Phase-sensitive laser detection by frequency-shifted optical feedback

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For further interferometric application on diffusive target, the phase fluctuation of a solid-state laser submitted to frequency shifted optical feedback is analyzed both theoretically and experimentally. As a drawback of the laser high sensitivity to optical feedback, the phase fluctuations induced by a strong phase-amplitude coupling noise are several orders of magnitude higher than the standard interferometric phase noise induced by the laser frequency width (Schawlow-Townes limit). Nevertheless, by sending a few milliwatts output power microchip laser beam on a diffusive target with an effective reflectivity of 10^{-9}, a target displacement precision of 0.1 Å/$\sqrt{\text{Hz}}$ has been experimentally determined.

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I. INTRODUCTION

Laser properties and behavior can be significantly affected and modified by optical feedback [1]. Since the discovery of lasers, parasitic coherent optical feedback has been the source of serious laser problems, increasing noise and creating laser instabilities [2,3]. On the other hand, controlled optical feedback can be of practical use [4]. For example, line width narrowing can be obtained with an external cavity laser diode [5]. Potential applications are also possible. One of these is laser feedback interferometry (LFI), where the steady state intensity of a laser is modified by coherent optical feedback from an external surface [6]. This phase sensitive technique is dependent on the reflectivity, distance and motion of the target [7].

The remote characterization of noncooperative targets such as diffusing surfaces is relevant for many applications [8,9]. In these cases, the reinjected light is only partially coherent. The nature of such light reduces drastically the interference contrast occurring inside the laser cavity. To overcome this problem, one solution is then to use the laser dynamic which is several order of magnitude more sensitive to optical feedback than the laser steady state properties [10,11]. Nowadays, the dynamical sensitivity of lasers to frequency shifted optical feedback is used in a self mixing laser Doppler velocimetry (LDV) experiment [12,13] and in a laser optical feedback imaging (LOFI) experiment [14]. Compared to conventional optical heterodyne detection, frequency shifted optical feedback allows intensity modulation contrast several order of magnitude higher (typically 10^3 for a diode laser and 10^6 for a microchip laser) [15]. The maximum of the modulation was obtained when the frequency shift was resonant with the laser relaxation oscillation frequency (typically 1 GHz for a diode laser and 1 MHz for a microchip laser).

For further interferometric application on diffusive target, the phase fluctuations of a laser submitted to frequency shifted optical feedback need to be analyzed both theoretically and experimentally [16]. The main objective of this paper is then to study the phase fluctuations of the LOFI signal induced by the laser quantum noise, in order to determine the ultimate displacement precision of a diffusive target for a given laser output power and a given detection bandwidth.

This paper is organized as follows. In the theoretical section, we recall the rate equations governing the dynamics of a laser submitted to frequency shifted optical feedback. These equations are then solved in a linear approximation. For weak optical feedback, we have compared the phase fluctuations induced by the laser frequency width with the phase fluctuations induced by the strong phase-amplitude coupling noise inherent to the LOFI detection principle. For strong optical feedback, the influence of the laser relaxation frequency on the laser phase fluctuations is also reported. In the experimental section, vibration measurements are realized to study the LOFI phase fluctuations versus the target effective reflectivity. The ultimate target displacement precision is then determined and compared to the theoretical prediction.

II. THEORY

Figure 1 shows the experimental setup for the detection of frequency-shifted optical feedback in a laser. The optical feedback is characterized by: the optical frequency shift (Ω_e), the distance (d_e) between the laser cavity and the target and the effective power reflectivity (R_e) of the target under investigation. For a microchip laser, typical operating parameters are given in Table I.

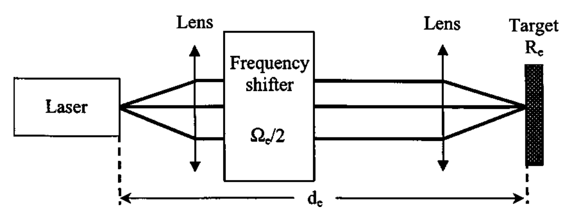

FIG. 1. Schematic of the laser detection with frequency shifted optical feedback. Ω_e is the total optical frequency shift, R_e is the power effective reflectivity of the target under investigation, and d_e is the feedback laser distance.

TABLE I. Typical operating parameters.

Laser parameters	
Laser wavelength	$\lambda_c = 1064$ nm
Cavity damping rate	$\gamma_c = 8 \times 10^9$ s^{-1}
Population inversion damping rate	$\gamma_n = 5 \times 10^3$ s^{-1}
Pumping parameters	$\eta = 1.7$
Relaxation oscillation frequency	$\Omega_R/2\pi = 0.84$ MHz
Relaxation oscillation damping rate	$\Delta\Omega_R/2\pi \approx 1.6$ kHz
Laser output power	$P_{\rm out} = 4$ mW ($n_{\rm out} = 2 \times 10^{16}$ photons/s)
Feedback parameters	
Feedback distance	$d_e = 80$ cm
Feedback time delay	$\tau_e = 2d_e/c = 5.33 \times 10^{-9}$ s
Feedback reflectivity	$10^{-14} < R_e < 10^{-8}$
Feedback coupling rate	8×10^2 s$^{-1} < \gamma_e = \gamma_c \sqrt{R_e} < 8 \times 10^5$ s^{-1}
Reduced parameters	
LOFI enhancement factor	$\gamma_c/\gamma_n = 10^6$
Coupling parameter	$4.3 \times 10^{-6} < \gamma_e \tau_e < 4.3 \times 10^{-3}$
Dynamical parameter	$\Omega_e \tau_e = 2.8 \times 10^{-2}$

A. Phase detection by frequency-shifted optical feedback

1. Basic equations

In the case of weak optical feedback, the dynamical behavior of a reinjected solid-state laser can be described by a simplified Lang and Kobayashi model [1,14,17]:

$$\frac{dN}{dt} = P - \gamma_1 N - BN|E(t)|^2,$$

$$\frac{d}{dt}[E(t)e^{i\omega t}] = \left[i\omega_c + \frac{1}{2}(BN - \gamma_c)\right]E(t)e^{i\omega t} + \gamma_e E(t-\tau_e)\exp[i\omega(t-\tau_e)] + F_E(t). \quad (1)$$

Here, N is the population inversion, $E(t)$ is the complex slowly varying amplitude of the electric field in reduced units (photon units), ω_c is the laser cavity frequency which is presumed resonant with the atomic frequency, ω is the optical running laser frequency, B is the Einstein coefficient, P is the pumping rate, γ_1 is the decay rate of the population inversion and γ_c is the laser cavity decay rate. The laser quantum fluctuations are described by the conventional Langevin noise function F_E [18,19]. From the dynamical point of view, the optical feedback is characterized by two parameters:

$$\tau_e = \frac{2d_e}{c}, \quad \gamma_e = \gamma_c\sqrt{R_e}. \quad (2)$$

The first one is the photon round-trip time between the laser and the target and the second one is the reinjection rate of the feedback electric field [1,14].

Let us consider now the case of a reinjected beam with an optical frequency shift Ω_e. After a round-trip time τ_e, the reinjected electric field inside the laser cavity is given by

$$E_c(t-\tau_e)e^{i\phi_c(t-\tau_e)}e^{i(\omega+\Omega_e)t}e^{-i(\omega+\Omega_e/2)\tau_e}, \quad (3)$$

where $E_c(t)$ and $\phi_c(t)$ are respectively the slowly varying amplitude and the optical phase of the laser electric field. The set of Eqs. (1) can then be rewritten:

$$\frac{dN}{dt} = P - \gamma_1 N - BN|E_c(t)|^2, \quad (4a)$$

$$\frac{dE_c(t)}{dt} = \frac{1}{2}(BN - \gamma_c)E_c(t) + \gamma_e E_c(t-\tau_e)\cos\left[\Omega_e t - \left(\omega + \frac{\Omega_e}{2}\right)\tau_e - \phi_c(t) + \phi_c(t-\tau_e)\right] + F_{E_c}(t), \quad (4b)$$

$$\frac{d\phi_c(t)}{dt} = \omega_c - \omega + \gamma_e\frac{E_c(t-\tau_e)}{E_c(t)}\sin\left[\Omega_e t - \left(\omega + \frac{\Omega_e}{2}\right)\tau_e - \phi_c(t) + \phi_c(t-\tau_e)\right] + F_{\Phi_c}(t). \quad (4c)$$

In the set of Eqs. (4a)–(4c), the periodic functions express the coherent interaction (beating) between the lasing and the feedback electric fields. The net laser gain is then modulated by the reinjected light at the optical frequency shift $\Omega_e/2\pi$.

The laser quantum fluctuations are now described by the Langevin noise terms [19]:

$$F_{E_c}(t) = \mathrm{Re}[F_E(t)\exp[-j\phi_c(t)]],$$

$$F_{\phi_c}(t) = \frac{\mathrm{Im}[F_E(t)\exp(-j\phi_c(t))]}{E_c(t)}, \quad (5a)$$

which are defined as having a zero mean value

$$\langle F_{E_c}(t)\rangle = 0, \quad \langle F_{\phi_c}(t)\rangle = 0, \quad (5b)$$

and a white noise type correlation function

$$\langle F_{E_c}(t)F_{E_c}(t')\rangle = \frac{\gamma_c}{2}\delta(t-t'), \quad (5c)$$

$$\langle F_{\phi_c}(t)F_{\phi_c}(t')\rangle = \frac{\gamma_c}{2E_S^2}\delta(t-t'), \quad (5d)$$

$$\langle F_{E_c}(t)F_{\phi_c}(t')\rangle = 0. \quad (5e)$$

2. LOFI signal

For weak optical feedback, the set of Eqs. (4a)–(4c) can be solved by linearization. If $\Delta N(t)$, $\Delta E_c(t)$, and $\Delta\Phi_c(t)$ are small modulations of the laser variable around the stationary values, we can write

$$N(t) = N_S + \Delta N(t), \quad (6a)$$

$$E_c(t) = E_S + \Delta E_c(t), \quad (6b)$$

$$\phi_c(t) = \phi_S + \Delta\phi_c(t), \tag{6c}$$

where the laser stationary values are given by

$$N_S = \gamma_c/B, \quad I_S = |E_S|^2 = \frac{\gamma_1}{B}(\eta - 1), \quad \phi_S = 2\pi, \tag{6d}$$

and where I_s is the stationary intensity of the laser field and $\eta = \mathrm{BP}/\gamma_c\gamma_1$ is the normalized pumping parameter [20].

We now suppose that the effect of the feedback on the detuning of the cavity is weak ($\omega \cong \omega_c$) and we only consider the case where the round trip time outside the cavity is much shorter than the period of the modulation ($\tau_e \ll 2\pi\Omega_e^{-1}$). From the dynamical point of view, this implies that $E_c(t - \tau_e) \cong E_c(t)$ and $\phi_c(t-\tau_e) \cong \phi_c(t)$.

In these conditions, by substituting Eqs. (6a)–(6d) into Eqs. (4a)–(4c) and neglecting both the second order term and the Langevin quantum noise, we obtain the following solution for the relative laser output power modulation:

$$\frac{\Delta P_{\mathrm{out}}(t,\Omega_e)}{P_{\mathrm{out}}} = \frac{2\Delta E_c(t,\Omega_e)}{E_s} = 2\sqrt{R_e}G(\Omega_e)\cos(\Omega_e t - \omega_c\tau_e + \Phi_R), \tag{7a}$$

where $G(\Omega_e)$ is an amplification gain defined by the laser parameters:

$$G(\Omega_e) = \frac{\gamma_c\sqrt{\Gamma_R^2 + \Omega_e^2}}{\sqrt{(\Omega_R^2 - \Omega_e^2)^2 + \Gamma_R^2\Omega_e^2}}, \tag{7b}$$

where Φ_R is a dynamical phase shift defined by

$$\tan\Phi_R = \frac{\Omega_e[(\Omega_R^2 - \Omega_e^2) - \Gamma_R]}{\Gamma_R\Omega_R^2}, \tag{7c}$$

and where $P_{\mathrm{out}}(t) = \gamma_c|E_c(t)|^2$ is the photon output rate (number of photon per second), $\Omega_R = \sqrt{\gamma_1\gamma_c(\eta-1)}$ is the laser relaxation oscillation frequency, and $\Gamma_R/2 = \gamma_1\eta/2$ is the damping rate of the relaxation oscillation.

As derived in the Appendix, the demodulation of the laser oscillations [Eq. (7a)] by the means of a lock-in amplifier gives us the amplitude and the phase of the LOFI signal [21]:

$$A_L(R_e) = 2\sqrt{R_e}G(\Omega_e)P_{\mathrm{out}}, \tag{8a}$$

and

$$\Phi_L(d_e) = \omega_c\frac{2d_e}{c}. \tag{8b}$$

Knowing the LOFI amplification gain [Eq. (7b)], the effective reflectivity R_e and the position d_e of the target can be determined.

B. LOFI phase noise

The main objective of this section is to study the phase fluctuations of the LOFI signal ($\sqrt{\langle\Delta\Phi_L^2\rangle}$) induced by the laser quantum noise [$F_{E_C}(t)$ and $F_{\Phi_C}(t)$], in order to determine the ultimate target displacement precision ($\sqrt{\langle\Delta d_e^2\rangle}$) for a given detection bandwidth ΔF.

1. Interferometric phase noise

In a first step, the set of Eqs. (4a)–(4c) suggests that the mean value of the LOFI phase($\Phi_L = \omega_c\tau_e$) need to be compared with the phase fluctuation induced by the inteferometric phase difference:

$$\delta\phi_c(t,\tau_e) = \phi_c(t) - \phi_c(t-\tau_e). \tag{9a}$$

From the dynamical point of view, if we suppose that the round trip time outside the cavity is shorter than the LOFI modulation period ($\tau_e \ll 2\pi\Omega_e^{-1}$), by using [Eq. (4c)], we assume the following approximation for the phase fluctuations:

$$\delta\phi_c(t,\tau_e) \approx \tau_e\dot{\phi}_c(t) = \tau_e F_{\phi_c}(t). \tag{9b}$$

From Eq. (9b), one easily obtains the power density spectrum of the phase difference fluctuation, by using the autocorrelation function of the Langevin phase noise [Eq. (5d)]:

$$\mathcal{P}_{\delta\phi_c}(\Omega) = \mathcal{F}[\langle\delta\phi_c(t,\tau_e)\delta\phi_c(t',\tau_e)\rangle] = \tau_e^2\frac{\gamma_c^2}{2\langle P_{\mathrm{out}}\rangle}, \tag{10a}$$

where $\mathcal{F}[y(t)]$ denotes the Fourier transform of $y(t)$ and where $\langle P_{\mathrm{out}}\rangle = \gamma_c E_S^2$ is the steady-state photon output rate. After the low pass filtering (ΔF) of the lock-in amplifier, the noise power of the LOFI phase is then given by

$$\langle\Delta\Phi_L^2\rangle_a = \mathcal{P}_{\delta\phi_c}(\Omega = 0)2\Delta F = \tau_e^2\frac{\gamma_c^2}{2P_{\mathrm{out}}}2\Delta F = \tau_e^2 2\pi\delta\nu 2\Delta F, \tag{10b}$$

where $\delta\nu = (1/2\pi)(\gamma_c^2/2P_{\mathrm{out}})$ is the laser optical frequency width first introduced by Shawlow and Townes [19,22].

2. Phase-amplitude coupling noise

The set of Eqs. (4a)–(4c) shows that for a laser with optical feedback, the time evolution of the amplitude and the phase of the laser electric field are coupled by the LOFI modulation (i.e., by the coherent interaction between the lasing and the feedback electric fields). As a consequence the LOFI amplitude noise and the LOFI phase noise are also coupled.

In order to be able to calculated the phase-amplitude coupling noise for a laser with optical feedback, let us recall the main results concerning the LOFI amplitude noise [14]. First, for the small laser fluctuations induced by the Langevin quantum noise, the power density spectrum of $E_c(t)$ is given by

$$\mathcal{P}_{E_c}(\Omega) = \frac{\gamma_c}{2}\frac{\Gamma_R^2 + \Omega^2}{[(\Omega_R^2 - \Omega^2)^2 + (\Gamma_R\Omega)^2]}. \tag{11a}$$

Second, for a detection bandwidth ΔF narrower than the laser relaxation resonance width ($2\pi\Delta F \ll \Gamma_R$), the noise power at the frequency shift Ω_e is given by

$$\langle\Delta E_c^2(\Omega_e)\rangle = \mathcal{P}_{E_c}(\Omega_e)2\Delta F. \tag{11b}$$

In these conditions, the signal to noise ratio (S/N) of the LOFI amplitude is obtain by using Eqs. (8a) and (11b) [14]:

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{4R_e G(\Omega_e) P_{\mathrm{out}}^2}{4\gamma_c P_{\mathrm{out}} \langle \Delta E_c^2(\Omega_e) \rangle} = \frac{R_e P_{\mathrm{out}}}{\Delta F}. \qquad (11c)$$

As we can see, the laser quantum noise [Eq. (11a)], as well as the LOFI signal [Eq. (8a)] exhibit a resonance at the laser relaxation frequency ($\Omega_e = \Omega_R$), but the signal to noise ratio is frequency independent [Eq. (11c)] and also shot noise limited. Indeed the ultimate sensitivity (S/N=1) is obtained when, during the integration time ($T = 1/\Delta F$), only one photon is re-injected inside the laser cavity. As an example, for an output beam of one milliwatt at a wavelength of one micrometer, the minimum detectable value of the effective reflection coefficient is $\min(R_e) = 2 \times 10^{-13}$ for an integration time of one millisecond (i.e., $\Delta F = 1$ KHz).

At this point, let us remark that Eq. (11c) can also be directly obtained from the comparison of the LOFI modulation driven function with the Langevin noise driven function obtained from the laser differential equation [Eq. (4b)]:

$$\gamma_e E_c(t-\tau_e) \cos\left[\Omega_e t - \left(\omega + \frac{\Omega_e}{2}\right)\tau_e - \phi_c(t) + \phi_c(t-\tau_e)\right], \qquad (12a)$$

$$F_{E_c}(t). \qquad (12b)$$

From the Appendix, one obtains directly the SNR given by Eq. (11c) by using for the signal $A_S^2 = \gamma_e^2 E_c^2(t-\tau) \approx \gamma_e^2 E_S^2$, and by using for the noise the diffusion coefficient given by Eq. (5c): $D_A^2 = \gamma_c/2$.

As also mentioned in the Appendix, phase measurement fluctuations are also induced by the laser amplitude noise [i.e., by Langevin noise function $F_{E_c}(t)$]. By using Eq. (A7b), the noise power of the LOFI phase induced by the laser amplitude noise is then given by

$$\langle \Delta \Phi_L^2 \rangle_b = \frac{D_A^2}{A_S^2} 2\Delta F = \frac{\gamma_c/2}{\gamma_e^2 E_S^2} 2\Delta F = \frac{1}{R_e P_{\mathrm{out}}} \Delta F. \qquad (13)$$

The phase-amplitude coupling noise is then frequency independent and inversely proportional to the amount of optical feedback.

3. Nonlinear phase-amplitude coupling noise

For higher feedback levels, a much more complicated scheme is also possible to obtain phase-amplitude coupling noise.

As previously mentioned (Sec. II B) the LOFI phase ($\Phi_L = \omega_c \tau_e$) need to be compared with the phase fluctuation induced by the inteferometric phase difference:

$$\delta\phi_c(t,\tau_e) = \phi_c(t) - \phi_c(t-\tau_e) \approx \tau_e \dot{\phi}_c(t). \qquad (14a)$$

For higher feedback levels these phase fluctuations are induced by the non linear interaction of the periodic LOFI modulation with the laser amplitude fluctuation. By using Eq. (4c), one obtains for the nonlinear phase fluctuations the following equality:

$$\delta\phi_c(t,\tau_e) = \tau_e \gamma_e \sin[\Omega_e t - \omega_c \tau_e] \frac{E_c(t-\tau_e)}{E_c(t)}. \qquad (14b)$$

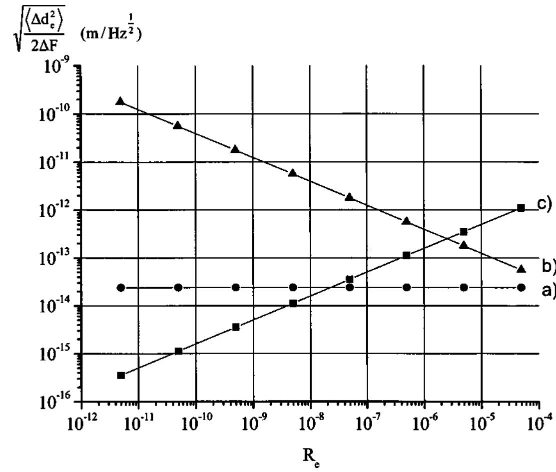

FIG. 2. Target displacement precision $\sqrt{\langle \Delta d_e^2 \rangle}$ versus the effective feedback reflectivity (R_e) for a LOFI detection system using a microchip laser (see Table I for operating laser parameters). ΔF is the lock-in filter bandwidth. (a) Interferometric phase noise [Eq. (10b)]. (b) Phase-amplitude coupling noise [Eq. (13)]. (c) Nonlinear phase-amplitude coupling noise [Eq. (15b)] with resonant optical feedback ($\Omega_e = \Omega_R$).

The power density spectrum of the nonlinear phase fluctuations is then simply obtained from the Fourier transform (FT) of the autocorrelation function of Eq. (14b):

$$\mathcal{P}_{\delta\phi_c}(\Omega) = \mathcal{F}(\langle \delta\phi_c(t,\tau_e)\, \delta\phi_c(t',\tau_e) \rangle)$$
$$\approx \left(\frac{\gamma_e \tau_e^2}{2E_S}\right)^2 (\Omega - \Omega_e)^2 \mathcal{P}_{E_C}(\Omega - \Omega_e), \qquad (15a)$$

where $P_{E_c}(\Omega)$ is the power density spectrum of the laser amplitude fluctuations given by Eq. (11c). After the low pass filtering ($2\pi\Delta F \ll \Gamma_R$) of the lock-in amplifier, the nonlinear noise power of the LOFI phase is then given by

$$\langle \Delta \Phi_L^2 \rangle_c = \mathcal{P}_{\delta\phi_c}(\Omega = 0) 2\Delta F$$
$$= \left(\frac{\gamma_c^4 \tau_e^4}{4}\right) \frac{R_e}{P_{\mathrm{out}}} \frac{\Omega_e^2(\Gamma_R^2 + \Omega_e^2)}{[(\Omega_R^2 - \Omega_e{}^2)^2 + (\Gamma_R \Omega_e)^2]} \Delta F. \qquad (15b)$$

Compared to the interferometric phase noise [Eq. (10b)] and to the linear phase-amplitude coupling noise [Eq. (13)], the nonlinear LOFI phase noise is frequency dependant and exhibits a strong resonance at the laser relaxation frequency ($\Omega_e = \Omega_R$).

4. Ultimate phase sensitivity

For a LOFI detection system using a microchip laser, Fig. 2 shows the ultimate precision of the target position calculated from the LOFI phase fluctuation:

$$\sqrt{\langle \Delta d_e^2 \rangle} = \frac{1}{2} \frac{\lambda_c}{2\pi} \sqrt{\langle \Delta \Phi_L^2 \rangle}. \qquad (16)$$

Trace (a) shows that the interferometric phase noise induced by the laser optical frequency width is only dependant

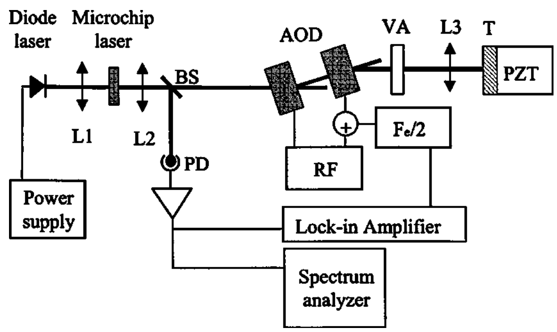

FIG. 3. Schematic diagram of the LOFI experiment: L1-L3, lenses; BS, beam splitter; AOD, acousto-optic-deflector; RF, radio frequency generator; F_e, frequency shift; VA, variable attenuator; T, target under investigation; PZT, piezoelectric transducers; PD, photodiode.

on the laser intrinsic parameters and consequently is independent on the amount of optical feedback (Re).

For the linear phase amplitude coupling noise (trace b), when the effective feedback reflectivity (Re) increases, the LOFI modulation is better defined, resulting in a more precise phase measurement. As a consequence the ultimate precision of the target position increases.

Inversely, for the nonlinear phase amplitude coupling noise (trace c), a strong laser interaction between the LOFI modulation and the laser quantum noise fluctuation is necessary. The ultimate precision of the target position decreases with the amount of optical feedback.

In conclusion, at low feedback level ($R_e < 10^{-6}$), the phase fluctuations are mainly control by the phase amplitude-coupling noise and are several order of magnitude higher than the standard interferometric phase noise ($\sqrt{\langle \Delta\Phi_L^2 \rangle_b} \gg \sqrt{\langle \Delta\Phi_L^2 \rangle_a}$). This result is a direct consequence of the high sensitivity of the microchip laser to optical feedback (i.e., the high value of the cavity damping rate γ_c).

At high feedback level ($R_e > 10^{-6}$), a strong interaction takes place between the LOFI modulation and the laser quantum noise in the vicinity of the laser relaxation frequency. In these conditions, the phase fluctuations increase with the amount of optical feedback, leading to the deterioration of the target displacement precision $\sqrt{\langle \Delta\Phi_L^2 \rangle_c} > \sqrt{\langle \Delta\Phi_L^2 \rangle_b}$.

In all cases [Eqs. (10b), (13), and (15b)] the phase measurement precision is inversely proportional to photon output rate (P_{out}) and then can be increased by increasing the laser output power.

III. EXPERIMENTAL RESULTS

A. Experimental setup

The experimental setup is shown schematically in Fig. 3. The laser is a Nd^{3+}:YAG microchip laser with a cavity length of 800 microns lasing at a wavelength of 1061.34 nm [23,24]. The pumping laser is a 810 nm diode laser. In typical operating conditions the threshold pump power is of the order of a few tens of mW. The maximum pump parameter available is about $\eta \approx 2$. For such conditions, the infrared output power is a few mW and the relaxation frequency $F_R = \Omega_R / 2\pi$ is in the range of 1 MHz.

The frequency shift is generated by means of two acousto-optic deflectors (AOD) respectively supplied by a RF at 81.5 MHz and 81.5 MHz$+F_e/2$ where $F_e = \Omega_e / 2\pi$ is the frequency shift. By combining the diffracted beam (order -1) of the first AOD with the diffracted beam (order $+1$) of the second AOD, the resulting optical frequency shift of the laser is then given by $F_e/2$ ($F_e/2 = -81.5$ MHz$+81.5$ MHz$+F_e/2$). The laser beam is then focused by a lens and sent to the target under investigation mounted on a piezoelectric transducer (PZT). All other beams are stopped by absorbing surfaces. The focused beam is diffracted and/or diffused by the target and only a small part of the retroreflected light is reinjected inside the laser cavity after a second pass through the frequency shifters. After this round trip the optical frequency of the reinjected beam is then shifted by F_e. This frequency can by adjusted and is typically of the order of the laser relaxation frequency. The amount of light coming back inside the laser cavity can be adjusted by means of a variable attenuator (VA). The PZT allows us to control the feedback time delay (i.e., the optical phase shift) between the laser cavity field and the feedback electric field.

A small fraction of the output beam of the microchip laser is sent to a Si-photodiode loaded by a 50 Ω resistor. The delivered voltage is analyzed by by a spectrum analyzer and/or a lock-in amplifier which gives directly the amplitude and the phase of the LOFI signal. All these signals are A/D converted and recorded by a PC for further analysis and/or imaging.

3D images can be obtained from the LOFI amplitude (reflectivity) [25] or from the LOFI phase (profilometry) [26]. Images are obained either by moving the target in three dimensions using micrometric motorized stages or by moving the laser beam using a galvanometric scanner.

B. Experimental observations

1. LOFI signal

Figure 4 shows a typical LOFI power spectra obtained experimentally by using a spectrum analyzer. As we can see, for weak optical feedback the laser fluctuations are principally composed of the LOFI signal at the optical frequency shift F_e and of the laser quantum noise at the relaxation frequency F_R. The LOFI oscillations are induced by the frequency shifted optical feedback while the noise fluctuations are mainly due to the resonant amplification of the Langevin quantum noise at the laser relaxation frequency.

Figure 4 also shows a good agreement between the experimental and the theoretical noise power spectrum of the microchip laser. The discrepancies between the two curves at high frequency comes from the harmonic noise induced by the nonlinear laser dynamics not included in our linear analytical development. For comparison, the power spectrum of the photodiode detector is also shown on this figure. As we can see, the laser quantum noise is several orders of magnitude higher than the detector noise. For a microchip laser, the LOFI detection system is then shot noise limited [14].

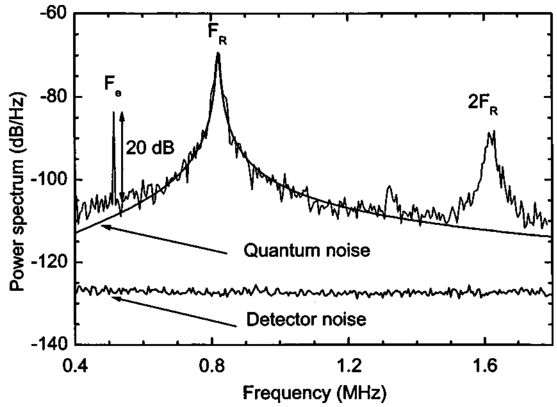

FIG. 4. Typical experimental power spectrum of a LOFI experiment with a Nd^{3+}:YAG microchip laser (see Table I for operating laser parameters). Acquisition time 100 μs. Feedback parameters: $F_e = 500$ kHz, $d_e = 80$ cm, and $R_e \approx 10^{-10}$. The power spectrum of the laser quantum noise and of the photodiode noise are shown for comparison.

At the optical frequency shift ($F_e = 500$ kHz), a direct comparison of the LOFI signal and of the experimental laser quantum noise allows us to determined a S/N of the order of 20 dB (i.e., S/N ≈ 100). Then, for a frequency bandwidth of $\Delta F = 10$ kHz, Eq. (11c) allows us to determine the effective feedback reflectivity of the target under investigation: $R_e \approx 10^{-10}$.

2. LOFI vibration measurement

The demodulation of the LOFI oscillations by means of a lock-in amplifier gives us the amplitude and the phase of the LOFI signal [Eq. (8)].

Figure 5(a) shows the time evolution of the LOFI phase when the optical feedback distance between the target and the laser is periodically modulated at a low frequency by means of a PZT:

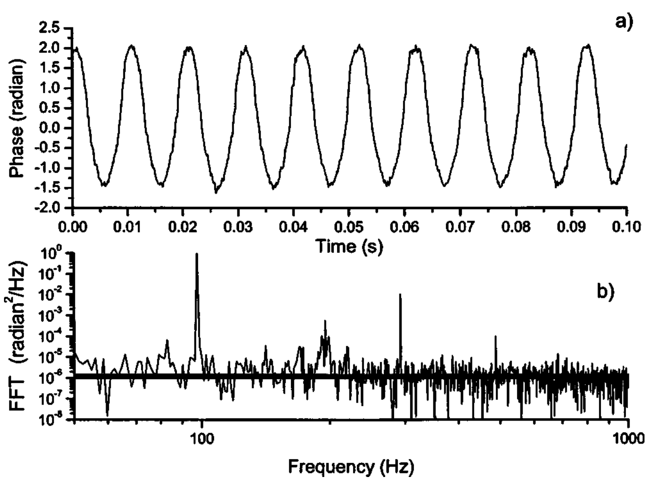

FIG. 5. (a) Experimental time evolution the LOFI phase. (b) Fast Fourier transform (FFT) of the LOFI phase. Acquisition time: 8192 samples $\times$ 100 μs. Feedback parameters: $F_0 = 500$ kHz, $d_e = 80$ cm, and $R_e \approx 10^{-10}$. Vibration parameters: vibration frequency, $F_v \approx 100$ Hz; vibration amplitude, $d_v \approx 150$ nm.

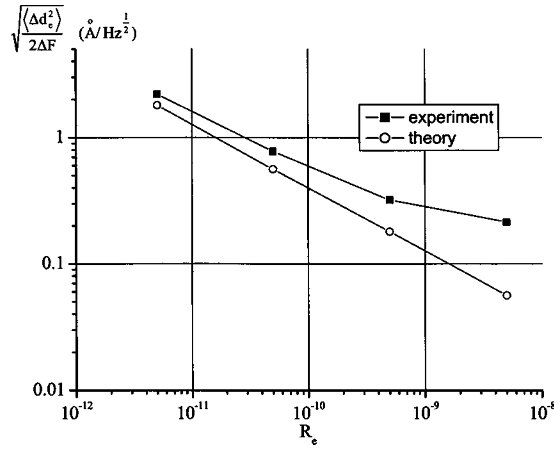

FIG. 6. Experimental and theoretical target displacement precision $\sqrt{\langle \Delta d_e^2 \rangle}$ versus the effective reflectivity R_e. Laser output power: $n_{\mathrm{out}} = 2 \times 10^{16}$ photon/s; lock-in integration time: $T = 100$ μs.

$$d_e = \langle d_e \rangle + d_v \cos(2\pi F_v t), \tag{17}$$

with $\langle d_e \rangle \approx 80$ cm, $d_v \approx 150$ nm, and $F_v \approx 100$ Hz. In good agreement with Eqs. (8b) and (17), the LOFI phase shows a steady periodic oscillation $(1/F_v = 0.01\ \mathrm{s})$ with a peak to peak amplitude given by $(2\pi/\lambda)4d_v \approx \pi$ rad.

Figure 5(b) shows the fast Fourier transform (FFT) of the time evolution of the LOFI phase. The power spectrum is principally composed of the LOFI phase modulation at the vibration frequency F_v and of the phase noise which is a first order approximation frequency independent (i.e, white noise type function). The integration of the power spectrum horizontal base line over the full frequency bandwidth (i.e., 10 kHz) allow us to determine the LOFI phase fluctuation ($\sqrt{\langle \Delta \Phi_L^2 \rangle} \approx 0.14$ rad) and then by using Eq. (16) the target displacement precision ($\sqrt{\langle \Delta d_e^2 \rangle} \approx 10$ nm) [27].

C. Phase measurement precision

Figure 6 shows the dependence of the target displacement precision $\sqrt{\langle \Delta d_e^2 \rangle}$ with the effective feedback reflectivity (R_e). As we can see, when the amount of optical feedback increases, more precise phase measurement are made resulting in a better target displacement precision (i.e., a lower value of $\sqrt{\langle \Delta d_e^2 \rangle}$). The experimental results also show a qualitative good agreement with the theoretical prediction obtained when the phase-amplitude coupling noise is taken into account in the LOFI experiment [Eq. (13)]. The LOFI phase fluctuations are therefore several order of magnitude higher than the standard interferometric phase noise induced by the laser frequency width (Schawlow-Twones limit).

When the amount of optical feedback increases, the disagreement between the experimental result and the theoretical prediction also increase. As we can see in the theoretical section, this effect can be explained by the nonlinear phase-amplitude coupling noise which increases with the amount of optical feedback [Eq. (15b)].

To confirm the effect of the nonlinear phase-amplitude coupling noise, we have studied the LOFI phase fluctuations

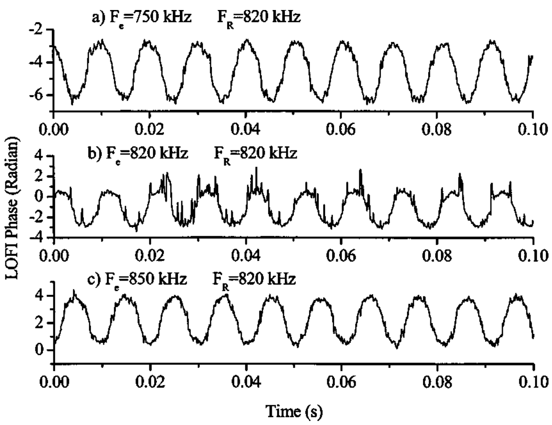

FIG. 7. Time evolution the LOFI phase (Φ_L) for strong optical feedback ($R_e > 10^{-7}$) and for different values of the optical frequency shift: (a) F_e=750 kHz$<F_R$; (b) $F_e=F_R$=820 kHz; (c) F_e =850 kHz$>F_R$. Vibration parameters: vibration frequency, $F_v \approx 100$ Hz; vibration amplitude, $d_v \approx 150$ nm at d_e=80 cm.

in the case of moderate optical feedback ($R_e \approx 10^{-7}$). In a good qualitative agreement with the theoretical predictions, the vibration measurements (Fig. 7) show clearly an increase of the LOFI phase noise when the optical frequency shift becomes resonant with the laser relaxation frequency ($F_e = F_R$).

For strong optical feedback ($R_e \gg 10^{-7}$), nonlinear effects appear in the laser dynamic (bursting, parametric oscillations and chaotic oscillations). The LOFI signal is then very unstable and difficult to analyze. In these conditions, the study of the nonlinear phase amplitude coupling noise is not relevant for further interferometric application. As a consequence the corresponding experimental results are not shown in this paper.

IV. CONCLUSIONS

We have studied both theoretically and experimentally the phase fluctuations of a laser submitted to frequency shifted optical feedback.

At low feedback level (typically $R_e < 10^{-7}$ when working with a Nd:YAG microchip laser), the LOFI phase fluctuations are mainly controlled by the phase-amplitude coupling noise induced by the optical reinjection and are several orders of magnitude higher than the standard interferometric phase noise induced by the laser frequency width (Schawlow-Twones limit). This result is a direct consequence of the high sensitivity of the microchip laser to optical feedback (i.e, the high value of the cavity damping rate γ_c). In this regime when the effective feedback reflectivity (Re) increases, the LOFI modulation is better defined, resulting in a more accurate phase measurement and as a consequence to a better target displacement precision.

At high feedback level ($R_e > 10^{-7}$), a strong interaction takes place between the LOFI modulation and the laser quantum noise in the vicinity of the laser relaxation frequency. In these conditions, the phase fluctuations increase with the amount of optical feedback, leading to the deterioration of the target displacement precision.

In all cases the phase measurement precision is inversely proportional to the photon output rate (P_{out}) and then can be enhanced by increasing the laser output power. Finally, by sending a few milliwatts output power microchip laser beam on a diffusive target with an effective reflectivity of 10^{-9}, a target displacement precision of 0.1 Å/$\sqrt{\text{Hz}}$ has been experimentally determined.

APPENDIX: LOCK-IN AMPLIFIER OUTPUT NOISE

By using a reference frequency given by the optical frequency shift ($\Omega_{\text{ref}} = \Omega_e$), the lock-in amplifier generates, from an input signal $S(t)$, two quadrature components given by

$$V_X = \frac{2}{T}\int_0^T S(t)\cos(\Omega_{\text{ref}}t + \Phi_{\text{ref}})dt,$$

$$V_Y = \frac{2}{T}\int_0^T S(t)\sin(\Omega_{\text{ref}}t + \Phi_{\text{ref}})dt, \tag{A1}$$

where Φ_{ref} is an adjustable reference phase and T is the integration time of the lock-in amplifier.

For a noisy input signal modulated at the optical frequency shift one can suppose that $S(t)$ can be written in the following form:

$$S(t) = A_S\cos(\Omega_e t + \Phi_S) + \delta A(t), \tag{A2}$$

where A_S and Φ_S are respectively the amplitude and the phase shift of the modulation signal and where we assume that $\delta A(t)$ is an additional noise with a zero mean value ($\langle \delta A(t)\rangle = 0$) and a white-noise type correlation functions

$$\langle \delta A(t)\,\delta A(t-\tau)\rangle = D_A^2\delta(\tau). \tag{A3}$$

By substituting Eq. (A2) into Eq. (A1) and after a low pass filtering ($T \gg \Omega_e^{-1}$), the mean values of the quadrature components are given by

$$\langle V_X\rangle \approx A_S\cos(\Phi_{\text{ref}} - \Phi_S)I, \tag{A4a}$$

$$\langle V_Y\rangle \approx A_S\sin(\Phi_{\text{ref}} - \Phi_S), \tag{A4b}$$

while their standard deviations are related to

$$\langle \Delta V_X^2\rangle = \langle (V_X - \langle V_X\rangle)^2\rangle = D_A^2 2\Delta F, \tag{A5a}$$

$$\langle \Delta V_Y^2\rangle = \langle (V_Y - V_Y)^2\rangle = D_A^2 2\Delta F, \tag{A5b}$$

$$\langle \Delta V_X \Delta V_Y\rangle = \langle (V_X - \langle V_X\rangle)(V_Y - \langle V_Y\rangle)\rangle = 0, \tag{A5c}$$

where $\Delta F = 1/T$ is the low-pass filter bandwidth.

With a lock-in amplifier, the modulation amplitude and the phase shift of the input signal are then simply extracted by using the following equations:

$$A_S = \sqrt{\langle V_X\rangle^2 + \langle V_Y\rangle^2}, \tag{A6a}$$

$$\Phi_S = \Phi_{\text{ref}} - a\tan\left(\frac{\langle V_Y\rangle}{\langle V_X\rangle}\right), \tag{A6b}$$

while the amplitude and the phase fluctuations are respectively given by

$$\Delta A_S^2 = \frac{\langle V_X^2\rangle}{\langle V_X^2\rangle + \langle V_Y^2\rangle}\langle \Delta V_X^2\rangle + \frac{\langle V_Y^2\rangle}{\langle V_X^2\rangle + \langle V_Y^2\rangle}\langle \Delta V_Y^2\rangle = D_A^2 2\Delta F, \tag{A7a}$$

$$\Delta \Phi_S^2 = \frac{\langle V_Y^2\rangle}{(\langle V_X^2\rangle + \langle V_Y^2\rangle)^2}\langle \Delta V_X^2\rangle + \frac{\langle V_X^2\rangle}{(\langle V_X^2\rangle + \langle V_Y^2\rangle)^2}\langle \Delta V_Y^2\rangle = \frac{D_A^2}{A_S^2} 2\Delta F. \tag{A7b}$$

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