travels in waveguides formed by optical fields [3], or by electric or magnetic interactions due to charged or current-carrying structures [2]. It can be also a good approximation in free space for atomic packets which are broad in the laser direction, perpendicular to the incident atomic direction, as demonstrated for time-of-arrival measurements by fluorescence [4].

In our models the atom is in an excited state after being transmitted and, in principle, excited atoms could cross the diode "backwards," i.e., from right to left. Nevertheless, an irreversible decay from the excited state to the ground state, will effectively block any backward motion.

Let us denote by $R_{\beta \alpha}^{l}(v)\left[R_{\beta \alpha}^{r}(v)\right]$ the scattering amplitudes for incidence with velocity $v>0$ from the left (right) in channel $\alpha$ and reflection in channel $\beta$. Similarly, we denote by $T_{\beta \alpha}^{l}(v)\left[T_{\beta \alpha}^{r}(v)\right]$ the scattering amplitude for incidence in channel $\alpha$ with velocity $v>0$ from the left (right) and transmission in channel $\beta$ to the right (left). The potential will be such that $\left|T_{31}^{l}(v)\right|^{2} \approx 1, \quad\left|R_{11}^{l}(v)\right|^{2} \approx 0$ and $\left|T_{31}^{r}(v)\right|^{2} \approx 0$, $\left|R_{11}^{r}(v)\right|^{2} \approx 1$. The basic idea is to combine two lasers that achieve stimulated Raman adiabatic passage (STIRAP) with a state-selective reflecting interaction for the ground state. The STIRAP method is well known [5] and consists of an adiabatic transfer of population between levels 1 and 3 by two partially overlapping (in time or space) laser beams (see Fig. 1). The pump laser couples the atomic levels 1 and 2 with Rabi frequency $\Omega_{P}$, and the Stokes laser couples the
states 2 and 3 with Rabi frequency $\Omega_{S}$. We assume here that these two lasers are on resonance with the corresponding transitions. We shall need, in addition, a third laser causing an effective reflecting potential $W(x) \hbar / 2$ for the ground-state component. It could be realized by an intense laser with a large positive (blue) detuning $\Delta$ (laser frequency minus the transition frequency) with respect to a transition with a fourth level, $W(x) \hbar / 2=\Omega_{14}(x)^{2} \hbar / 4 \Delta, \Omega_{14}$ being the corresponding Rabi frequency. Due to the large detuning, there is no pumping so that this type of coupling has a purely mechanical effect. Even a large detuning is very small compared with the optical frequencies between the different levels, so we can neglect the effect of this third laser on the other levels.

Neglecting decay first (we will take it into account later on), the resulting Hamiltonian for the atomic state, within the rotating wave approximation, and in the interaction picture to get rid of any time dependence, is

$$
H_{3 L}=\frac{p_{x}^{2}}{2 m}+\frac{\hbar}{2}\left(\begin{array}{ccc}
W(x) & \Omega_{P}(x) & 0  \tag{1}\\
\Omega_{P}(x) & 0 & \Omega_{S}(x) \\
0 & \Omega_{S}(x) & 0
\end{array}\right)
$$

where $p_{x}=-i \hbar(\partial / \partial x)$ is the momentum operator. The shapes of the Rabi frequencies and the reflecting potential in the model are Gaussian, $\Omega_{P}(x)=\hat{\Omega} \Pi\left(x, x_{P}\right), \Omega_{S}(x)=\hat{\Omega} \Pi\left(x, x_{S}\right)$, $W(x)=\hat{W} \Pi\left(x, x_{W}\right)$ with


FIG. 1. Schematic connection of the atom levels by the different lasers (left figure) and location of the different lasers (right figure).


FIG. 2. (a) Reflection probability $\left|R_{11}^{l / r}(|v|)\right|^{2}$ and (b) transmission probability $\left|T_{31}^{I / r}(|v|)\right|^{2}$; negative $v$ correspond to incidence from the right, positive $v$ correspond to incidence from the left; the mass is the mass of neon, $\Delta x=15 \mu \mathrm{~m}, x_{S}=140 \mu \mathrm{~m}, x_{P}=170 \mu \mathrm{~m}$; threelevel atom: $x_{W}=260 \mu \mathrm{~m}, \hat{\Omega}=0.2 \times 10^{6} s^{-1}, \hat{W}=20 \times 10^{6} s^{-1}$ (thin dashed line), $\hat{\Omega}=1 \times 10^{6} s^{-1}, \hat{W}=100 \times 10^{6} s^{-1}$ (thick dashed line); two-level atom: $\hat{f}^{2}=100 \times 10^{6} s^{-1}$ (solid line, coincides with thick dashed line).

$$
\Pi\left(x, x_{0}\right)=\exp \left(-\frac{\left(x-x_{0}\right)^{2}}{2 \Delta x^{2}}\right)
$$

but similar shapes do not alter the results in any significant way. We shall also assume for simplicity that the shapes and widths of pump laser, Stokes laser, and state-selective reflecting laser potentials are all equal. The location of the three laser beams is shown in Fig. 1.

If the atom is incident from the left in the ground state, it will be transfered by STIRAP to the third state so it is not affected by $W(x)$, and will be transmitted, i.e., the transmission probability $\left|T_{31}^{l}(v)\right|^{2} \approx 1$, while the other reflection and transmission amplitudes for left incidence in the first state $\left(R_{11}^{l}, R_{21}^{l}, R_{31}^{l}, T_{11}^{l}, T_{21}^{l}\right)$ will be approximately zero. If the atom is incident from the right, in the ground state, and with low enough velocity, it is reflected by the potential $W(x) \hbar / 2$. Therefore $\left|T_{31}^{r}(v)\right|^{2} \approx 0 \neq\left|T_{31}^{l}(v)\right|^{2}$ and $\left|R_{11}^{r}(v)\right|^{2} \approx 1$. The other reflection and transmission amplitudes $\left(R_{21}^{r}, R_{31}^{r}, T_{11}^{r}\right.$, $T_{21}^{r}$ ) will be also approximately zero.

This behavior is indeed observed solving numerically the stationary Schrödinger equation with Eq. (1) by the invariant imbedding method [6,7]. The results are shown in Fig. 2. In a velocity range, the "diodic" behavior holds, i.e., $\left|R_{11}^{l}\right|^{2} \approx 0$, $\left|T_{31}^{l}\right|^{2} \approx 1$ and $\left|R_{11}^{r}\right|^{2} \approx 1,\left|T_{31}^{r}\right|^{2} \approx 0$. In this range the other transmission and reflection coefficients for incidence in the


FIG. 3. Limit $v_{\text {max }}$ for "diodic" behavior, $\epsilon=0.01$; three-level atom, the mass is the mass of neon, $\Delta x=15 \mu \mathrm{~m}, x_{S}=140 \mu \mathrm{~m}, x_{P}$ $=170 \mu \mathrm{~m}, x_{W}=260 \mu \mathrm{~m}$.
first state $\left(R_{21}^{l / r}, R_{31}^{l / r}, T_{11}^{l / r}, T_{21}^{l / r}\right)$ are zero. The left-incidence velocity boundary for diodic behavior, $v_{\text {left }}$, is due to the breakdown of the STIRAP effect [8]. The addition of a spontaneous decay rate $\Gamma$ from state 2 does not alter $v_{\text {left }}$ significantly for $\hat{\Omega} / \Gamma \gtrsim 100$. This boundary can be increased by increasing $\hat{\Omega}$. The velocity boundary $v_{\text {right }}$ for right incidence, due to the inability of the reflecting laser to block fast atoms, increases when $\hat{W}$ increases, so that both boundaries can be adjusted independently from each other. We may define $v_{\max }$ as the minimum of $v_{\text {left }}$ and $v_{\text {right }}$.

There is also a lower, positive-velocity boundary for the STIRAP effect, i.e., the STIRAP effect breaks down at extremely low velocities, $v<v_{\text {min }} \approx 0.05 \mathrm{~cm} / s$ with the laser intensities (Rabi frequencies) of the numerical examples. This may appear contradictory since one expects better adiabatic transfer at lower velocities. Indeed this is the case, but only as long as the semiclassical approximation is valid for the translational motion. For sufficiently low velocities the quantum aspects of translational motion become important and atomic reflection occurs.

More precisely, $v_{\max }$ and $v_{\text {min }}$ are defined by imposing that all scattering probabilities from the ground state be small except the ones that define the diode (the probability for transmission to 3 from the left and for reflection to 1 from the right), i.e., they are the limiting upper and lower values for which $\quad \sum_{\alpha=1}^{3}\left(\left|R_{\alpha 1}^{l}\right|^{2}+\left|T_{\alpha 1}^{r}\right|^{2}\right)+\sum_{\alpha=1}^{2}\left(\left|R_{\alpha+1,1}^{r}\right|^{2}+\left|T_{\alpha 1}^{l}\right|^{2}\right)+(1$ $\left.-\left|T_{31}^{l}\right|^{2}\right)+\left(1-\left|R_{11}^{r}\right|^{2}\right)<\epsilon$ is satisfied. In Fig. 3, $v_{\text {max }}$ is plotted versus $\hat{\Omega}$ and $\hat{W}$. For the intensities considered $v_{\max }$ is in the ultracold regime below $1 \mathrm{~m} / \mathrm{s}$. In the $v_{\max }$ surface, $v_{\text {right }}$ due to reflection failure is more restrictive in the hillside represented by circles, whereas $v_{\text {left }}$, due to "high-velocity" STIRAP failure, is more restrictive in the hillside with triangles. From the numerical scales used for $\hat{\Omega}$ and $\hat{W}$, it becomes clear that, generally, reflection failure will be more problematic than STIRAP failure in practice.

Notice that a unidirectional transmission can also be obtained for a two-level atom. It is well known [9] that the three-level Hamiltonian (1) with $W=0$ can be reformulated as a two-level one, but here we use a different idea to construct directly a two-level potential with the "diodic" prop-


FIG. 4. Schematic connection of the atom levels by the different lasers (left figure) and the order of the functions $f_{S}, f_{P}$ (right figure).
erty. Assume first that we can neglect the kinetic term in the Hamiltonian and that the motion in $x$ direction is classical. Let us define the two-position-dependent eigenvectors of the two-level potential as

$$
\begin{gathered}
\zeta_{1}(x)=\frac{1}{\sqrt{f_{P}^{2}(x)+f_{S}^{2}(x)}}\binom{f_{S}(x)}{-f_{P}(x)}, \\
\zeta_{2}(x)=\frac{1}{\sqrt{f_{P}^{2}(x)+f_{S}^{2}(x)}}\binom{f_{P}(x)}{f_{S}(x)}
\end{gathered}
$$

With the order of $f_{S}, f_{P} \geqslant 0$ shown on the right-hand side of Fig. 4, we get for Gaussian (or similar) functions $f_{S}$ and $f_{P}$ the asymptotic properties

$$
\begin{gathered}
\zeta_{1}(-\infty)=\binom{1}{0}, \quad \zeta_{1}(+\infty)=\binom{0}{-1}, \\
\zeta_{2}(-\infty)=\binom{0}{1}, \quad \zeta_{2}(+\infty)=\binom{1}{0} .
\end{gathered}
$$

This means that ground and excited states are asymptotically swapped. $\zeta_{1}$ should correspond to the eigenvalue $\lambda_{1}=0$ which results in adiabatic transfer from ground to excited state if the atom impinges from the left, and $\zeta_{2}$ should correspond to $\lambda_{2}=(\hbar / 2)\left[f_{P}^{2}(x)+f_{S}^{2}(x)\right] \gg 0$, so there will be nearly full reflection if the atom impinges from the right. The eigenfunctions and eigenvalues define the potential, and the two-level Hamiltonian is

$$
H_{2 L}=\frac{p_{x}^{2}}{2 m}+\frac{\hbar}{2}\left(\begin{array}{cc}
f_{P}^{2}(x) & f_{P}(x) f_{S}(x)  \tag{2}\\
f_{P}(x) f_{S}(x) & f_{S}^{2}(x)
\end{array}\right)
$$

We have calculated the scattering amplitudes numerically with $f_{P}(x)=\hat{f} \Pi\left(x, x_{P}\right)$ and $f_{S}(x)=\hat{f} \Pi\left(x, x_{S}\right)$ for right and left incidence and observed the diodic behavior, (see Fig. 2.) The two-level Hamiltonian can be also used as a diode for incidence in the excited state. Then it works in the opposite


FIG. 5. Scheme for the time-dependent simulation including decay. The mass is the mass of neon, $x_{S}=140 \mu \mathrm{~m}, \hat{\Omega}_{S}=0.2 \times 10^{6} \mathrm{~s}^{-1}$, $x_{P}=170 \mu \mathrm{~m}, \quad \hat{\Omega}_{P}=0.2 \times 10^{6} \mathrm{~s}^{-1}, \quad x_{W}=260 \mu \mathrm{~m}, \quad \hat{W}=10 \times 10^{6} \mathrm{~s}^{-1}$, $\Delta x=15 \mu \mathrm{~m}, x_{0}=40 \mu \mathrm{~m}\left(v_{0}>0\right)$ or $x_{0}=360 \mu \mathrm{~m}\left(v_{0}<0\right)$, and $\Delta v_{0}$ $=0.1 \mathrm{~cm} / \mathrm{s}$.
direction, i.e., $\left|T_{13}^{r}(v)\right|^{2} \approx 1,\left|R_{33}^{r}(v)\right|^{2} \approx 0$ and $\left|T_{13}^{l}(v)\right|^{2} \approx 0$, $\left|R_{33}^{l}(v)\right|^{2} \approx 1$. This is not the case for the Hamiltonian (1) unless an additional potential acting on the third level is added.

Let us return to the three-level atom to study the possible effect of decay from the third state to the first state with a relatively small decay rate $\gamma$. This is unlikely a spontaneous process but it can be forced by a laser coupling of the third state to an auxiliary state decaying to the ground state. The process may be characterized by an effective decay rate from 3 to 1 [11]. We examine the time-dependent case, see Fig. 5, by means of a one-dimensional master equation which includes the effect of recoil, see [10],

$$
\begin{align*}
\frac{\partial}{\partial t} \rho= & -\frac{i}{\hbar}\left[H_{3 L}, \rho\right]_{-}-\frac{\gamma}{2}\{|3\rangle\langle 3|, \rho\}_{+} \\
& +\gamma \int_{-1}^{1} d u \frac{3}{8}\left(1+u^{2}\right) \exp \left(i \frac{m v_{r e c}}{\hbar} u x\right)|1\rangle \\
& \times\langle 3| \rho|3\rangle\langle 1| \exp \left(-i \frac{m v_{r e c}}{\hbar} u x\right) \tag{3}
\end{align*}
$$

The initial state at $t=0$ is $\rho(0)=\left|\Psi_{0}\right\rangle\left\langle\Psi_{0}\right|$, namely a Gaussian wave packet with mean velocity $v_{0}$,

$$
\Psi_{0}(x)=\frac{1}{N}\left(\begin{array}{l}
1 \\
0 \\
0
\end{array}\right) \exp \left(-\frac{\Delta v_{0} m}{2 \hbar}\left(x-x_{0}\right)^{2}+i \frac{v_{0} m}{\hbar} x\right)
$$

where $N$ is a normalization constant. We solve the master equation by using the quantum jump technique [12]. Let $t_{\max }$ be a large time such that the resulting wave packet $\Psi_{j}\left(t_{\max }\right)$ of nearly every quantum "trajectory" $j$ separates into right and left moving parts far from the interaction region but possibly with third-state components, not decayed yet at $t_{\max }$. By averaging over all trajectories we get

$$
\begin{equation*}
\hat{p}_{r}=\int_{0}^{\infty} d v\left(\langle v| \rho_{11}\left(t_{\max }\right)|v\rangle+\langle v| \rho_{33}\left(t_{\max }\right)|v\rangle\right) \tag{4}
\end{equation*}
$$

which is plotted in Fig. 6 as a function of $v_{0}$ for different $\gamma$ and $v_{\text {rec }}$. The error bars, defined by the absolute difference between averaging over $n / 2$ and $n$ trajectories, are smaller than the symbol size.


FIG. 6. Probability $\hat{p}_{r}$ of traveling to the right after $t_{\max }$ $=400 \mu \mathrm{~m} / v_{0} ; v_{\text {rec }}=3 \mathrm{~cm} / \mathrm{s}, \gamma=20 \mathrm{~s}^{-1}$ (down-pointing triangles); $v_{\text {rec }}=3 \mathrm{~cm} / \mathrm{s}, \gamma=40 \mathrm{~s}^{-1}$ (up-pointing triangles); $v_{\text {rec }}=6 \mathrm{~cm} / \mathrm{s}, \gamma$ $=20 s^{-1}$ (circles); $n=1000$ trajectories; the dashed line indicates $\hat{p}_{r}=0.95$; other parameters are the same as in Fig. 5.

A value $\hat{p}_{r}\left(v_{0}\right) \approx 1$ for $v_{0}<0$ means that nearly all atoms coming from the right are reflected. The reflection probability is not affected by the decay since the reflected atoms are rarely excited during the collision.

A value $\hat{p}_{r}\left(v_{0}\right) \approx 1$ for $v_{0}>0$ means that nearly all atoms coming from the left are transmitted and will be finally in the
ground state moving to the right. This is true for $v_{0}$ $\geqslant 8 \mathrm{~cm} / \mathrm{s}$ [with $\hat{p}_{r}\left(v_{0}\right) \geqslant 0.95$ ] for all examined combinations of decay rate $\gamma$ and recoil velocity $v_{\text {rec }}$. Therefore, for not-too-low velocities, a large part of the atoms will be transmitted and stay finally in the ground state, i.e., the atom diode works also with decay and recoil, with the advantage that decay prevents the backward motion of excited atoms. The decrease of $\hat{p}_{r}$ for low, positive velocities is due to the atom decay before passing the potential $W(x) \hbar / 2$.

In summary, we have presented a simple model for an atom diode that can be realized with laser interactions, a device which can be passed by the atom in one direction but not in the opposite direction.

Note added. Recently, we received a manuscript of Raizen et al.[13] in which, independently of our paper, a similar idea for unidirectional atomic transmission is discussed.

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