

Orthogonality of biphoton polarization states

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Orthogonality of two-photon polarization states belonging to a single frequency and spatial mode is demonstrated experimentally, in a generalization of the well-known anticorrelation “dip” experiment.

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Orthogonality, one of the basic mathematical concepts, plays an important role in physics, especially quantum physics and, in particular, quantum optics. A well-known example including both classical and quantum cases is orthogonality of two polarization modes of electromagnetic radiation. Physically, orthogonality of two arbitrary polarization states means that if light is prepared in a certain (in the general case, elliptic) polarization state it will not pass through a filter selecting the orthogonal state. (A filter selecting an arbitrary polarization state can be made of a rotatable quarter-wave plate and a rotatable linear polarization filter [1].) Orthogonality of polarization states has an explicit representation on the Poincaré sphere where each polarization state is depicted by a point. The state orthogonal to a given one is shown by a point placed on the opposite side of the same diameter. This concept of orthogonality relates to both classical polarization states of light and single-photon quantum states of polarized light [2]. Mathematically, orthogonality of two quantum states means that the inner product of their state vectors is equal to zero.

However, in addition to single-photon states there are other types of nonclassical light. In quantum optics, one of the central roles is played by two-photon states, which are most easily generated via spontaneous parametric down-conversion (SPDC) [3]. In a two-photon state, radiation consists of photon pairs, often called biphotons, that are correlated in frequency, wave vector, moment of birth, and polarization. Focusing on the case of collinear frequency-degenerate SPDC, here we will discuss the so-called single-mode biphotons. Although even in the frequency-degenerate collinear case SPDC has a finite frequency and angular spectrum, under certain experimental conditions such biphotons can be treated as relating to a single frequency and angular mode.

One can show that the polarization state of a single-mode biphoton [4] can be described as a qutrit, a three-state quantum system. Using qutrits instead of qubits for the transmission of quantum information has been previously discussed [5], in particular, in connection with ternary cryptography protocols [6–8]. The general case of a qutrit represented by a biphoton in an arbitrary pure polarization state is given by the state vector

$$|\psi\rangle = c_1|2,0\rangle + c_2|1,1\rangle + c_3|0,2\rangle, \quad (1)$$

where c_i are complex amplitudes satisfying the normalization condition, $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1$, and $|n,m\rangle$ denotes a two-photon ($n+m=2$) state with n photons polarized horizontally

and m photons polarized vertically [9]. In [10], it is shown that the state (1) allows an explicit representation on the Poincaré sphere. It can be written in the form of two arbitrarily polarized correlated single-photon states,

$$|\psi\rangle = \frac{a^\dagger(\theta, \phi)a^\dagger(\theta', \phi')|\text{vac}\rangle}{\|a^\dagger(\theta, \phi)a^\dagger(\theta', \phi')|\text{vac}\rangle\|}. \quad (2)$$

Here, $a^\dagger(\theta, \phi)$ and $a^\dagger(\theta', \phi')$ are operators of photon creation in arbitrary polarization modes given by the coordinates $\theta, \phi, \theta', \phi'$ on the Poincaré sphere:

$$a^\dagger(\theta, \phi) = \cos(\theta/2)a_{H}^\dagger + e^{i\phi}\sin(\theta/2)a_{V}^\dagger, \quad (3)$$

where $a_{H,V}^\dagger$ are photon creation operators in the horizontal and vertical linear polarization modes, and similarly for $a^\dagger(\theta', \phi')$. Axial angles θ, θ' and azimuthal angles ϕ, ϕ' [11] are in one-to-one correspondence with the four parameters describing the state (1), which are, for instance, $d_1 = |c_1|, d_3 = |c_3|, \phi_1 = \arg(c_1) - \arg(c_2), \phi_3 = \arg(c_3) - \arg(c_2)$. Representation (2) means that a biphoton of arbitrary polarization can be shown as a pair of points on the Poincaré sphere. It turns out that the Stokes vector of a biphoton is simply a normalized sum of the Stokes vectors of photons forming it (“biphoton halves”), and the polarization degree P of the pair (in its classical definition, see Ref. [1]) is given by the angle σ at which the pair can be seen from the sphere center.

A question arises: what does orthogonality of two biphoton polarization states mean? As usually in quantum mechanics, it means that the product of their state vectors is equal to zero. For instance, orthogonality of two biphotons $\psi_{ab} \equiv a^\dagger b^\dagger|\text{vac}\rangle/\|a^\dagger b^\dagger|\text{vac}\rangle\|$ and $\psi_{cd} \equiv c^\dagger d^\dagger|\text{vac}\rangle/\|c^\dagger d^\dagger|\text{vac}\rangle\|$, with the operators of photon creation in arbitrary polarization modes denoted now by $a^\dagger, b^\dagger, c^\dagger, d^\dagger$, means that

$$\langle \text{vac} | c d a^\dagger b^\dagger | \text{vac} \rangle = 0. \quad (4)$$

What does orthogonality of two biphotons mean from the viewpoint of physics? This question was answered in [12] where an operational criterion of orthogonality for arbitrarily polarized biphotons was formulated. Namely, orthogonality of two biphotons can be tested using a simple setup consisting of a nonpolarizing beamsplitter, two detectors installed in its two output ports, with an arbitrary polarization filter inserted at the input of each detector, and a coincidence circuit. A biphoton is registered if there is a coincidence of photo-counts from the two detectors. Let a biphoton $|\psi_{ab}\rangle$ be at the input and the filters in the output ports of the beamsplitter select polarization states corresponding to photon creation

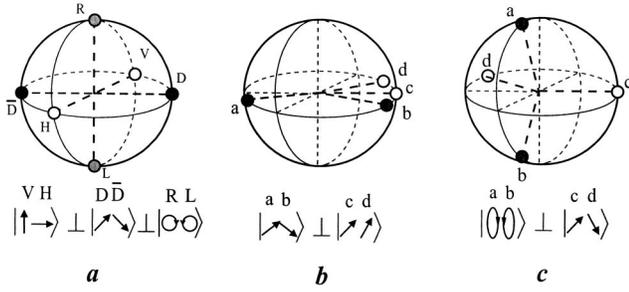


FIG. 1. Different cases of orthogonal biphotons: (a) three orthogonal nonpolarized biphotons, $|HV\rangle$, $|RL\rangle$, and $|D\bar{D}\rangle$; (b) two orthogonal biphotons formed by linearly polarized photons. The input biphoton has polarization degree $P=0.5$ and the polarizers are at 45° and 60° to the horizontal axis. (c) The “nonplane” version: the input state also has $P=0.5$ but is formed by elliptically polarized photons.

operators c^\dagger, d^\dagger . Then orthogonality of $|\psi_{ab}\rangle$ and $|\psi_{cd}\rangle$ is equivalent to the absence of coincidences [13] in such a setup. Note that orthogonality of any two polarization states among $a^\dagger|vac\rangle$, $b^\dagger|vac\rangle$, $c^\dagger|vac\rangle$, $d^\dagger|vac\rangle$ is not required.

Such an experiment is the most general case of the well-known anticorrelation experiment [14]. Earlier, a particular case of this polarization version of anticorrelation experiment has been performed for type-II SPDC [15]. The absence of coincidences in the anticorrelation experiment [15] can be interpreted as orthogonality of the states $|HV\rangle \equiv a_H^\dagger a_V^\dagger |vac\rangle$ and $|D\bar{D}\rangle \equiv a_{45}^\dagger a_{-45}^\dagger |vac\rangle$, a pair of photons polarized linearly at angles $\pm 45^\circ$ to the vertical axis. Similarly, in [16], orthogonality of the states $|HV\rangle$ and $|D\bar{D}\rangle$ to the state of right- and left polarized photons, $|RL\rangle$, has been demonstrated. In both cases, orthogonal biphotons had zero polarization degree, which means that they were pairs of orthogonally polarized photons. Another example of a basis formed by three mutually orthogonal biphotons with zero polarization degree was demonstrated in [17].

Several examples of orthogonal biphotons of other polarization degrees are shown in Fig. 1. Figure 1(a) shows three mutually orthogonal states $|HV\rangle, |D\bar{D}\rangle, |RL\rangle$ of biphotons with zero polarization degree studied in [15,16]. All three states are biphotons consisting of two orthogonal photons; at the same time, no photon forming a biphoton is orthogonal to any photon of the other two biphotons. Three biphotons shown in Fig. 1(a) form an orthogonal basis.

Whenever a biphoton is fixed (two points are fixed on the Poincaré sphere), there are infinitely many biphotons orthogonal to it. Replacing the Poincaré sphere by the globe, one can pick two spots denoting a biphoton to be, for instance, Moscow (Russia) and Turin (Italy). Then, one should make a choice for the third point. Let it be Baltimore (MD, USA). Then the fourth point is found from Eq. (4), and it turns out to be near New Zealand and the Bounty isles. So, the biphoton “Moscow-Turin” is orthogonal to the biphoton “Baltimore-Bounty!”

The idea of our experiment was to prepare some arbitrary input biphoton state, to make the registration part select the state orthogonal to it, and to demonstrate orthogonality by

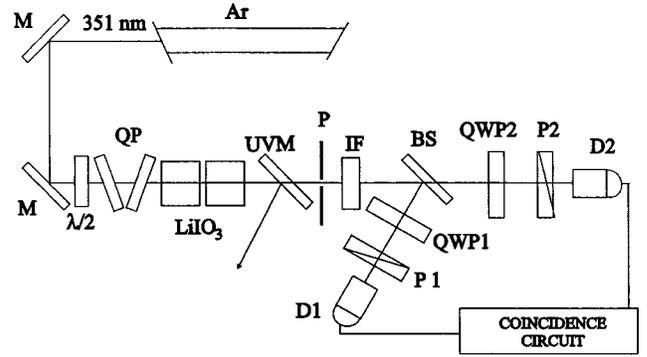


FIG. 2. The experimental setup. SPDC is excited in two type-I LiIO_3 crystals with the optic axes in orthogonal planes; the first crystal generates $|2,0\rangle$ and the second one, $|0,2\rangle$. Quartz plates QP enable variation of the phase between the two states from 0 to π . UVM is a UV mirror, P a pinhole, IF an interference filter, BS a nonpolarizing beamsplitter, QWP1,2 are quarter-wave plates, P1,2 rotatable polarizers, and D1,2 detectors.

scanning the parameters of the input and registered states around the minimum of coincidence counting rate. We chose the input biphoton state to be a pair of photons polarized linearly at the opposite angles to the horizontal direction, the polarization degree being $P=0.5$. On the Poincaré sphere, this is shown as two points on the equator placed symmetrically at the angles $\pm 74.5^\circ$ with respect to the “H” axis, which corresponds to photons polarized linearly at the angles 37.25° to the horizontal axis. For the reasons that we will explain later, it is convenient to make one of the polarization filters in the registration part select the state polarized linearly at the angle 45° to the horizontal axis. The other filter, as one can easily find, should then select linear polarization at the angle 60° to the horizontal axis. This configuration, which is used in one of our experiments, is shown in Fig. 1(b). Finally, Fig. 1(c) gives an example of two orthogonal biphotons with polarization degree $P=0.5$ in a “nonplane” configuration. This time, the points denoting the biphoton “ab” are placed on the “Greenwich meridian,” if we follow the globe terminology. Again, one of the polarizers selects the state polarized linearly at 45° . However, the position of the other polarizer is changed: now, it should select linear polarization at the angle -60° to the horizontal axis. Under each Poincaré sphere in Fig. 1, the corresponding polarization states are shown schematically. In all these examples, there is no orthogonality between separate photons, or “halves of biphotons.”

The experimental setup is shown in Fig. 2. Collinear frequency-degenerate SPDC is generated in two similar type-I lithium iodate crystals of length 1 cm, cw radiation of argon laser at wavelength 351 nm used as the pump. The optic axis of the first crystal is in the vertical plane while the optic axis of the other crystal is in the horizontal plane. The two-photon state generated after the crystals is of the form (1), with $c_2=0$, the amplitudes d_1 and d_3 can be varied by rotating the $\lambda/2$ plate in the pump beam, and the phase $\Delta\phi \equiv \phi_3 - \phi_1$ can be varied by tilting the two quartz plates QP, whose optic axes are oriented in the vertical plane. The pump after the crystals is cut off by a UV mirror.

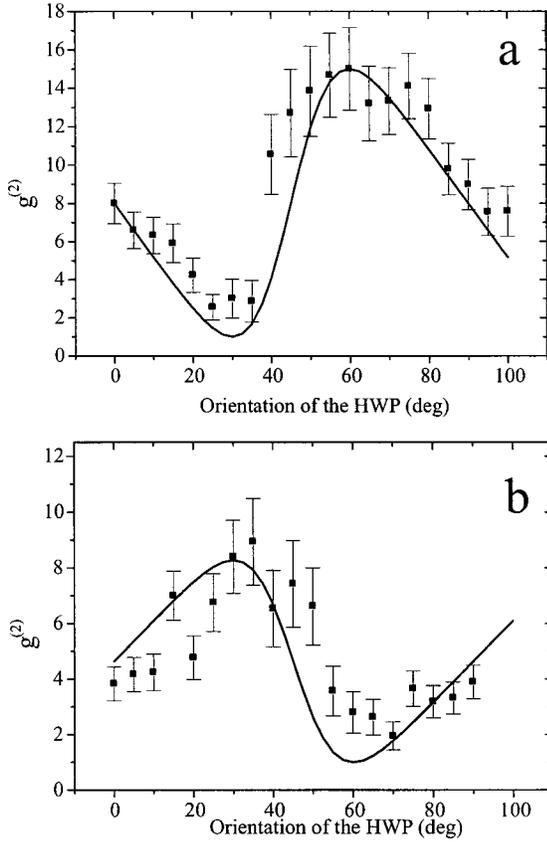


FIG. 3. Normalized second-order correlation function versus the angle χ for $\Delta\phi=\pi$ and the polarizers P1,2 fixed at 45° and 60° , respectively (a) and at 45° and -60° , respectively (b).

Spatial and frequency filtering of the SPDC radiation is performed by a pinhole P and an interference filter IF with 702 nm central wavelength and 3 nm bandwidth. The right-hand side of the setup shows the registration part (the Brown-Twiss interferometer). It includes a nonpolarizing beamsplitter BS and two detectors (photomultiplier tubes) D1, D2 inserted in its output ports. At the input of each detector, there is a polarization filter consisting of a rotatable quarter-wave plate QWP1,2 [18] and a rotatable polarizer P1,2. Coincidences between the photocounts of the detectors are registered using a coincidence circuit with a resolution $T_c = 5.5$ ns.

The measurements were performed for two configurations shown in Figs. 1(b) and 1(c). The plates QP were tilted so that the phase $\Delta\phi$ was equal to π . Then the angle χ of the $\lambda/2$ plate was scanned from 0° to 90° . As a result, the two points corresponding to the produced biphoton state traveled on the Poincaré sphere: first, from the “VV” point ($\chi=0^\circ$) to the “HH” point ($\chi=45^\circ$) symmetrically along the opposite sides of the equator, then again to the “VV” point ($\chi=90^\circ$) but this time, along the opposite sides of the “Greenwich meridian.” This way, both cases shown in Figs. 1(b) and 1(c) were realized. In the first run, the polarizer P1,2 orientations were fixed as in Fig. 1(b): $\zeta_1=45^\circ$ and $\zeta_2=60^\circ$. In the dependence of coincidence counting rate on χ [Fig. 3(a)], the minimum was at $\chi=30^\circ$, which corresponded to $d_1^2/d_3^2=3$. For this point, the ratio d_1^2/d_3^2 was measured using the tomogra-

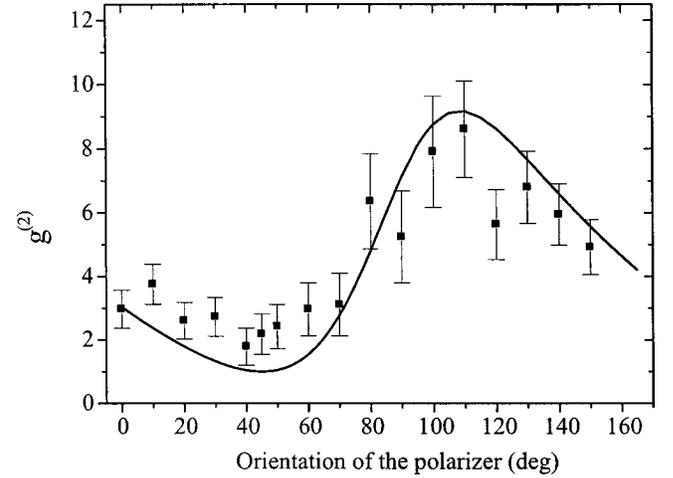


FIG. 4. Normalized second-order correlation function versus the angle of the polarizer P1 orientation with the half-wave plate fixed at $\chi=30^\circ$ and the phase $\Delta\phi=\pi$.

phy procedure developed in [19]; this ratio turned out to be 3.4 ± 0.8 . The 45° orientation of P1 is convenient since in this case, rotation of the half-wave plate in the pump beam does not lead to the variation of D1 single counting rate R_1 .

In the next run, the orientations of P1, P2 were fixed as in Fig. 2: $\zeta_1=45^\circ$ and $\zeta_2=-60^\circ$. In this case, the minimum was achieved for $\chi=60^\circ$ [Fig. 3(b)], corresponding to $d_1^2/d_3^2=1/3$.

Finally, orthogonality of two biphoton states was checked by fixing all parameters ($\chi, \zeta_{1,2}, \Delta\phi$) in the configuration shown in Fig. 1(b) and then scanning them around their optimal values. The plot in Fig. 4 shows the dependence of the coincidence counting rate on the orientation of polarizer P1, with the other polarizer fixed at 60° and the half-wave plate in the pump beam fixed at $\chi=30^\circ$. Similar dependencies were obtained for scanning ζ_2 and $\Delta\phi$.

Two important notes should be made about the measurement procedure. First, when the angle χ is scanned with fixed positions of the two polarizers [Fig. 3(a)], a considerable modulation in the singles counting rate R_2 of detector D2 is observed (Fig. 5). This is quite natural since in the course of χ variation the state of biphoton light changes from being vertically polarized through completely nonpolarized state (at $\chi=22.5^\circ$) to being horizontally polarized. With the polarizer P1 oriented at $\pm 60^\circ$, the intensity of transmitted light should vary three times, as it indeed does in Fig. 5. However, the minimum of the coincidence counting rate R_c does not correspond to the point where R_2 is minimal; according to the calculation, it occurs at $\chi=30^\circ$. The second remark is that in all kinds of anticorrelation experiments, the coincidence counting rate in the minimum is given by the level of accidental coincidence counting rate, which corresponds to the normalized second-order Glauber’s correlation function $g^{(2)}=1$. To demonstrate this, instead of the coincidence counting rate, in all plots we present $g^{(2)}$ [20] instead of R_c .

To find the dependence of the coincidence counting rate R_c on all parameters, one can write

$$R_c \sim |\langle \text{vac} | cda^\dagger b^\dagger | \text{vac} \rangle|^2. \quad (5)$$

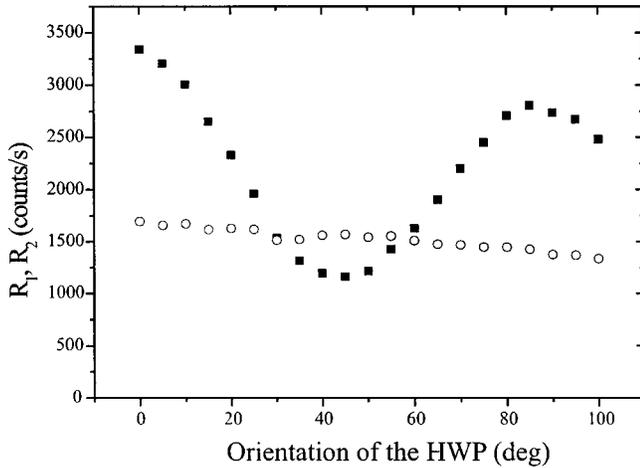


FIG. 5. Single-photon counting rates at detectors D1 (circles) and D2 (squares) versus the angle χ for $\Delta\phi=\pi$ and the polarizers P1,2 fixed at 45° and 60° , respectively. The total decrease in the counting rates is caused by a gradual decrease in the pump power.

The operators $a^\dagger, b^\dagger, c, d$ are substituted in Eq. (5) in the form (3), with the angles ϕ taking the values $0, \pi, 0, 0$, respectively. The axial angles for a, b can be expressed as functions of χ via the relations given in [12] and the formulas $d_1=\sin(2\chi)$, $d_3=\cos(2\chi)$. The axial angles for c, d can be calculated as functions of $\zeta_{1,2}$, the angles of P1,P2 orientations. Then we obtain the coincidence counting rate,

$$R_c \sim [\cos \zeta_1 \cos \zeta_2 \sin(2\chi) - \sin \zeta_1 \sin \zeta_2 \cos(2\chi)]^2, \quad (6)$$

and the single-photon counting rate in detector 2,

$$R_2 \sim \cos^2 \zeta_2 \sin^2(2\chi) + \sin^2 \zeta_2 \cos^2(2\chi). \quad (7)$$

Equations (6) and (7) were used to plot the theoretical dependencies shown in Figs. 3 and 4.

Since in the cases of both “planar” and “nonplanar” geometry, the experimentally measured coincidence counting rates in the minima exceed the theoretically predicted ones, it is important to find the measured value of the squared scalar product of the prepared state and the measured state. This can be done by using as references the maxima of dependencies shown in Figs. 3(a) and 3(b). Calculation shows that for the case of Fig. 3(a), the value of $|\langle \psi_{ab} | \psi_{cd} \rangle|^2$ in the maximum is equal to 0.39 ± 0.04 , and in the case of Fig. 3(b), to 0.74 ± 0.1 . Hence, the observed values of squared scalar products in the minima are equal, respectively, to 0.05 ± 0.015 and 0.09 ± 0.02 .

To conclude, we have experimentally demonstrated orthogonality of two biphotons having polarization degree between 0 and 1. Our experiment is a generalization of the “anti-correlation dip” experiment to the case of arbitrarily polarized photon pairs. The observed effect can find applications in ternary quantum cryptography protocols [21].

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