Field correlations in electromagnetically induced transparency

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The interaction of a bichromatic quantized field with three-level atoms in a Λ configuration is analyzed. We calculate the correlation functions of the field emitted by the atoms when the driven field is in a coherent state. We consider the cases where the atoms are inside and outside a cavity.

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I. INTRODUCTION

When we illuminate an atom with light which has a frequency that matches the frequency of a particular atomic transition, a resonance condition occurs and there is larger fluorescence. Electromagnetically induced transparency (EIT) [1] is a technique that can be used in certain cases to eliminate this fluorescence. This means transforming an otherwise opaque medium into a transparent one. This phenomenon has been observed in systems of three-level atoms in a Λ configuration, where the two quasidegenerate lower levels may perform optical transitions to the upper level by interacting with the electromagnetic field. The physical explanation of this phenomenon is related to the existence of a dark state, which is a state of the atom that is decoupled from the electromagnetic field when resonance occurs. This dark state is given by a certain linear combination of the two lower levels [2]. If the atom is in this state, the absorption probability is zero. For real atoms the radiative spontaneous emission rates are different from zero. However, if we can neglect the decoherence between the two lower levels, we may show the very interesting property that the steady state of this system is still the same dark state [3].

When the system is not exactly in a dark state, radiation is emitted from the atoms. The steady-state quadratures of the irradiated fields for driven three-level atoms in a Λ configuration have been extensively studied. Most of these studies were oriented in finding squeezing in one of the quadratures. For two-level atoms, the largest squeezing is found in the out-of-phase quadrature of the fluorescence field and is proportional to the difference between the upper-level population and the probability of a transition between the two levels [4]. In EIT, the upper-level population and the dipole moments between the upper and lower levels vanish at resonance and there is no fluorescence. Nevertheless, squeezing is expected [5,6] when the system has rather asymmetric parameters and is outside the two-photon resonance. Squeezing is also expected when the two lower levels are in resonance with the upper level but decoherence among them is so high that the atom behaves almost as a two-level atom [7]. From these considerations it appears that coherence between the two lower levels, which is responsible for the EIT phenomena, excludes the appearance of significant squeezing. However, coherence between atomic levels may be useful in enhancing squeezing as was demonstrated recently by Li, Gao, and Zhu [8]. They numerically studied the steady-state spectrum of the optimum phase quadrature in the resonance fluorescence of a coherently driven four-level atom. They found quadrature squeezing for a range of spectrum frequencies and related enhancing of squeezing to the coherence of the two upper levels.

In a recent paper [9], an experiment to measure the spectrum of the intensity-intensity correlations of the probe and pump fields in EIT was reported. The authors found, at resonance, super-Poissonian statistics in the intensity fluctuations and that the two initially uncorrelated fields become correlated after interacting with the atoms. In this paper we analyze in detail the steady-state spectrum of the correlation function fluctuations of the field emitted by three-level atoms in a Λ configuration interacting with two fields (pump and probe) modes. We focus our analysis on the case of Λ systems with intrinsic features. By this we mean that the two Rabi frequencies associated with the two optical transitions are equal, the two spontaneous emission rates from the upper to the lower levels are equal, and the decoherence of the two lower levels is small compared with that due to the spontaneous emission rates. In Sec. II we obtain analytical expressions for the steady-state spectrum of the correlation functions of the resonance fluorescence emitted by the atom. We study two cases: (a) each mode of the field (pump and probe) is in resonance with an optical transition of the atom, and (b) only one mode (pump field) is in resonance and the other (probe field) is scanned in frequency. In Sec. III we again consider the two cases of Sec. II and numerically study the correlation functions of the output field when the atoms are inside a two-mode cavity, which is fed by input coherent-state fields. In Sec. IV we present our conclusions.

II. FLUORESCENCE FIELD CORRELATIONS

Consider the atom as if it were a three-level atom in a Λ configuration (see Fig. 1). The two lower-energy levels are designated by $|1\rangle$ and $|2\rangle$ while the upper level is designated by $|0\rangle$. Levels 1 and 2 are quasidegenerate, level 1 is metastable, and electromagnetic optical transitions from the upper level to levels 1 and 2 are allowed. This is a good approximation, for example, in experiments involving ^{85}Rb vapor,

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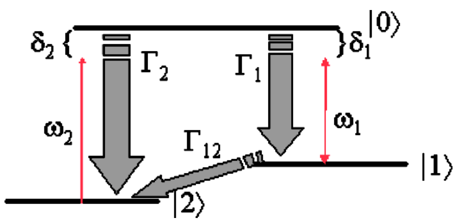

FIG. 1. An atom in Λ configuration. Γ_1 and Γ_2 represent radiative decaying constants. Γ_{12} represents a phenomenological phase decay.

where electromagnetic transitions among the ground state ($|5S_{1/2},F=3\rangle,|5S_{1/2},F=2\rangle$) and the excited ($|5P_{3/2},F'=3\rangle$) levels are important.

The semiclassical atom-field interaction Hamiltonian, in the rotating-wave approximation and in the interaction representation, is given by [10]

$$H_{\mathrm{int}} = \hbar(g_1\hat{\sigma}'_{10}\alpha_1^* e^{i\delta_1 t} + g_2\hat{\sigma}'_{20}\alpha_2^* e^{i\delta_2 t} + \mathrm{H.c.}) + H_{\mathrm{res}}, \quad (1)$$

where $\hat{\sigma}'_{ij}=|i\rangle\langle j|$, and α_1 and α_2 are the amplitudes of the field modes with frequencies ω_1 and ω_2, which couple with the transitions among levels 1,0 and 2,0, respectively. $\delta_1=\omega_1-\omega_{01}$ and $\delta_2=\omega_2-\omega_{02}$ are the frequency detunings of the fields with respect to the atomic transitions with frequency ω_{01} and ω_{02}, respectively. H_{res} describes the interaction of the atom with the modes of the quantized field and will be treated as a reservoir.

We consider an approach similar to the usual one used in resonance fluorescence [11] of two-level atoms. The field is decomposed into two: the initial field plus the irradiated field from the atoms. The far-field expression of the latter depends on the dipole operators $\hat{\sigma}'_{0i}$ and is given by

$$\hat{E}(r,t) = \eta_1\hat{\sigma}'_{01}(t-r/c) + \eta_2\hat{\sigma}'_{02}(t-r/c) + \mathrm{H.c.}, \quad (2)$$

where

$$\eta_i = -\frac{\omega_{0i}^2}{4\pi\epsilon_0 c^2 r^3}[(\mathbf{p_{0i}}\times\mathbf{r})\times\mathbf{r}], \quad (3)$$

$\mathbf{p_{0i}}$ being the atom dipole moment, ϵ_0 the electric permeability and $\mathbf{r}$ the position vector. We want to note that if the initial quantum state that describes the pump and probe fields is a product of coherent states with amplitudes α_1 and α_2, the results obtained using the above semiclassical formalism coincide with those obtained from the quantum formalism [12,13].

Following the usual resonance fluorescence approach for the time evolution of the Heisenberg operators, the reservoir operators in H_{res} are interpreted as Langevin fluctuation operators, analogous to the Langevin fluctuation forces in classical statistical mechanics. Making the additional transformation $\hat{\sigma}_{0j}=\hat{\sigma}'_{0j}\exp(i\delta_j)$ and $\hat{\sigma}_{12}=\hat{\sigma}'_{12}\exp\, i(\delta_1-\delta_2)$, the following Heisenberg-Langevin equations for the atom and field operators are obtained:

$$\frac{d}{dt}\hat{w}_1 = \frac{1}{3}(-2\Gamma_1-\Gamma_2)(1+\hat{w}_1+\hat{w}_2) - 2i\Omega_1\hat{\sigma}_{01} + 2i\Omega_1\hat{\sigma}_{10} - i\Omega_2\hat{\sigma}_{02} + i\Omega_2\hat{\sigma}_{20} + \hat{f}_{w_1}, \quad (4a)$$

$$\frac{d}{dt}\hat{w}_2 = \frac{1}{3}(-\Gamma_1-2\Gamma_2)(1+\hat{w}_1+\hat{w}_2) - i\Omega_1\hat{\sigma}_{01} + i\Omega_1\hat{\sigma}_{10} - 2i\Omega_2\hat{\sigma}_{02} + 2i\Omega_2\hat{\sigma}_{20} + \hat{f}_{w_2}, \quad (4b)$$

$$\frac{d}{dt}\hat{\sigma}_{10} = \left(-\frac{\Gamma_1+\Gamma_2}{2} + i\delta_1\right)\hat{\sigma}_{10} + i\Omega_1\hat{w}_1 - i\Omega_2\hat{\sigma}_{12} + \hat{f}_{10}, \quad (4c)$$

$$\frac{d}{dt}\hat{\sigma}_{20} = \left(-\frac{\Gamma_1+\Gamma_2}{2} + i\delta_2\right)\hat{\sigma}_{20} + i\Omega_2\hat{w}_2 - i\Omega_1\hat{\sigma}_{21} + \hat{f}_{20}, \quad (4d)$$

$$\frac{d}{dt}\hat{\sigma}_{21} = (-\Gamma_{12} - i\delta_r)\hat{\sigma}_{21} - i\Omega_1\hat{\sigma}_{20} + i\Omega_2\hat{\sigma}_{01} + \hat{f}_{21}, \quad (4e)$$

where $\hat{w}_j=\hat{\sigma}_{00}-\hat{\sigma}_{jj}$ and $\delta_r=\delta_1-\delta_2$ and we define the phases such that $\Omega_i=g_i\alpha_i$ is real and positive. Γ_1 and Γ_2 are the decaying constants from level $|0\rangle$ to levels $|1\rangle$ and $|2\rangle$, respectively. We also have included a phenomenological phase decay term, Γ_{12}, which accounts for the loss of coherence among the lower levels mainly due to collisions. We also added the Langevin fluctuation operator $\hat{f}_{12}$, associated with the decay term Γ_{12}, in order to satisfy the fluctuation-dissipation theorem and maintain the correct commutation relations of the operators.

The Langevin fluctuation operators $\hat{f}$'s are assumed to be δ correlated, with zero mean:

$$\langle\hat{f}_x\rangle = 0, \quad (5)$$

$$\langle\hat{f}_x(t)\hat{f}_y(t')\rangle = D_{xy}\delta(t-t'), \quad (6)$$

where x and y label the fluctuation operators. It will be useful to rewrite Eqs. (4) and the Hermitian conjugate of Eqs. (4c)–(4e), in a more compact form, as a matrix equation

$$\frac{d}{dt}\hat{\boldsymbol{o}} = \mathbf{A}\cdot\hat{\boldsymbol{o}} + \hat{\boldsymbol{F}}, \quad (7)$$

where the column vectors $\hat{\boldsymbol{o}}$ and $\hat{\boldsymbol{F}}$ have components

$$\hat{\boldsymbol{o}}^T = (\hat{\sigma}_{02},\hat{\sigma}_{01},\hat{\sigma}_{12},\hat{w}_1,\hat{w}_2,\hat{\sigma}_{21},\hat{\sigma}_{10},\hat{\sigma}_{20}),$$

$$\hat{\boldsymbol{F}}^T = (\hat{f}_{02},\hat{f}_{01},\hat{f}_{12},\hat{f}_{w_1},\hat{f}_{w_2},\hat{f}_{21},\hat{f}_{10},\hat{f}_{20}), \quad (8)$$

and the matrix $\mathbf{A}$ is easily obtained from Eqs. (4) and their Hermitian conjugates.

The atom diffusion coefficient can be obtained using the generalized Einstein relations [14]. The nonzero diffusion coefficients are given by

$$D_{w_1w_1} = (4\Gamma_1 + \Gamma_2)\langle\hat{\sigma}_{00}\rangle,$$

$$D_{w_2w_2} = (\Gamma_1 + 4\Gamma_2)\langle\hat{\sigma}_{00}\rangle,$$

$$D_{w_1w_2} = D_{w_2w_1} = (2\Gamma_1 + 2\Gamma_2)\langle\hat{\sigma}_{00}\rangle,$$

$$D_{w_1\sigma_{01}} = D^*_{\sigma_{10}w_1} = (2\Gamma_1 + \Gamma_2)\langle\hat{\sigma}_{01}\rangle,$$

$$D_{w_1\sigma_{02}} = D^*_{\sigma_{20}w_1} = (2\Gamma_1 + \Gamma_2)\langle\hat{\sigma}_{02}\rangle,$$

$$D_{w_2\sigma_{01}} = D^*_{\sigma_{10}w_2} = (\Gamma_1 + 2\Gamma_2)\langle\hat{\sigma}_{01}\rangle,$$

$$D_{w_2\sigma_{02}} = D^*_{\sigma_{20}w_2} = (\Gamma_1 + 2\Gamma_2)\langle\hat{\sigma}_{02}\rangle,$$

$$D_{\sigma_{10}\sigma_{01}} = \Gamma_1\langle\sigma_{00}\rangle + (\Gamma_1 + \Gamma_2)\langle\sigma_{11}\rangle,$$

$$D_{\sigma_{20}\sigma_{02}} = \Gamma_2\langle\sigma_{00}\rangle + (\Gamma_1 + \Gamma_2)\langle\sigma_{22}\rangle,$$

$$D_{\sigma_{01}\sigma_{12}} = D^*_{\sigma_{21}\sigma_{10}} = \Gamma_{12}\langle\hat{\sigma}_{02}\rangle,$$

$$D_{\sigma_{02}\sigma_{21}} = D^*_{\sigma_{12}\sigma_{20}} = \Gamma_{12}\langle\hat{\sigma}_{01}\rangle,$$

$$D_{\sigma_{12}\sigma_{21}} = \Gamma_1\langle\hat{\sigma}_{00}\rangle + 2\Gamma_{12}\langle\hat{\sigma}_{11}\rangle,$$

$$D_{\sigma_{21}\sigma_{12}} = \Gamma_2\langle\hat{\sigma}_{00}\rangle + 2\Gamma_{12}\langle\hat{\sigma}_{22}\rangle,$$

$$D_{\sigma_{10}\sigma_{02}} = D^*_{\sigma_{20}\sigma_{01}} = (\Gamma_1 + \Gamma_2 - \Gamma_{12})\langle\sigma_{12}\rangle. \quad (9)$$

The steady-state solutions of the operator equations for the average of the variables are obtained by setting the time derivatives equal to zero and using the fact that the average of the Langevin fluctuation forces is zero. In this case we have an algebraic linear system of equations that can be solved by a straightforward calculation. The general solution of this system of equations is given in the Appendix.

The absorption coefficient of the field i is proportional to the imaginary part of σ_{i0}. Using Eq. (A3) of the Appendix and in the case of $\delta_1=0$ and $\Gamma_{12},\delta_2 \ll \Omega_1,\Omega_2$ we have

$$\mathrm{Im}(\sigma_{20}) \approx \frac{\Gamma_2\Omega_1{}^2\Omega_2\left(\Gamma_{12} + \dfrac{(\Gamma_1+\Gamma_2)\delta_2{}^2}{2(\Omega_1{}^2+\Omega_2{}^2)}\right)}{(\Omega_1{}^2+\Omega_2{}^2)(\Gamma_2\Omega_1{}^2+\Gamma_1\Omega_2{}^2)}. \quad (10)$$

In this case the absorption is zero if we do not have any source of decoherence in the lower levels ($\Gamma_{12}=0$) and $\delta_2=0$. In Fig. 2 we show a typical EIT absorption curve, when the pump (field 1) is in resonance ($\delta_1=0$) and the probe (field 2) frequency is scanned.

We are mostly interested in calculating the spectrum of the field quadrature correlation functions $\Delta Y^i_\theta(\omega)$ and $\Delta C(\theta,\phi,\omega)$, defined as

$$\Delta Y^i_\theta(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle :\hat{Y}^i_\theta(t), \hat{Y}^i_\theta(t+\tau):\rangle, \quad (11)$$

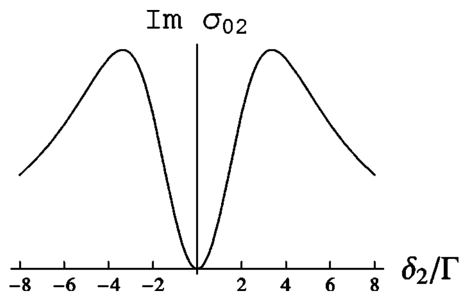

FIG. 2. EIT absorption curve. The parameters are $\Gamma_{12}=0$, $\Omega_1=\Omega_2=2\Gamma$, and $\delta_1=0$.

$$\Delta C(\theta_1,\theta_2,\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle :\hat{Y}^1_{\theta_1}(t), \hat{Y}^2_{\theta_2}(t+\tau):\rangle, \quad (12)$$

where $\langle :\hat{A},\hat{B}:\rangle = \langle :\hat{A}\hat{B}:\rangle - \langle\hat{A}\rangle\langle\hat{B}\rangle$, $:\ldots:$ meaning normal ordering. The normal ordering we choose here is the one shown in the components of the vector operators defined in Eq. (8). The θ quadrature of the field is given by

$$\hat{Y}^i_\theta(t) = [\hat{E}^{i(+)}(t)\exp(i\theta) + \hat{E}^{i(-)}\exp(-i\theta)]/\eta_i, \quad (13)$$

where $\hat{E}^{i(+)} \sim \hat{\sigma}_{0i}$ and $\hat{E}^{i(-)} \sim \hat{\sigma}_{i0}$ are the operators for the positive- and negative-frequency parts of the ith mode of the radiated field.

The dynamics of fluctuations around the steady state is independent of the initial conditions. Therefore, we can use Fourier transforms to solve Eqs. (4) for the operators $\hat{\sigma}_{ij}$ as a function of the Fourier transforms of the Langevin fluctuation operators. Taking the Fourier transform of Eq. (7), we obtain the matrix equation

$$\hat{\boldsymbol{o}}(\omega) = (i\omega\mathbb{1} - \boldsymbol{A})^{-1}\hat{\boldsymbol{F}}(\omega), \quad (14)$$

where $\hat{o}_i(\omega) = (1/\sqrt{2\pi})\int_{-\infty}^{\infty}\hat{o}_i(t)\exp(i\omega t)dt$ and $\hat{F}_i(\omega) = (1/\sqrt{2\pi})\int_{-\infty}^{\infty}\hat{F}_i(t)\exp(i\omega t)dt$ are the Fourier transforms of $\hat{o}_i(t)$ and $\hat{F}_i(t)$, respectively.

The Fourier transforms of the quadrature autocorrelation function $\Delta Y^i_\theta(\omega)$ and of the quadrature probe-pump correlation function $\Delta C(\theta,\phi,\omega)$ may be obtained through linear combinations of functions $S_{ij}(\omega)$, which are the Fourier transforms of the two-time atom operator correlation functions. They may be obtained using the Wiener-Khinchine theorem and are given by [10]

$$\langle\hat{o}_i(\omega)\hat{o}_j(\omega')\rangle = S_{ij}(\omega)\delta(\omega+\omega'). \quad (15)$$

From Eq. (14) we may easily calculate the functions $S_{ij}(\omega)$:

$$\boldsymbol{S}(\omega) = (\boldsymbol{A} + i\omega\mathbb{1})^{-1}\cdot\boldsymbol{D}\cdot(\boldsymbol{A}^\dagger - i\omega\mathbb{1})^{-1}, \quad (16)$$

where $\boldsymbol{D}\delta(\omega+\omega') = \langle\boldsymbol{F}\cdot\boldsymbol{F}^T\rangle$. The components of the diffusion matrix $\boldsymbol{D}$ are the diffusion coefficients given in Eq. (9).

Using these results, we obtain the analytical solution for the spectrum of quadrature correlation functions:

$$\Delta Y_{\theta}^{2}(\omega) = S_{11}(\omega)\exp(i2\theta) + S_{88}(\omega)\exp - (i2\theta) + 2S_{18}(\omega) \tag{17}$$

and

$$\Delta C(\theta_1,\theta_2,\omega) = S_{12}(\omega)e^{i(\theta_1+\theta_2)} + S_{78}(\omega)e^{-i(\theta_1+\theta_2)} + S_{17}(\omega)e^{i(\theta_2-\theta_1)} + S_{28}(\omega)e^{i(\theta_1-\theta_2)}. \tag{18}$$

In the Appendix we give some general results for these correlations.

When the system is in resonance and $\Gamma_{12}=0$, it is easy to see from Eq. (A5) of the Appendix that the steady state is the dark state [2]

$$|DS\rangle = \frac{\Omega_2|1\rangle - \Omega_1|2\rangle}{\sqrt{|\Omega_1|^2 + |\Omega_2|^2}}. \tag{19}$$

Therefore, in this situation, the medium is completely transparent and $\Delta C(\theta_1,\theta_2,\omega)$ and $\Delta Y_{\theta}^{i}(\omega)$ vanish, as can be easily seen from Eqs. (A9) and (A10).

A. Observation at the laser frequency

We start by studying the spectrum of the correlation functions at the laser frequency—that is, when $\omega=0$.

When the system is in resonance and $\Gamma_{12}>0$ the correlations are different from zero. In Eqs. (A9) and (A10) of the Appendix we give general results for the correlations at $\omega=0$ and for $\Gamma_{12}\ll\Gamma_1,\Gamma_2$. We consider here the interesting and special case when $\Omega_1=\Omega_2=\Omega$, $\Gamma_1=\Gamma_2=\Gamma$ and $\Gamma_{12}\ll\Gamma$:

$$\Delta Y_{\theta}^{2}(\omega=0) = \Delta Y_{\theta}^{1}(\omega=0) = \frac{\Gamma_{12}}{2|\Omega|^2\Gamma^2}[|\Omega|^2 + 2\Gamma^2 + (|\Omega|^2 - \Gamma^2)\cos(2\theta)], \tag{20}$$

$$\Delta C(\theta_1,\theta_2,\omega=0) = \frac{\Gamma_{12}}{2|\Omega|^2\Gamma^2}[(2|\Omega|^2 - \Gamma^2)\cos\,\theta_1\cos\,\theta_2 + \Gamma^2\sin\,\theta_1\sin\,\theta_2]. \tag{21}$$

We can easily see that $\Delta Y_{\theta}^{i}(\omega=0)$ is always positive. Therefore, there is no squeezing at the frequency of the laser in any quadrature $Y_{\theta}^{i}(\omega=0)$.

In the case of small fluctuations, the spectrum of the quadrature correlation functions in phase with the fields ($\theta=0$) is approximately proportional to the Fourier transforms of the intensity-intensity correlations. This means that $\Delta Y_0^2(\omega)$ in Eq. (20) is proportional to the Fourier transform of the field intensity autocorrelation and $\Delta C(0,0,\omega)$ in Eq. (21) is proportional to the Fourier transform of the probe-pump intensity correlation. These quantities have been recently measured in laboratory [9].

From Eq. (21), we easily see that for $\Gamma<\sqrt{2}|\Omega|$, the spectrum of the probe-pump intensity correlation at $\omega=0$, which is proportional to $\Delta C(0,0,0)$, is positive, while for $\Gamma>\sqrt{2}|\Omega|$, these correlations are negative. If there is no damping ($\Gamma=0$) or dephasing ($\Gamma_{12}=0$), the system (unless it would be prepared in the dark state) makes Rabi oscillations between the state $|0\rangle$ and a state perpendicular to the dark state, with a frequency $\sqrt{2}|\Omega|$ [2]. Using this fact, the results for the correlation might be explained as follows: When $\Gamma_{12}=0$ the steady state is a dark state and the correlation is zero. When $\Gamma_{12}>0$, the system is most of the time in the dark state but from time to time a fluctuation causes the atom to leave it. Before it goes back to the dark state, in the typical time $1/\Gamma$, it tries to visit the state $|0\rangle$ with a frequency $\sqrt{2}|\Omega|$. When $\Gamma<\sqrt{2}|\Omega|$, the atom visits the state $|0\rangle$, on average, more than once, emitting photons to both modes of the field, which should then be positively correlated. When $\Gamma>\sqrt{2}|\Omega|$, the atom does not have time to make a complete transition to the state $|0\rangle$ and should emit less than one photon, on average, to one of the fields. In this case, the correlation is negative. As $\Gamma_{12}\ll\Gamma$ the absolute value of the correlations increases linearly with Γ_{12}, as is shown in Eqs. (20) and (21), due to the increasing probability to leave the dark state, afterwards it goes through an extremum and then it decreases as the dissipation process begins to become important. Figures 3(a) and 3(b) show how the normalized probe-pump correlation

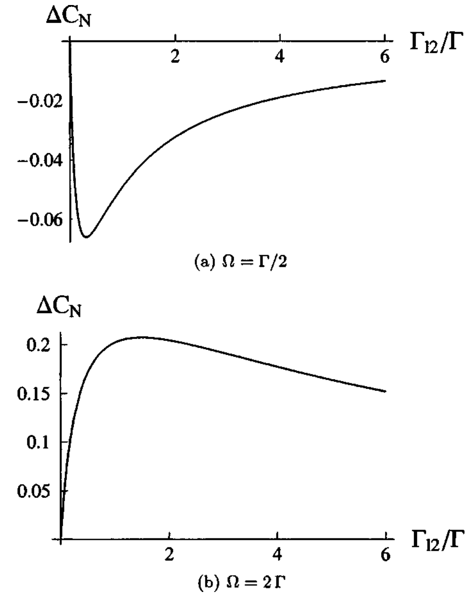

FIG. 3. Normalized probe-pump correlation ΔC^N as a function of the dephasing constant Γ_{12}/Γ. $\delta_1=\delta_2=0$.

$$\Delta C^N = \Delta C/\sqrt{(1+\Delta Y^1)(1+\Delta Y^2)} \tag{22}$$

varies with Γ_{12}, for $\Omega=\Gamma/2$ and $\Omega=2\Gamma$, respectively.

We now consider the case when the pump field is in resonance ($\delta_1=0$) and study the correlation functions as we vary the detuning, δ_2, of the probe field. In Eqs. (A11) and (A12) of the Appendix we give the correlation functions for any quadrature in the case $\Omega_1=\Omega_2=\Omega$ and $\Gamma_1=\Gamma_2=\Gamma$ and $\Gamma_{12}=0$. Here we give the results for the spectrum of the in-phase and out-of-phase quadrature autocorrelations,

$$\Delta Y_0^2(\omega=0) = 8\delta_2^2[16|\Omega|^{10} + 4|\Omega|^4(2\Gamma^4 + 10\Gamma^2|\Omega|^2 + 21|\Omega|^4)\delta_2^2 + 8|\Omega|^4(3\Gamma^2 + 4|\Omega|^2)\delta_2^4 + (\Gamma^4 + 2\Gamma^2|\Omega|^2 + 9|\Omega|^4) + \delta_2^6]/N, \tag{23}$$

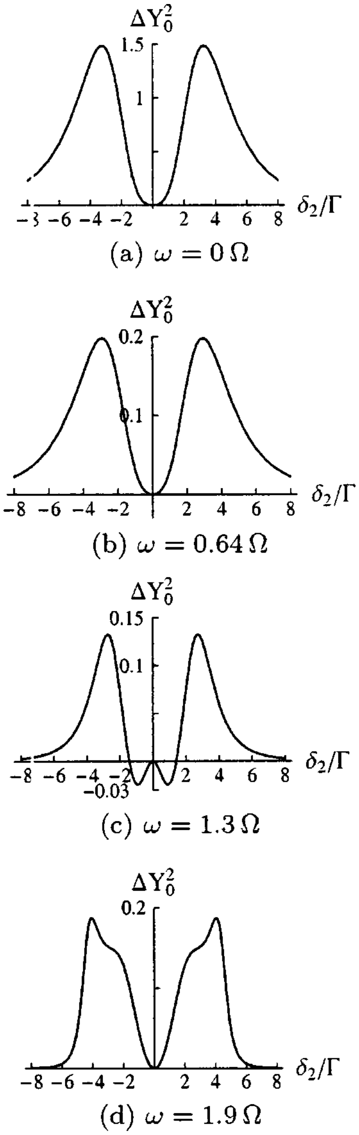

FIG. 4. In-phase quadrature ΔY_0^2 as a function of δ_2, for different values of ω. $\delta_1=0$, $\Omega=2\Gamma$.

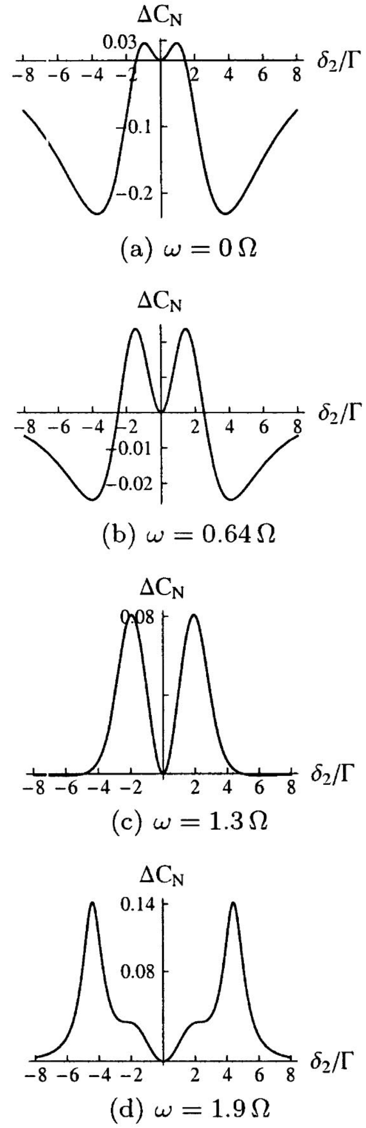

FIG. 5. Normalized probe-pump correlation ΔC^N as a function of δ_2, for different values of ω. $\delta_1=0$, $\Omega=2\ \Gamma$.

$$\Delta Y_{\pi/2}^2(\omega=0) = 4\Gamma^2\delta_2^2(128|\Omega|^8 + 8|\Omega|^4(3\Gamma^2 + 7|\Omega|^2)\delta_2^2 + 2(\Gamma^4 + 3\Gamma^2|\Omega|^2 + 9|\Omega|^4)\delta_2^4 + 3|\Omega|^2\delta_2^6)/N, \tag{24}$$

and the spectrum of the probe-pump in-phase quadrature correlation function,

$$\Delta C(0,0,\omega=0) = 4|\Omega|^2\delta_2^2[32|\Omega|^8 - 8|\Omega|^2(2\Gamma^4 + 3\Gamma^2|\Omega|^2 + 5|\Omega|^4)\delta_2^2 - 2(-\Gamma^4 + 11\Gamma^2|\Omega|^2 + 10|\Omega|^4)\delta_2^4 - (\Gamma^2 + 8|\Omega|^2)\delta_2^6]/N, \tag{25}$$

where

$$N = \Gamma(8|\Omega|^4 + 2|\Omega|^2\delta_2^2 + 2\Gamma^2\delta_2^2 + \delta_2^4)^3. \tag{26}$$

In Fig. 4(a) we show $\Delta Y_0^2(\omega=0)$ as a function of δ_2/Γ. We observe that the curve follows approximately the transparency curve (see Fig. 2). This is expected since fluorescence emission (at the frequency of the incoming field) should increase with the absorption.

In Fig. 5(a) we show $\Delta C(0,0,\omega=0)$ as a function of δ_2/Γ, for $\Omega_1=\Omega_2=2\Gamma$, which are close values to typical used in a recent experiment [9]. For large $\delta_2 \gg \Omega$ we should have anticorrelation since the atom remains most of the time in states $|0\rangle$ and $|1\rangle$. When δ_2 is very small, the system behaves close to the resonant situation and we should expect anticorrelation for $\sqrt{2}\Omega < \Gamma$ and positive correlation for $\sqrt{2}\Omega > \Gamma$. When $\sqrt{2}\Omega > \Gamma$, the initial positive correlation turns into negative correlation for δ_2^2 at the value of the real root of Eq. (25). For $\Omega \gg \Gamma$, this value is $\delta_2 \approx 0.76|\Omega|$.

The initial curvature, for $\Gamma \ll \Omega$, of Figs. 4(a) and 5(a), when $\omega=0$, can be easily calculated from the above equations:

$$\Delta C(0,0,\omega=0) \approx \frac{\delta_2^2}{4\Gamma\Omega^2}, \tag{27}$$

$$\Delta Y_0^2(\omega=0) \approx \frac{\delta_2^2}{4\Gamma\Omega^2}, \tag{28}$$

$$\Delta Y_{\pi/2}^2(\omega=0) \approx \frac{\Gamma\delta_2^2}{\Omega^4}. \tag{29}$$

B. Dependence of the correlations on the frequency of observation

We begin by studying the case of both fields in resonance. Following the procedure outlined in the beginning of Sec. II, we have calculated the general expressions for ΔY^i and ΔC. These expressions are cumbersome and here we will give only the results in the case $\Gamma_1=\Gamma_2=\Gamma$, $\Omega_1=\Omega_2=\Omega$, and $\Gamma_{12} \ll \Omega, \Gamma$. The result for the spectrum of the in-phase quadrature autocorrelation function is

$$\Delta Y_0^2(\omega) = \frac{\Gamma_{12}[4|\Omega|^4+\omega^2(\Gamma^2+\omega^2)+2|\Omega|^2(\Gamma^2-\omega^2)]}{(\Gamma^2+\omega^2)[4|\Omega|^2(|\Omega|^2-\omega^2)+\omega^2(\Gamma^2+\omega^2)]}. \tag{30}$$

When $\Omega \gg \Gamma$ this last equation has three maxima, one at $\omega=0$ and the other two at $\omega=\pm\sqrt{2}\Omega$. As we mentioned before, ΔY_0^2 is proportional to the Fourier transform of the field intensity autocorrelation variance. Its inverse Fourier transform added to the field intensity squared gives, as a function of time, the conditional probability of detecting a second photon after the measuring of a first photon. Since the frequency of visiting the $|0\rangle$ state is $\sqrt{2}\Omega$, we expect that after a photon is measured, we should wait an average time equal to $1/(\sqrt{2}\Omega)$ before another photon be measured. The second maxima in Eq. (30) is a consequence of this fact. There is no squeezing ($\Delta Y_0^2<0$) in the intensity quadrature for any value of ω, as can be seen from Eq. (30).

The spectrum of the out-of-phase quadrature autocorrelation can be written as

$$\Delta Y_{\pi/2}^2(\omega) = \Gamma_{12}\frac{-\omega^4(\Gamma^2+\omega^2)(4\Gamma^2+\omega^2)+48\Omega^6(2\Gamma^2+5\omega^2)-4\Omega^4\omega^2(16\Omega^2+37\omega^2)+2\Omega^2\omega^2(12\Gamma^4+35\Gamma^2\omega^2+17\omega^4)}{(4\Omega^4-4\Omega^2\omega^2+\Gamma^2\omega^2+\omega^4)[4\Gamma^4\omega^2+5\Gamma^2\omega^4+\omega^6+8\Omega^2\omega^2(\Gamma^2-2\omega^2)+16\Omega^4(\Gamma^2+4\omega^2)]}. \tag{31}$$

When $\Omega>\Gamma$ this autocorrelation is negative only if ω^2 is very large.

The spectrum of the probe-pump in-phase quadrature correlation is given by

$$\Delta C(0,0,\omega) = \Gamma_{12}\frac{4|\Omega|^4+\omega^2(\Gamma^2+\omega^2)-2|\Omega|^2(\Gamma^2+3\omega^2)}{(\Gamma^2+\omega^2)[4|\Omega|^2(|\Omega|^2-\omega^2)+\omega^2(\Gamma^2+\omega^2)]}. \tag{32}$$

When $\Gamma<\sqrt{2}\Omega$ the last equation shows that the correlation is positive, except when the value of ω is inside the interval (ω_-, ω_+), where

$$\omega_{\mp} = \sqrt{3\Omega^2-\Gamma^2/2 \mp \sqrt{5\Omega^4-\Gamma^2\Omega^2+\Gamma^4/4}}. \tag{33}$$

Now we turn our attention to the case in which only one of the fields is in resonance ($\delta_1=0$). We also have obtained general results in the situation of negligible phase decoherence ($\Gamma_{12}=0$). Up to second order in δ_2 they reduce to

$$\Delta Y_0^2(\omega) \approx \Gamma\delta_2^2\frac{8\Omega^8-16\Omega^6\omega^2-4\Omega^2\omega^4(\Gamma^2+\omega^2)+\omega^4(\Gamma^2+\omega^2)^2+2\Omega^4\omega^2(\Gamma^2+5\omega^2)}{2\Omega^2(\Gamma^2+\omega^2)(4\Omega^4-4\Omega^2\omega^2+\Gamma^2\omega^2+\omega^4)^2}. \tag{34}$$

In Fig. 4 we show the results for the full expression of ΔY_0^2 (no perturbation expansion on δ_2) as a function of δ_2/Γ, for several values of the frequency of observation ω. We observe in the figure that ΔY_0^2 is negative when $\omega=1.3\Omega$ and δ_2/Γ is small. In fact it is easy to see from Eq. (34) , valid for small δ_2, that there is always a region of ω where ΔY_0^2 is negative. When $\Omega \gg \Gamma$ this happens for $0.89\,\Omega \lesssim \omega \lesssim \sqrt{2}\Omega$.

In Fig. 5 we show the results for the full expression of $\Delta C(0,0,\omega)$ as a function of δ_2/Γ, for several values of the frequency of observation ω. A simple expression for $\Delta C(0,0,\omega)$ may be obtained also for small δ_2:

$$\Delta C(0,0,\omega) \approx \frac{\Gamma\delta_2^2(2\Omega^4-2\Omega^2\omega^2+\Gamma^2\omega^2+\omega^4)}{2\Omega^2(\Gamma^2+\omega^2)(4\Omega^4-4\Omega^2\omega^2+\Gamma^2\omega^2+\omega^4)}, \tag{35}$$

which shows that $\Delta C(0,0,\omega)$ is always positive when δ_2 is small.

III. CORRELATIONS IN THE CAVITY OUTPUT FIELD

Now we consider the case of N three-level atoms, occupying a volume of dimensions small compared with the wavelength of the input lasers, inside a cavity that sustains two modes of the electromagnetic field with frequencies $\omega_{01}+\delta_1$ and $\omega_{02}+\delta_2$ quasiresonant with the transitions between levels $|1\rangle$ and $|0\rangle$ and between levels $|2\rangle$ and $|0\rangle$ of the atom, respectively. The annihilation field operators inside the cavity are denoted by $\hat{a}_1$ and $\hat{a}_2$. We use input-output theory to relate the inside field with the outside field [15] and work in the interaction representation, where the Hamiltonian is given by Eq. (1). The intracavity field operators satisfy the equations

$$\frac{d}{dt}\hat{a}_1(t) = -ig_1\hat{\Sigma}_{10}(t) - \frac{\gamma_1}{2}\hat{a}_1(t) + \sqrt{\gamma_1}\hat{a}_{1\text{in}}(t),$$

$$\frac{d}{dt}\hat{a}_2(t) = -ig_2\hat{\Sigma}_{20}(t) - \frac{\gamma_2}{2}\hat{a}_2(t) + \sqrt{\gamma_2}\hat{a}_{2\text{in}}(t), \quad (36)$$

where $\Sigma_{ij}=\Sigma_{k=1}^{N}\sigma_{ij}^{k}$ are the collective operators that represent the sum of individual atomic operators, $\sigma_{ij}^{k}=|i\rangle\langle j|^{k}$ associated with the kth atom. The input and output annihilation operators, which are associated with the pumping and output fields, are related by [15]

$$\hat{a}_{1\text{in}}(t) + \hat{a}_{1\text{out}}(t) = \sqrt{\gamma_1}\hat{a}_1(t),$$

$$\hat{a}_{2\text{in}}(t) + \hat{a}_{2\text{out}}(t) = \sqrt{\gamma_2}\hat{a}_2(t). \quad (37)$$

In the above equations, we define γ_i as the damping rate of the cavity mode i. Writing $\hat{a}_{i\ \text{in}}=\langle\hat{a}_{i\ \text{in}}\rangle+\hat{f}_{ai}$, one can show that, for a cavity at zero temperature, the operators $\hat{f}_{ai}$ satisfy [15]

$$\langle\hat{f}_{ai}^{\dagger}(t)\hat{f}_{aj}(t')\rangle = 0,$$

$$\langle\hat{f}_{ai}(t)\hat{f}_{aj}^{\dagger}(t')\rangle = \gamma_i\delta(t-t')\delta_{ij},$$

$$\langle\hat{f}_{ai}^{\dagger}(t)\hat{f}_{aj}^{\dagger}(t')\rangle = 0,$$

$$\langle\hat{f}_{ai}(t)\hat{f}_{aj}(t')\rangle = 0. \quad (38)$$

From Eqs. (36) it is easy to see that $\hat{f}_{ai}$ represents the Langevin fluctuation operator associated with the field i inside the cavity.

The Langevin equations for the atomic operators are obtained using the interaction Hamiltonian. These equations are analogous to Eqs. (4) of Sec. II and may be written by substituting α_1 (α_2) by $\hat{a}_1$ ($\hat{a}_2$), $\hat{\sigma}_{ij}$ by $\hat{\Sigma}_{ij}$, and the one-atom Langevin fluctuation operator $\hat{f}_x$ by the collective Langevin fluctuation operator $\hat{F}_x$. As the atoms are assumed to be independent of each other, the new collective diffusion coefficients are given by the sum of all one-atom diffusion coefficients.

These new differential operator equations are not as easy to solve as the system of equations in Sec. II, since now we have a set of first-order nonlinear equations, where products of two operators appear on the right-hand side of the equations. What is usually done is to transform these operator differential stochastic equations into c-number Ito stochastic differential equations. These latter equations are equivalent to the original ones up to second order in the operators [16]. These c-number equations can then be solved using usual differential stochastic methods [17]. To define this transformation uniquely we define an order, which we call "normal" order, for the operators in the differential equations. The normal order we choose is

$$\hat{a}_2^{\dagger},\hat{a}_1^{\dagger},\hat{\Sigma}_{02},\hat{\Sigma}_{01},\hat{\Sigma}_{12},\hat{W}_1,\hat{W}_2,\hat{\Sigma}_{21},\hat{\Sigma}_{10},\hat{\Sigma}_{20},\hat{a}_1,\hat{a}_2. \quad (39)$$

We will use the c-number variables Σ_{ij}, W_1, W_2, α_i^{*}, and α_i for the corresponding operators $\hat{\Sigma}_{ij}$, $\hat{W}_1$, $\hat{W}_2$, $\hat{a}_i^{\dagger}$, and $\hat{a}_i$. The stochastic average of a c-number variable is equal to the mean value of the corresponding operator and the stochastic average of the product of two c-number variables corresponds to the mean value of the normal-ordered multiplication of the two corresponding operators—for example, $\Sigma_{02}(t)\Sigma_{12}(t')=\Sigma_{12}(t')\Sigma_{02}(t)$ is equal to $\langle\hat{\Sigma}_{02}(t)\hat{\Sigma}_{12}(t')\rangle$.

The new c-number equations for the system look the same as the operator equations except that we should replace the Langevin fluctuation operators by modified Langevin fluctuation forces. These modified Langevin fluctuation forces still have zero mean and are still δ function correlated. However, the diffusion coefficients associated with these new Langevin fluctuation forces are modified so that the operator equations of normal-ordered products coincide with the c-number equations of the corresponding products. A very clear explanation of this procedure is given by Davidovich [16].

We have calculated these new normal-ordered diffusion coefficients. Of course they satisfy the symmetry relation $D_{xy}=D_{yx}$. The nonzero coefficients (and the symmetrical ones) are given by

$$D_{W_1W_1} = (4\Gamma_1+\Gamma_2)\Sigma_{00} - i(4\Omega_1\Sigma_{01}+\Omega_2\Sigma_{02}-\text{c.c.}),$$

$$D_{W_2W_2} = (\Gamma_1+4\Gamma_2)\Sigma_{00} - i(4\Omega_2\Sigma_{02}+\Omega_1\Sigma_{01}-\text{c.c.}),$$

$$D_{W_1W_2} = (2\Gamma_1+2\Gamma_2)\Sigma_{00} - 2i(\Omega_2\Sigma_{02}+\Omega_1\Sigma_{01}-\text{c.c.}),$$

$$D_{\Sigma_{12}\Sigma_{21}} = \Gamma_1\Sigma_{00} + 2\Gamma_{12}\Sigma_{11} - (i\Omega_2\Sigma_{12}+\text{c.c.}),$$

$$D_{\Sigma_{02}\Sigma_{12}} = D_{\Sigma_{20}\Sigma_{21}}^{*} = -i\Omega_2^{*}\Sigma_{12},$$

$$D_{\Sigma_{02}\Sigma_{21}} = D_{\Sigma_{20}\Sigma_{12}}^{*} = \Gamma_{12}\Sigma_{01},$$

$$D_{W_1\Sigma_{10}} = D_{W_1\Sigma_{01}}^{*} = i\Omega_2\Sigma_{12},$$

$$D_{W_1\Sigma_{21}} = D_{W_1\Sigma_{12}}^{*} = -2i\Omega_1^{*}\Sigma_{20} + 2i\Omega_2\Sigma_{01},$$

$$D_{W_2\Sigma_{10}} = D_{W_2\Sigma_{01}}^{*} = -i\Omega_2\Sigma_{12},$$

$$D_{W_2\Sigma_{21}} = D_{W_2\Sigma_{12}}^{*} = -i\Omega_1^{*}\Sigma_{20} + i\Omega_2\Sigma_{01},$$

$$D_{\Sigma_{10}\Sigma_{10}} = D_{\Sigma_{01}\Sigma_{01}}^{*} = 2i\Omega_1\Sigma_{10},$$

$$D_{\Sigma_{01}\Sigma_{02}} = D^*_{\Sigma_{20}\Sigma_{10}} = -i\Omega_1^*\Sigma_{02} - i\Omega_2^*\Sigma_{01},$$

$$D_{\Sigma_{20}\Sigma_{20}} = D^*_{\Sigma_{02}\Sigma_{02}} = 2i\Omega_2\Sigma_{20},$$

$$D_{\Sigma_{01}\Sigma_{12}} = D^*_{\Sigma_{21}\Sigma_{10}} = \Gamma_{12}\Sigma_{02} + i\Omega_2(W_1 - W_2) + i\Omega_1\Sigma_{12}. \tag{40}$$

A similar calculation of c-number diffusion coefficients for three-level atoms interacting with traveling waves was done by Fleischhauer and Richter [18].

The steady-state solutions of the c-number equations for the average of the stochastic variables are obtained by setting the time derivatives equal to zero and using the fact that the average of the modified Langevin fluctuation forces is zero. Of course one should verify if these solutions are stable.

As before, we are mostly interested in the dynamics of fluctuations around the steady-state. In order to calculate this dynamics, we express the solutions of the stochastic variables as the sum of the steady state value plus fluctuations, that is, for any stochastic variable O, we make $O(t)\approx\langle O\rangle + \delta O(t)$, with $\delta O \ll \langle O\rangle$. In the system equations, the atomic operators scale as the number of atoms, N, and the fluctuation forces scale as $\sqrt{N}$ [16], if the number of atoms inside the cavity is big enough ($\sqrt{N}\gg 1$) and the fluctuations are small and we can neglect terms of order greater than 1 in δO. Neglecting those terms, the differential equations for the corresponding stochastic c-number equations can be easily written in a matrix form

$$\frac{d}{dt}\boldsymbol{\delta O} = \boldsymbol{B}\cdot\boldsymbol{\delta O} + \boldsymbol{G}, \tag{41}$$

where the column vectors $\boldsymbol{\delta O}$ and $\boldsymbol{G}$ have components:

$$\boldsymbol{\delta O}^T = (\delta a_2^*, \delta a_1^*, \delta\Sigma_{02}, \delta\Sigma_{01}, \delta\Sigma_{12}, \delta w_1, \delta w_2, \delta\Sigma_{21}, \delta\Sigma_{10}, \delta\Sigma_{20}, \delta a_1, \delta a_2),$$

$$\boldsymbol{G}^T = (f^*_{a1}, f^*_{a2}, f_{02}, f_{01}, f_{12}, f_{w_1}, f_{w_2}, f_{21}, f_{10}, f_{20}, f_{a1}, f_{a2}).$$

The $\boldsymbol{B}$ matrix can be easily obtained from the expansion up to first order in δO of the system's equations. The spectrum of the double correlation of the irradiated fields, $S_{ij}(\omega)$, is now given by

$$\langle \delta O_i(\omega)\delta O_j^*(\omega')\rangle = S_{ij}(\omega)\delta(\omega+\omega'), \tag{42}$$

where $\langle\cdots\rangle$ means stochastic average and $O_i(\omega) = (1/\sqrt{2\pi})\int_{-\infty}^{\infty} O_i(t)\exp(i\omega t)dt$ is the Fourier transform of $O_i(t)$.

Taking the Fourier transform of Eq. (41), multiplying by $\boldsymbol{\delta O}^\dagger$, and taking the stochastic average, we obtain

$$\boldsymbol{S}(\omega) = (\boldsymbol{B} + i\ \omega\ \mathbb{1})^{-1}\cdot\boldsymbol{D}\cdot(\boldsymbol{B}^\dagger - i\ \omega\ \mathbb{1})^{-1}, \tag{43}$$

where $\boldsymbol{D}\ \delta(\omega+\omega') = \langle\boldsymbol{G}\cdot\boldsymbol{G}^T\rangle$. The components of the symmetric diffusion matrix $\boldsymbol{D}$ are the c-number diffusion coefficients given in Eq. (40).

We numerically calculate the spectrum functions given by Eq. (43). By using the Fourier transform of Eqs. (37), we obtain the spectrum of the output field correlation functions ΔY and ΔC, given in Eqs. (11) and (12). The operators $\hat{Y}_\theta$ are now defined as

$$\hat{Y}^i_\theta(t) = \hat{a}_{i\ \text{out}}(t)\exp(i\theta) + \hat{a}^\dagger_{i\ \text{out}}(t)\exp(-i\theta). \tag{44}$$

Of course the above results are only valid if the steady-state solutions are stable. This can be verified by calculating the eigenvalues of the matrix $\boldsymbol{B}$. If the real part of all the eigenvalues is negative, all the fluctuations decrease as time increases. Below we study regions of stability of the solutions in the case $\Gamma_1=\Gamma_2=\Gamma$ and $\Omega_1=\Omega_2=\Omega$. For $\delta_1=0$ and $\Gamma_{12}=0$ the stability of the system is characterized by only three parameters δ_2/Γ, Ω/Γ, and the cooperative parameter $C=N\,g^2/(\gamma\,\Gamma)$. When $\delta_2=0$ the system is stable for any C and $\Omega/\Gamma>0$ since the medium is transparent. In Fig. 6 we have studied the regions of stability for the parameters δ_2/Γ and C, when $\Omega/\Gamma=5,10,15,20$. The figure shows an increase of the region of stability, in the vicinity of $\delta_2=0$ as Ω/Γ increases. This is connected with the increasing of the EIT window [see Eq. (10)].

We have checked numerically that the behavior of the output correlation functions is characterized by the cooperative parameter C defined above and by Ω/Γ when $\omega=0$. When $\omega\neq 0$ the output correlation functions also depend on γ. For a small cooperation number $C=Ng^2/(\gamma\Gamma)\ll 1$, the atoms do not interact strongly with the radiation emitted and the qualitative behavior of the output correlation functions ΔY^i_θ is very similar to the correlation functions $\Delta Y^i_{\theta+\pi/2}$ of the resonance fluorescence case, discussed in Sec. II.

In Fig. 7 we show how the spectrum of the in-phase probe-pump quadrature correlation, $\Delta C(0,0,\omega=0)$, varies as we increase C. We notice that in Figs. 7(a)–7(c) the correlation function $\Delta C(0,0,\omega=0)$ is positive in the EIT window $(-10\leq\delta_2/\Gamma\leq 10)$ and follows approximately the EIT absorption curve, shown in Fig. 8. Negative correlations appear for larger values of the cooperative parameter C [see Fig. 7(d)]. The appearance of these negative correlations, in the EIT window, can be explained as follows: an excess of quadrature intensity in any one of the output fields—say, field mode 1—implies necessarily that field 1 momentarily increases inside the cavity. This makes the EIT window transparency wider [2] temporarily and therefore fluctuations in field 2 decrease.

The spectrum of the in-phase quadrature correlation function ΔY_0^2 is shown in Fig. 9 for several values of C. Up to C around 0.5, the curves follow the general trend of the absorption curve. As we increase C, new maxima appear closer to $\delta_2=0$. In order to understand the origin of it, we studied the value of the incoming field needed to build up an amplitude α in both modes inside the cavity [see Eq. (36)]. For fixed amplitude α of the pump field inside the cavity, we expected (and found numerically) that any increment in the incoming probe field corresponds to an increment in fluorescence. Therefore the maxima and minima of ΔY_0^2, as a function of δ_2/Γ, observed in Fig. 9(d) coincide with the maxima and minima in the probe incoming field $[\langle\hat{a}_{2\text{in}}(t)\rangle]$ needed to build up the probe field with amplitude α inside the cavity. Using these results and the analytical solutions for the mean values of the operators, we found that in the case that N

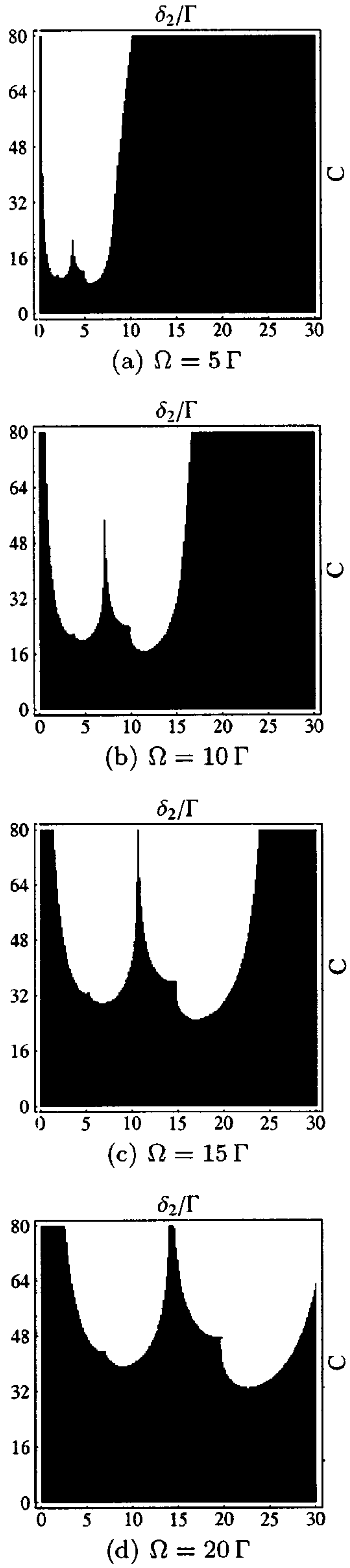

FIG. 6. Regions of stability (black) and instability (white) of the stationary solutions. (a) $\Omega=5\ \Gamma$, (b) $\Omega=10\ \Gamma$, (c) $\Omega=15\ \Gamma$, (d) $\Omega=20\ \Gamma$.

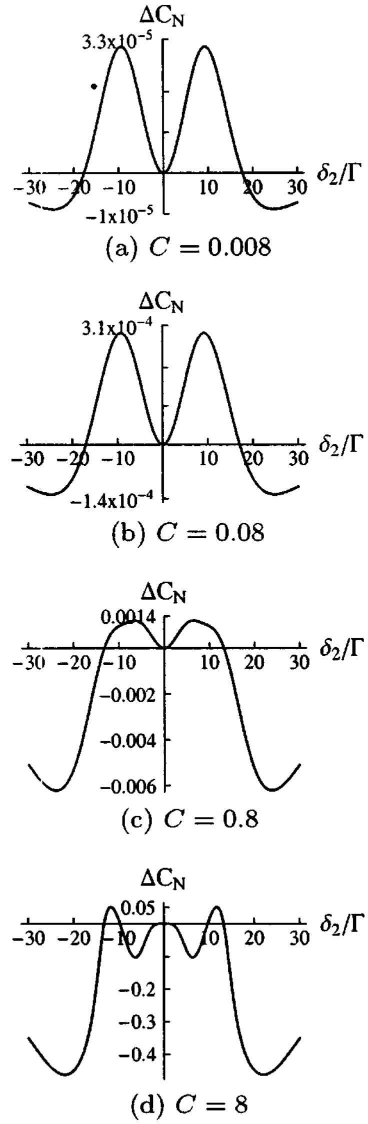

FIG. 7. Normalized probe-pump correlation ΔC^N as a function of δ_2, for different values of C. $\delta_1=0, \omega=0$, $\Omega=10\ \Gamma$, and $\gamma_1=\gamma_2=0.15\ \Gamma$.

$\gg 1$, $\Omega \gg \Gamma$, the minima (besides the one at zero, associated with the dark state) are at $\delta_2 \approx \pm\sqrt{2}\ \Omega$, $\approx 14\Gamma$ in Fig. 9(d). The maxima closer to zero are in the positions $\delta_2 \approx \pm 0.7\ \Omega$.

Figure 10 shows the spectrum of the in-phase quadrature correlation function ΔY_0^2 for $C=8$ as a function of δ_2/Γ for

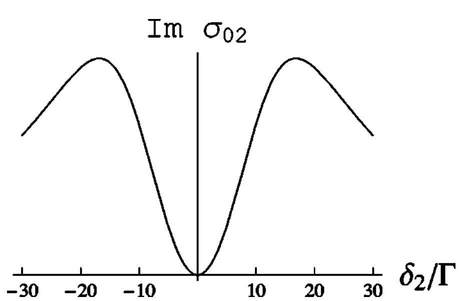

FIG. 8. Cavity EIT absorption curve. $\Gamma_{12}=0, \Omega=10\ \Gamma$, and $\delta_1=0$.

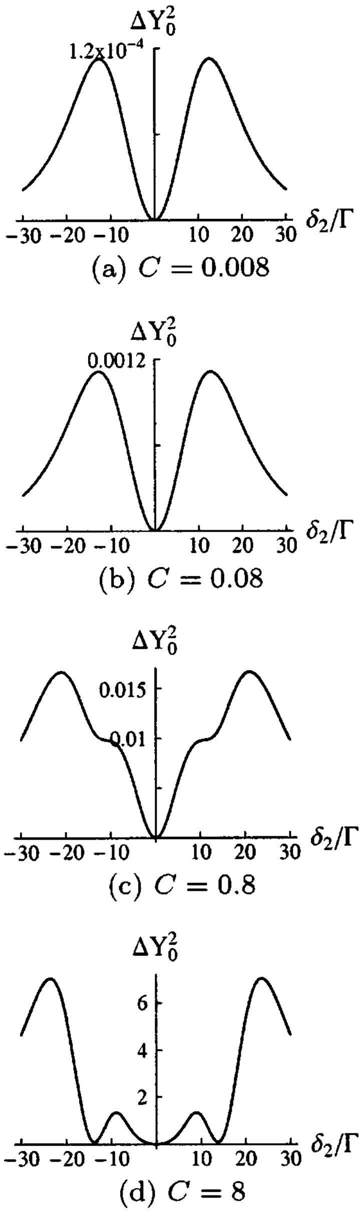

FIG. 9. In-phase quadrature function ΔY_0^2 as a function of δ_2, for different values of C. $\delta_1=0, \omega=0$, $\Omega=10\ \Gamma$, and $\gamma_1=\gamma_2=0.15\ \Gamma$.

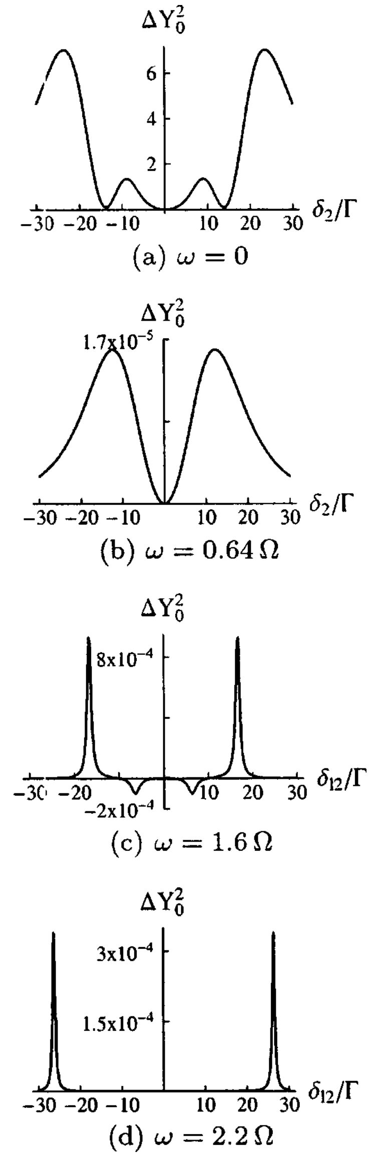

FIG. 10. In-phase quadrature ΔY_0^2 as function of δ_2, for different values of ω. $\delta_1=0$, $\Omega=10\ \Gamma$, $\gamma_1=\gamma_2=0.15\ \Gamma$, and $C=8$.

several values of the observation frequency ω. We obtain a very small squeezing, as can be seen from the figure.

The spectrum of the out-of-phase quadrature correlation function $\Delta Y_{\pi/2}^2$ was also calculated and presents a behavior similar to ΔY_0^2.

IV. CONCLUSIONS

We have studied the correlation functions of the fields emitted by three-level atoms, interacting with two modes of the electromagnetic field in a coherent state, outside and inside a cavity. This was done when both modes of the field are in resonance and when one of the modes is in resonance and the other one is scanned.

When the atoms are outside the cavity, simple analytical expressions were obtained when the decoherence in the lower levels is small and the detuning of one of the modes of the fields is small. We found that the correlations between the two modes of the field can be positive or negative and characterize the regions of parameters in which this happens. We also found a very small squeezing in the in-phase and out-of-phase quadratures for some regions of the observation frequencies. This does not contradict previous results [5] which refer to total squeezing.

For fixed Ω/Γ, if the cooperative parameter C is small, the cavity correlation functions have, in general, the same shape as a function of δ_2/Γ as the corresponding correlation function rotated by $\pi/2$ in the fluorescence case. If C is large, this does not happen. We found that when the cooperative parameter of the cavity increases, positive correlations may turn into negative correlations in certain regions of the probe frequency. We also found that new maxima appear in the spectrum of the in-phase autocorrelation function as we increase the cooperative parameter.

Our results for the intensity-intensity fluctuations of the fields do not agree with the theoretical predictions presented in [9]. In fact the authors analyzed the equations for the steady state of the system in a parameter region where the solutions are unstable. We also did not obtain any result showing either super-Poissonian statistics in the intensity-intensity fluctuations or large positive probe-pump intensity correlation at resonance, in contradiction with the experimental results of Ref. [9]. These discrepancies are probably due to phase noise in the fields [19].

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APPENDIX: GENERAL RESULTS

The steady-state solutions of the operator equations (4) for the average of the operators are obtained by setting their time derivatives equal to zero and using the fact that the average of the Langevin fluctuation operators is zero. We obtain an algebraic linear system of equations that is solved using Wolfram's Mathematica computer program and get

$$\langle w_1\rangle = \{-\Gamma_1\Gamma_t(\Gamma_t^2+4\delta_1^2)(\Gamma_{12}^2+\delta_r^2)\Omega_2^2+[-2\Gamma_{12}(\Gamma_t^3+4\Gamma_1\delta_1^2+4\Gamma_2\delta_1\delta_2)+4\Gamma_2\Gamma_t\delta_r^2]\Omega_1^2\Omega_2^2-4\Gamma_2\Gamma_t\Omega_1^4\Omega_2^2 \\ -4\Gamma_1\Gamma_t(\Gamma_{12}\Gamma_t-2\delta_1\delta_r)\Omega_2^4-4\Gamma_t^2\Omega_1^2\Omega_2^4-4\Gamma_1\Gamma_t\Omega_2^6\}/M, \tag{A1}$$

$$\langle w_2\rangle = \{-\Gamma_2\Gamma_t(\Gamma_t^2+4\delta_2^2)(\Gamma_{12}^2+\delta_r^2)\Omega_1^2+[-2\Gamma_{12}(\Gamma_t^3+4\Gamma_1\delta_1\delta_2+4\Gamma_2\delta_2^2)+4\Gamma_1\Gamma_t\delta_r^2]\Omega_1^2\Omega_2^2-4\Gamma_1\Gamma_t\Omega_1^2\Omega_2^4 \\ -4\Gamma_2\Gamma_t(\Gamma_{12}\Gamma_t+2\delta_2\delta_r)\Omega_1^4-4\Gamma_t^2\Omega_1^4\Omega_2^2-4\Gamma_2\Gamma_t\Omega_1^6\}/M, \tag{A2}$$

$$\langle\sigma_{10}\rangle = (2\Gamma_1\Gamma_t(2\delta_1-i\Gamma_t)(\Gamma_{12}^2+\delta_r^2)\Omega_1\Omega_2^2+4\Omega_1^3\Omega_2^2\{\Gamma_{12}[\Gamma_1(2\delta_1-i\Gamma_t)+2\Gamma_2\delta_2]-\Gamma_2\Gamma_t\delta_r\}-4\Gamma_1\Gamma_t(-i\Gamma_{12}-\delta_r)\Omega_1\Omega_2^4)/M, \tag{A3}$$

$$\langle\sigma_{20}\rangle = (2\Gamma_2\Gamma_t(2\delta_2-i\Gamma_t)(\Gamma_{12}^2+\delta_r^2)\Omega_1^2\Omega_2+4\Omega_1^2\Omega_2^3\{\Gamma_{12}[\Gamma_2(2\delta_2-i\Gamma_t)+2\Gamma_1\delta_1]+\Gamma_1\Gamma_t\delta_r\}+4\Gamma_2\Gamma_t(-i\Gamma_{12}+\delta_r)\Omega_1^4\Omega_2)/M, \tag{A4}$$

$$\langle\sigma_{21}\rangle = [2\Gamma_2\Gamma_t(2\delta_2-i\Gamma_t)(-\delta_r-i\Gamma_{12})\Omega_1^3\Omega_2-4\Gamma_2\Gamma_t\Omega_1^5\Omega_2+2\Gamma_1\Gamma_t(2\delta_1+i\Gamma_t)(\delta_r+i\Gamma_{12})\Omega_1\Omega_2^3-4\Gamma_t^2\Omega_1^3\Omega_2^3-4\Gamma_1\Gamma_t\Omega_1\Omega_2^5]/M, \tag{A5}$$

where

$$M = \Gamma_2\Gamma_t(\Gamma_t^2+4\delta_2^2)(\Gamma_{12}^2+\delta_r^2)\Omega_1^2+4\Gamma_2\Gamma_t(\Gamma_{12}\Gamma_t+2\delta_2\delta_r)\Omega_1^4+4\Gamma_2\Gamma_t\Omega_1^6+\Gamma_1\Gamma_t(\Gamma_t^2+4\delta_1^2)(\Gamma_{12}^2+\delta_r^2)\Omega_2^2 \\ +(4\Gamma_{12}\{\Gamma_1^3+3\Gamma_1^2(\Gamma_2+\Gamma_{12})+\Gamma_1[3\Gamma_2(\Gamma_2+2\Gamma_{12})+2\delta_1(\delta_1+\delta_2)]+\Gamma_2[\Gamma_2^2+3\Gamma_2\Gamma_{12}+2\delta_2(\delta_1+\delta_2)]\}+8\Gamma_t^2\delta_r^2)\Omega_1^2\Omega_2^2 \\ +4(\Gamma_1+2\Gamma_2+6\Gamma_{12})\Gamma_t\Omega_1^4\Omega_2^2+4\Gamma_1\Gamma_t(\Gamma_{12}\Gamma_t-2\delta_1\delta_r)\Omega_2^4+4(2\Gamma_1+\Gamma_2+6\Gamma_{12})\Gamma_t\Omega_1^2\Omega_2^4+4\Gamma_1\Gamma_t\Omega_2^6 \tag{A6}$$

and $\Gamma_t=\Gamma_1+\Gamma_2$.

The correlation functions $\Delta Y_\theta^2(\omega)$ and $\Delta C(\theta_1,\theta_2,\omega)$ are obtained from Eq. (15) and are expressed as

$$\Delta Y_\theta^2(\omega) = S_{11}(\omega)e^{i2\theta}+S_{88}(\omega)e^{-i2\theta}+2S_{18}(\omega) \tag{A7}$$

and

$$\Delta C(\theta_1,\theta_2,\omega) = S_{12}(\omega)e^{i(\theta_1+\theta_2)}+S_{78}(\omega)e^{-i(\theta_1+\theta_2)}+S_{17}(\omega)e^{i(\theta_2-\theta_1)}+S_{28}(\omega)e^{i(\theta_1-\theta_2)}. \tag{A8}$$

We calculated the elements of the spectra of the double correlation functions of the irradiated fields S_{ij} when both fields, probe and pump, are in resonance and at the observation laser frequency. After manipulating the elements of S_{ij} and using Eqs. (A7) and (A8), we obtain a lengthy general expression for the correlations. When $\Gamma_{12}\ll\Gamma_1,\Gamma_2,\Omega_1,\Omega_2$, they reduce to

$$\Delta Y_\theta^2(\omega=0) = 2\Gamma_{12}\Omega_1^2\Omega_2^2(2\Gamma_2(\Gamma_1+\Gamma_2)^2\Omega_1^2+[(\Gamma_1+\Gamma_2)^3+4\Gamma_2\Omega_1^2]\Omega_2^2+4\Gamma_1\Omega_2^4+\cos(2\theta)\{-2\Gamma_2^2(\Gamma_1+\Gamma_2)\Omega_1^2 \\ +[(\Gamma_1+\Gamma_2)(\Gamma_1^2-2\Gamma_1\Gamma_2-\Gamma_2^2)+4\Gamma_2\Omega_1^2]\Omega_2^2+4\Gamma_1\Omega_2^4\})/[(\Gamma_1+\Gamma_2)(\Omega_1^2+\Omega_2^2)^2(\Gamma_2\Omega_1^2+\Gamma_1\Omega_2^2)^2] \tag{A9}$$

and

$$\Delta C(\theta_1,\theta_2,\omega=0) = 2\Gamma_{12}\Omega_1\Omega_2 \frac{4\cos(\theta_1-\theta_2)\Omega_1^2\Omega_2^2 + \cos(\theta_1+\theta_2)[4\Omega_1^2\Omega_2^2 - (\Gamma_1+\Gamma_2)(\Gamma_1\Omega_2^2+\Gamma_2\Omega_1^2)]}{(\Gamma_1+\Gamma_2)(\Omega_1^2+\Omega_2^2)^2(\Gamma_2\Omega_1^2+\Gamma_1\Omega_2^2)}. \tag{A10}$$

We also calculated S_{ij} when only field 1 is in resonance. Taking $\Gamma_{12}=0$, $\Gamma_1=\Gamma_2=\Gamma$, and $\Omega_1=\Omega_2=\Omega$ and using Eqs. (A7) and (A8), we obtain

$$\begin{aligned}\Delta C(\theta_1,\theta_2,\omega=0) = {}& 4\delta_2^2[\Gamma\sin\theta_1\{-2\Gamma\sin\theta_2[-32\Omega^8+4\Gamma^2\Omega^4\delta_2^2+(\Gamma^4+4\Gamma^2\Omega^2)\delta_2^4+(\Gamma^2+\Omega^2)\delta_2^6]+\cos\theta_2\delta_2\\ &\times[-8\Omega^6(5\Gamma^2+12\Omega^2)+2\Omega^2(-\Gamma+\Omega)(\Gamma+\Omega)(\Gamma^2+4\Omega^2)\delta_2^2+(2\Gamma^4+11\Gamma^2\Omega^2+2\Omega^4)\delta_2^4+(2\Gamma^2+5\Omega^2)\delta_2^6]\}\\ &+\Omega^2\cos\theta_1(\Gamma\sin\theta_2\delta_2[8\Omega^4(5\Gamma^2+2\Omega^2)+2(\Gamma^4+5\Gamma^2\Omega^2+10\Omega^4)\delta_2^2+(5\Gamma^2+4\Omega^2)\delta_2^4+\delta_2^6]+\cos\theta_2[32\Omega^8\\ &-8(2\Gamma^4\Omega^2+3\Gamma^2\Omega^4+5\Omega^6)\delta_2^2+2(\Gamma^4-11\Gamma^2\Omega^2-10\Omega^4)\delta_2^4-(\Gamma^2+8\Omega^2)\delta_2^6])]/[\Gamma(8\Omega^4+2\Gamma^2\delta_2^2+2\Omega^2\delta_2^2+\delta_2^4)^3]\end{aligned} \tag{A11}$$

and

$$\begin{aligned}\Delta Y_\theta^2(\omega=0) = {}& 2\delta_2\{4\Gamma^4\delta_2^5[-(\Gamma\sin\theta)+\cos\theta\delta_2]^2+64\Omega^{10}\cos\theta(4\Gamma\sin\theta+\cos\theta\delta_2)+8\Omega^6\delta_2^2[17\Gamma^2\delta_2+8\delta_2^3-2\Gamma\sin2\theta(\Gamma^2\\ &+4\delta_2^2)\\ &+\cos2\theta\delta_2(3\Gamma^2+8\delta_2^2)]+8\Omega^8\delta_2[16\Gamma^2-20\Gamma\sin2\theta\delta_2+21\delta_2^2+\cos2\theta(-16\Gamma^2+21\delta_2^2)]+\Gamma\Omega^2\delta_2^4[6\Gamma^3\delta_2+7\Gamma\delta_2^3\\ &+\Gamma\cos2\theta\delta_2(-6\Gamma^2+\delta_2^2)-2\sin2\theta(-2\Gamma^4+3\Gamma^2\delta_2^2+\delta_2^4)]+2\Omega^4\delta_2^3[20\Gamma^4+33\Gamma^2\delta_2^2+9\delta_2^4-2\Gamma\sin2\theta\delta_2(9\Gamma^2+5\delta_2^2)\\ &+\cos2\theta(-4\Gamma^4+15\Gamma^2\delta_2^2+9\delta_2^4)]\}/\{\Gamma[8\Omega^4+2(\Gamma^2+\Omega^2)\delta_2^2+\delta_2^4]^3\}.\end{aligned} \tag{A12}$$

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