Multiple beam splitter for single photons

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We propose a method using "light storage" and fractional stimulated Raman adiabatic passage (F-STIRAP) to get entangled multiple Fock states from a single photon. A light storage technique is used to store the quantum information of a single-photon pulse in atoms. F-STIRAP pulses then split the stored coherence, such that reading pulses retrieve the quantum information from this new coherence. Since each reading pulse only retrieves part of the total coherence, we can obtain entangled multiple Fock states with arbitrary relative amplitude. This method to create entanglement is versatile for obtaining frequency, time, and/or spatial entanglement. Indeed, we obtain a multiple beam splitter with easily adjustable parameters.

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I. INTRODUCTION

Quantum entanglement is one of the most striking features of quantum physics. The entanglement of the quantum states of separate particles, such as the entanglement of photon pairs, plays a crucial role in quantum information science. Entangled states explicitly demonstrate the nonlocal character of quantum theory, having potential applications in high-precision spectroscopy, quantum communication, cryptography, and computation. Maximally entangled (Greenberger-Horne-Zeilinger) states have been reported [1–4] for testing quantum nonlocality. Recently, single-particle entanglement attracted attention. Although it lacks the nonlocal character, it has been shown to be useful for simulating certain quantum algorithms [5–8]. Two entangled Fock states of a single photon [9] (also called a single-photon two-mode state [10], a single-photon two-qubit state, or a single-photon Bell state [11]) were proposed to realize quantum cryptography [11,12] and quantum teleportation [10].

EIT (electromagnetically induced transparency) [13] is the basis for "light storage" or "stopping" [14–19]. While being "stopped," the photons are transformed to atomic Raman coherence, which can be transformed back into photons (possibly at different frequency and/or different direction) when the reading pulse arrives. The storage time is only limited by the decay of the Raman coherence, which could be very long, of the order of milliseconds [14,15]. During the storage, we can use some electromagnetic fields or even phonons to manage the coherence [19–21]. This is in essence multiwave mixing with the fields not being applied simultaneously [22]. Since we can manipulate the coherence at different times, we have more freedom than normal multiwave mixing, and manipulating the coherence can be very efficient.

STIRAP (stimulated Raman adiabatic passage) means robust population transfer from one level to another in atomic and molecular systems. In recent years, fractional STIRAP (F-STIRAP) has been used to partially transfer population and create superpositions between the initial and final levels [23–27] or to transfer Raman coherence between pairs of Raman levels [28]. In this paper, we point out that F-STIRAP can be used to split Raman coherence, and thus entangled multiple Fock states from a single photon can be obtained. Payne and Deng [9] have already proposed a scheme using two simultaneous reading pulses at two different frequencies to get two frequency entangled Fock states $|\Psi(z,t)\rangle = a_{10}(z,t)|1_{\omega_1}\rangle|0_{\omega_2}\rangle + a_{01}(z,t)|0_{\omega_1}\rangle|1_{\omega_2}\rangle$. In that scheme, the entanglement process and the light propagation process are intertwined. In contrast, our method separates these two processes and makes it possible to obtain flexible frequency, time, and/or space entangled states $|\Psi(z,t)\rangle = \Sigma_i a_i(t_i)|0_{\omega_0}\rangle\cdots|1_{\omega_i}\rangle\cdots|0_{\omega_n}\rangle$. The time entangled photons could be important in quantum communication [29–31]. Because of its flexibility for producing entangled multiple Fock states from a single photon and the robustness of the photon propagation, we expect its application in quantum information.

This paper is organized as follows. In Sec. II, we describe the splitting of the Raman coherence using F-STIRAP. This is used to get entangled multiple Fock states from a single photon in Sec. III. Discussion and conclusion follow in Sec. IV.

II. COHERENCE SPLIT USING F-STIRAP

We consider a four-level atomic system with lasers acting on the transitions (Fig. 1). We assume that due to their polarizations and frequencies, each laser field drives only one transition and each transition is on resonance. The input probe field $\hat{\mathcal{E}}_{12}$ is assumed to be weak and quantized, while Ω_{13} and Ω_{14} are strong classical fields.

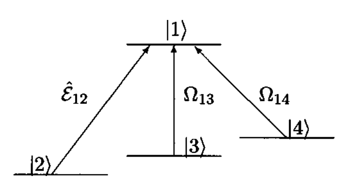

FIG. 1. Four-level atomic system coupled with a probe field $\hat{\mathcal{E}}_{12}$ and two strong classical fields: pump Ω_{13} and Stokes Ω_{14}.

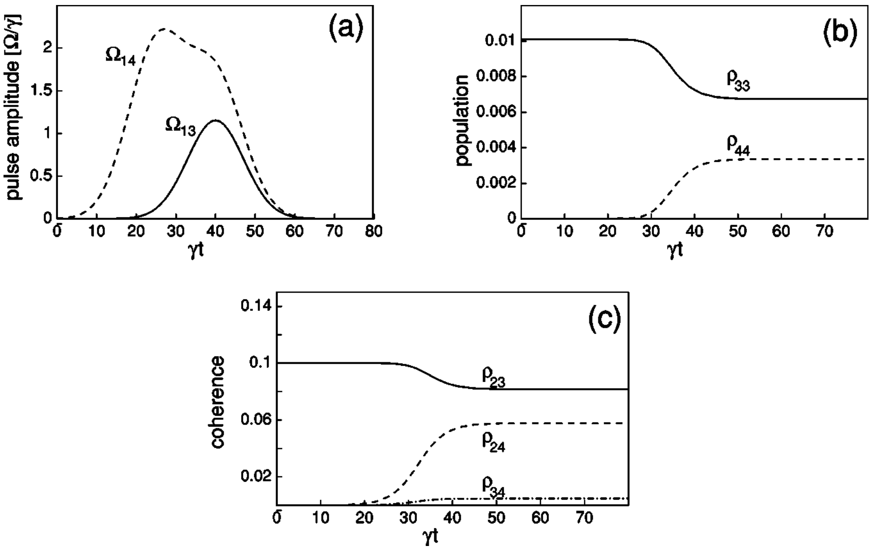

FIG. 2. Split population and Raman coherence for a single atom using fractional STIRAP. (a) Rabi frequencies for F-STIRAP pump Ω_{13} and Stokes Ω_{14} pulses [Eq. (2) with $\tan\alpha = 1/\sqrt{2}$, $\Omega_0 = \gamma$, and $\tau = 10\gamma^{-1}$] as a function of time; time is given in terms of the inverse of decay rate γ out of $|1\rangle$ into each $|2\rangle$, $|3\rangle$, and $|4\rangle$; Rabi frequencies are given in units of γ. (b) Population evolution as a result of the pulses in (a). (c) Evolution of coherences as a result of the pulses in (a).

When the weak probe laser $\hat{\mathcal{E}}_{12}$ is not turned on, the interaction picture Hamiltonian of this system under the dipole and rotating-wave approximation is

$$H(t) = \frac{\hbar}{2}(\Omega_{13}|1\rangle\langle 3| + \Omega_{14}|1\rangle\langle 4|) + \text{H.c.} \quad (1)$$

Here and in the rest of this paper, resonance for each laser is assumed. One of the eigenvalues of $H(t)$ is equal to zero and the corresponding eigenstate is the dark state $|\Phi_-(t)\rangle = [\Omega_{14}(t)|3\rangle - \Omega_{13}(t)|4\rangle]/\Omega(t)$, where $\Omega(t) = \sqrt{\Omega_{13}^2(t) + \Omega_{14}^2(t)}$. The bright state is $|\Phi_+(t)\rangle = [\Omega_{13}(t)|3\rangle + \Omega_{14}(t)|4\rangle]/\Omega(t)$. In fractional STIRAP, as in STIRAP, the Stokes pulse arrives before the pump pulse, but unlike STIRAP, the two pulses terminate simultaneously, while maintaining a constant ratio of amplitudes. The pulses we use have the following forms [24] [Fig. 2(a)]:

$$\Omega_{13}(t) = \Omega_0 e^{-i\phi_{\Omega_{13}}}\sin(\alpha)\exp[-(t-\tau_0)^2/\tau^2],$$

$$\Omega_{14}(t) = \Omega_0 e^{-i\phi_{\Omega_{14}}}\exp[-(t-\tau_0+1.5\tau)^2/\tau^2] + \Omega_0 e^{-i\phi_{\Omega_{14}}}\cos(\alpha)\exp[-(t-\tau_0)^2/\tau^2]. \quad (2)$$

The ratio of the final populations on $|3\rangle$ and $|4\rangle$ is determined by the mixing angle α. Then the dark state has the limits $|\Psi_-(t_0)\rangle = e^{-i\phi_{\Omega_{14}}}|3\rangle$ before the pulse and $|\Psi_-(t_1)\rangle = e^{-i\phi_{\Omega_{14}}}(\cos\alpha|3\rangle - \sin\alpha e^{i\phi}|4\rangle)$ after the pulse with $\phi = \phi_{\Omega_{13}} + \phi_{\Omega_{14}}$. The phase ϕ is a constant which is purely determined by input laser phases [23]. Subsequently, if necessary, this phase can be altered by applying a pulsed magnetic field or off-resonant laser pulses. In what follows, we neglect this phase and assume that all coefficients are real. Suppose the system is initially in state $|\Psi(t_0)\rangle = c_2|2\rangle + c_3|3\rangle$. Since $|2\rangle$ is uncoupled in the above process, c_2 does not change and thus, after the F-STIRAP pulses [Eq. (2)] are applied, the system goes into the superposition

$$|\Psi(t_1)\rangle = c_2|\psi_2\rangle + c_3(\cos\alpha|\psi_3\rangle - \sin\alpha|\psi_4\rangle). \quad (3)$$

Equation (3) shows that not only is the population on $|3\rangle$ split, but also the coherence $\rho_{23}(t_0)$, which, accordingly, is split into ρ_{23} and ρ_{24},

$$\rho_{23}(t_1) = c_2 c_3^* \cos\alpha = \rho_{23}(t_0)\cos\alpha,$$

$$\rho_{24}(t_1) = -c_2 c_3^* \sin\alpha = -\rho_{23}(t_0)\sin\alpha. \quad (4)$$

The ratio of these two coherences is $\rho_{24}(t_1)/\rho_{23}(t_1) = -\tan\alpha$. Note that a coherence $\rho_{34}(t_1) = -c_3^2\sin\alpha\cos\alpha$ is also built during F-STIRAP. This arises because F-STIRAP is a coherent process and can be used to create a superposition state between initial and final Raman levels [24]. However, $\rho_{34}(t_1)$ only depends on the initial population of $|3\rangle$ and not on the initial coherence ρ_{23} and therefore is not interesting here.

A typical example of an F-STIRAP process with $\tan\alpha = 1/\sqrt{2}$, $\Omega_0 = \gamma$, $t_0 = 0$, and $\tau = 10\gamma^{-1}$ for Eq. (2) is shown in Fig. 2. It shows that the population and coherence are split according to Eqs. (3) and (4).

In the next section, we will show that this splitting of Raman coherence makes it possible to get entangled multiple Fock states from a single photon. Since the optical field is quantized, we need atomic flip operators $\sigma_{jk} = |j\rangle\langle k|$ [32], instead of density matrix operators ρ_{jk} as above. Under adiabatic conditions, we can show that

$$\dot{\sigma}_{2-} = 0. \quad (5)$$

Equation (5) shows that σ_{2-} is a constant of motion during the F-STIRAP pulses. So we have

$$\sigma_{23}(t_1) = \sigma_{2+}(t_1)\sin\alpha + \sigma_{2-}(t_0)\cos\alpha,$$

$$\sigma_{24}(t_1) = \sigma_{2+}(t_1)\cos\alpha - \sigma_{2-}(t_0)\sin\alpha. \quad (6)$$

Equation (4) follows from the more general form Eqs. (6), which are the desired form for splitting the coherence created from a single photon in the following section.

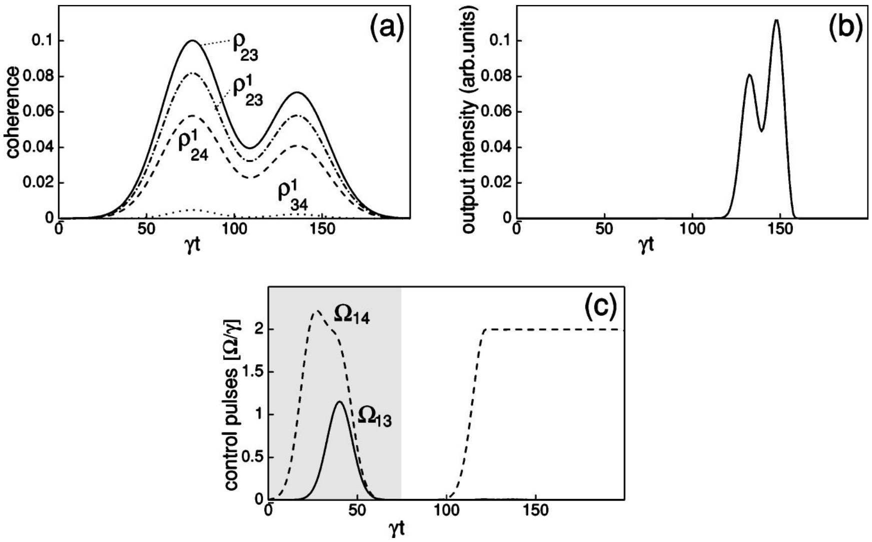

FIG. 3. F-STIRAP pulses split the Raman coherence of atoms in which a single photon has been stored. Pulses used are shown in Fig. 2(a) and the initial Raman coherence ρ_{32} is caused by storage input photon $\hat{\mathcal{E}}_{12}$. $\eta=3\lambda^2 N\gamma/8\pi$. (a) Initial coherence ρ_{23}; final coherence ρ^1_{23}, $-\rho^1_{24}$, and ρ^1_{34} in the medium. (b) Output wave packet from first reading pulse Ω_{14} on coherence ρ^1_{24} as a function of time. (c) Overall F-STIRAP pulses and first reading pulse as a function of time. The shaded area covers the F-STIRAP pulse. Units as in Fig. 2.

III. ENTANGLED FOCK STATES FROM A SINGLE PHOTON

In this section, we will apply F-STIRAP to atoms to split the Raman coherence created by a probe photon of quantized field $\hat{\mathcal{E}}_{12}$. Suppose using the light storage technique [16] on $|1\rangle$, $|2\rangle$, and $|3\rangle$ with writing pulse Ω_{13}. We have stored the probe field $\hat{\mathcal{E}}_{12}$ of a single photon, which has pulse shape function $P_0(t)$, in the atoms. In this process, the polariton has been changed from photonlike to matterlike by doing the following transformation [17,18]: $\hat{\mathcal{E}}_{12}\Rightarrow\sigma_{23}$. Note that the shape and coherence of the photon are preserved when it is stored.

Now, F-STIRAP pulses Ω_{13} and Ω_{14} (shaded areas in Fig. 3) are applied at $t=0$ to split the coherence σ_{23} into σ^1_{23} and σ^1_{24} as Eq. (6),

$$\sigma^1_{23}(t_1) = \sigma^1_{2+} \sin \alpha_1 + \sigma^0_{2-} \cos \alpha_1,$$
$$\sigma^1_{24}(t_1) = \sigma^1_{2+} \cos \alpha_1 - \sigma^0_{2-} \sin \alpha_1. \quad (7)$$

Here and in the following, the superscript i refers to the result of the ith pair of F-STIRAP pulses. Figure 3(a) shows the Raman coherence in the atoms after the first pair of F-STIRAP pulses. Note that both $\langle\sigma^1_{23}\rangle$ and $\langle\sigma^1_{24}\rangle$ have the same shape as the initial coherence σ_{23}, with their relative amplitudes determined by Eq. (4) with $\tan \alpha=1/\sqrt{2}$.

Once we apply reading pulse Ω_{14}, the coherence σ^1_{24} is transformed to the photon wave packet through the dark state polariton [17,18]

$$\hat{\Psi}^1(z,t) = \cos \theta(t)\hat{\mathcal{E}}^1_{12}(z,t) - \sin \theta(t)\sqrt{N}\sigma^1_{24}(z,t) \quad (8)$$

with $\theta(t)$ given by $\tan^2 \theta(t)=g^2N/\Omega^2_{14}(t)$,, where g is the atom-field coupling constant (vacuum Rabi frequency). By changing the strength of the external driving field Ω_{14}, we can rotate θ from $\theta_i=\pi/2$ at time t_{1r} to θ_f at time $t_{1r}+T_r$ with the ramping time of the reading pulse T_r, and thus the polariton makes a transformation $\sigma^1_{24}(z)\Rightarrow\hat{\mathcal{E}}^1_{12}(z')$ with $z'=z+\int_{t_{1r}}^t dtc \cos^2 \theta$ in the medium. If the rotation is fast enough, we have $z'=z+v_g(t-t_{1r})$ with v_g being the group velocity. Once the wave packet exits the medium, it moves at the speed of light in vacuum. This in essence transfers a quantum state from Raman coherence to photon wave packet. After the transfer, the Raman coherence σ^1_{23} is entangled with the output wave packet $\hat{\mathcal{E}}^1_{12}$.

Then we can apply other F-STIRAP pulses and reading pulses in a row. The coherence σ_{23} is split again and again in exactly the same way as above except with increasing τ_0. At the end of the ith F-STIRAP pulses, the amplitude of the coherence σ^i_{24} in the atoms is

$$\langle\sigma^i_{24}(z,t_i)\rangle = -\langle\sigma_{23}(z,0)\rangle\prod_{j=1}^{i-1} \cos \alpha_j \sin \alpha_i, \quad (9)$$

which means the relative magnitude of each wave packet can be arbitrary. This coherence is transformed to photon wave packets when the reading pulse Ω_{14} is applied on $|1\rangle$ and $|4\rangle$. For example, Fig. 4 shows that when there are two F-STIRAP pulses with $\tan \alpha_1=1/\sqrt{3}$ and $\tan \alpha_2=1$, we get three identical output photon wave packets from the initial Raman coherence. As this coherence is obtained from a single photon, we get three entangled Fock states from a single photon. In general, the whole process gives entangled multiple Fock states from a single photon,

$$|\Psi(z,t)\rangle = \sum_i a_i(z,t_i)|0\rangle\cdots|1_i\rangle\cdots|0\rangle, \quad (10)$$

where

$$a_i(z,t_i) = P_0\left(t_i = t - t_{ir} - \frac{z_0}{V_g} - \frac{z-z_0}{c}\right)\prod_{j=1}^{i-1} \cos \alpha_j \sin \alpha_i,$$

t_{ir} is the starting time of the ith reading pulse, and z_0 is the length of the medium. Since the amplitude of each Fock state is determined by the mixing angle α_i, we get *entangled multiple Fock states from a single photon* with arbitrary ratio of amplitudes and separation between Fock states limited only by the duration of F-STIRAP pulses. As an example, Fig. 5 shows the output for $\tan \alpha_1=1/\sqrt{2}$ and $\tan \alpha_2=1$ with the

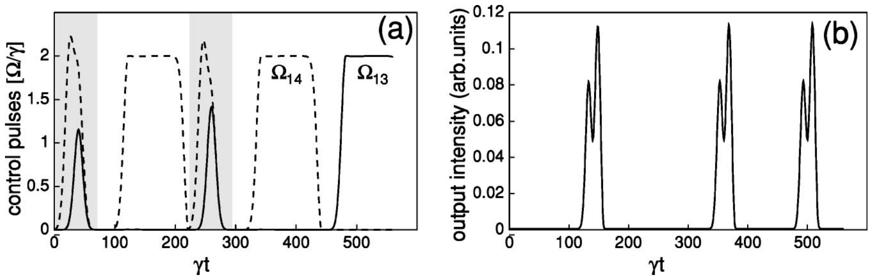

FIG. 4. Illustration of the output of a three-pulse Fock state with equal amplitudes. The initial Raman coherence is caused by storage input photon $\hat{\mathcal{E}}_{12}$ as in Fig. 3. (a) F-STIRAP (shaded parts) and reading pulses Ω_{13} (solid line) and Ω_{14} (dash-dot line) applied to the medium. The equal amplitudes are determined by setting $\tan\alpha_1=\sqrt{2}$ and $\tan\alpha_2=1$. (b) Output wave packets as a result of (a) applied on the initial coherence. Units as in Fig. 2.

amplitude ratio of the three wave packets being $\sqrt{3}:\sqrt{2}:2$. In a word, the total number of output wave packets is the number of F-STIRAP pulses plus 1. The wave packets are identical in shape to the input photon while their relative amplitudes are determined by the termination ratios $\tan\alpha_i$ of the F-STIRAP pulses. This is the major result of this paper.

Note that if we change the frequency and/or direction of the reading pulses [20,21], the corresponding output photon packets would accordingly have different carrier frequencies, and thus we can have time, frequency, and/or spatial entanglement of multi-Fock states from a single photon.

As absorption and dispersions of polaritons are negligible when the spectrum of the probe pulse is in the transparency window of the medium [17,18], the fidelity of "light storage" F_s is bigger than 0.99 if the whole process is done within $0.01\tau_s$ [33], where τ_s is the lifetime of Raman coherence, which is in the order of milliseconds; and the fidelity of F-STIRAP F_f can be bigger than 0.99 when the adiabatic conditions $|\dot{\vartheta}(t)|\ll\frac{1}{2}\Omega(t)$ and $|\dot{\vartheta}(t)|\ll 1/\tau$, where $\vartheta=\arctan[\Omega_{13}(t)/\Omega_{14}(t)]$, are satisfied [24]. So the fidelity of the outcome entanglement, which is the product of F_s and F_f, could be very close to 1. If each of the entangled photon wave packets is transferred to a different qubit, then these qubits are entangled [33] and could be used for quantum communication.

IV. DISCUSSION AND CONCLUSION

Similarly, our scheme allows us to obtain entangled states from an input pulse that has more than one photon or to split coherent pulses. So the whole system works as a multiple beam splitter with easily adjustable parameters. This is also true in [9]. Both methods have mixing angles to determine the amplitudes of the entangled Fock states. The mixing angle in [9] is determined by the product of the length of medium and interaction parameters, while the mixing angle in this paper is determined by the termination ratio of Ω_{13} and Ω_{14}, which makes it considerably easier to control the mixing angles. This is possible in our scheme because the entanglement and propagation processes are separated. Therefore, our scheme allows us to obtain entangled multiple Fock states instead of only two entangled Fock states.

Now we discuss experimental issues. The four-level system can be provided by the D1 line of ^{87}Rb. To decouple the propagation and entanglement processes, F-STIRAP pulses should be transverse to the probe field and thus also transverse to the writing and reading pulses; thus, ultracold atoms should be used to reduce the Doppler effect. Of course, all the pulses should be finished within the storage time of the Raman coherence. Note that homogeneous Raman pulses can also split the coherence, however homogeneous Raman pulses are difficult to realize experimentally, which is why STIRAP and F-STIRAP are used to transfer population in atomic systems.

In conclusion, we proposed a robust scheme using light storage and fractional STIRAP to obtain entanglement of multiple Fock states in both time, frequency, and spatial domains, using single- or few-photon states. We showed that F-STIRAP not only splits the population, but also the coherence. After the input photon is stored in the medium as Raman coherence via dark state polaritons, F-STIRAP pulses are applied to coherently split the atomic Raman coherence into pairs. Reading pulses are then used to retrieve the quantum information in this coherence. Since each reading pulse only retrieves part of the total Raman coherence, we obtain entangled multiple Fock states from a single photon. If each wave packet is stored in a different atomic sample, then these atomic samples are entangled. In our scheme, the entanglement process and the propagation process are separated,

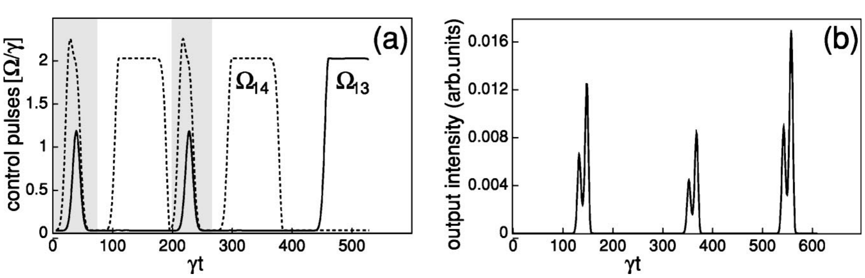

FIG. 5. The same as in Fig. 4, except for output with amplitude ratio $\sqrt{3}:\sqrt{2}:2$. This is caused by setting $\tan\alpha_1=\tan\alpha_2=1/\sqrt{2}$.

which makes our scheme very flexible. In general, we obtain a controllable beam splitter with easily adjustable parameters. We expect interesting quantum information applications based on this scheme.

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[1] C. A. Sackett *et al.*, Nature (London) **404**, 256 (2000).
[2] L. M. Duan, Phys. Rev. Lett. **88**, 170402 (2002).
[3] D. Bouwmeester, J. W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **82**, 1345 (1999).
[4] S. B. Zheng, Phys. Rev. Lett. **87**, 230404 (2001).
[5] N. J. Cerf, C. Adami, and P. G. Kwiat, Phys. Rev. A **57**, R1477 (1998).
[6] G. Brassard, S. L. Braunstein, and R. Cleve, Physica D **120**, 43 (1998).
[7] P. Knight, Science **287**, 441 (2000).
[8] B. G. Englert, C. Kurtsiefer, and H. Weinfurter, Phys. Rev. A **63**, 032303 (2001).
[9] M. G. Payne and L. Deng, Phys. Rev. Lett. **91**, 123602 (2003).
[10] D. Fattal, E. Diamanti, K. Inoue, and Y. Yamamoto, Phys. Rev. Lett. **92**, 037904 (2004).
[11] Y. H. Kim, Phys. Rev. A **67**, 040301(R) (2003).
[12] A. Beige, B. G. Englert, C. Kurtsiefer, and H. Weinfurter, J. Phys. A **35**, L407 (2002).
[13] S. E. Harris, Phys. Today **50**(7), 36 (1997).
[14] D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).
[15] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409**, 490 (2001).
[16] M. Fleischhauer, S. F. Yelin, and M. D. Lukin, Opt. Commun. **179**, 395 (2000).
[17] M. Fleischhauer and M. D. Lukin, Phys. Rev. A **65**, 022314 (2002).
[18] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000).
[19] R. Walsworth, S. F. Yelin, and M. D. Lukin, Opt. Photonics News **13**, 50 (2002).
[20] A. B. Matsko, Y. V. Rostovtsev, O. Kocharovskaya, A. S. Zibrov, and M. O. Scully, Phys. Rev. A **64**, 043809 (2001).
[21] A. S. Zibrov, A. B. Matsko, O. Kocharovskaya, Y. V. Rostovtsev, G. R. Welch, and M. O. Scully, Phys. Rev. Lett. **88**, 103601 (2002).
[22] G. Juzeliūnas and H. J. Carmichael, Phys. Rev. A **65**, 021601(R) (2002).
[23] P. Marte, P. Zoller, and J. L. Hall, Phys. Rev. A **44**, R4118 (1991).
[24] N. V. Vitanov, K. A. Suominen, and B. W. Shore, J. Phys. B **32**, 4535 (1999).
[25] V. A. Sautenkov, C. Y. Ye, Y. V. Rostovtsev, G. R. Welch, and M. O. Scully, e-print quant-ph/0311076.
[26] R. G. Unanyan, B. W. Shore, and K. Bergmann, Phys. Rev. A **63**, 043401 (2001).
[27] R. Unanyan, M. Fleischhauer, B. W. Shore, and K. Bergmann, Opt. Commun. **155**, 144 (1999).
[28] F. Renzoni and S. Stenholm, Opt. Commun. **189**, 69 (2001).
[29] L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) **414**, 413 (2001).
[30] C. H. V. der Wal, M. D. Eisaman, A. Andre, R. L. Walsworth, D. F. Phillips, A. S. Zibrov, and M. D. Lukin, Science **301**, 196 (2003).
[31] A. Kuzmich, W. P. Bowen, A. D. Boozer, A. Boca, C. W. Chou, L. M. Duan, and H. J. Kimble, Nature (London) **423**, 731 (2003).
[32] σ_{jk} are actually collective atomic operators $\sigma_{jk} = (1/N_z)\Sigma_{z_i \in N_z} \sigma^i_{jk}$, where $\sigma^i_{jk} = |j\rangle_{ii}\langle k|$ are the atomic operators for atom i, and $N_z \gg 0$ is the number of atoms in a length interval Δz in which the slowly varying quantum amplitude does not change. Since the collective atomic operators are a linear superposition of the single atomic operators and homogeneous F-STIRAP pulses are assumed for all atoms, we get Eq. (6).
[33] M. D. Lukin, S. F. Yelin, and M. Fleischhauer, Phys. Rev. Lett. **84**, 4232 (2000).