Adiabatic path to fractional quantum Hall states of a few bosonic atoms

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(Received 10 May 2004; published 17 November 2004)

We propose a realistic scheme to create motionally entangled states of a few bosonic atoms. It can experimentally be realized with a gas of ultracold bosonic atoms trapped in a deep optical lattice potential. By simultaneously deforming and rotating the trapping potential on each lattice site it is feasible to adiabatically create a variety of entangled states on each lattice well. We fully address the case of $N=2$ and 4 atoms per well and identify a sequence of fractional quantum Hall states: the Pfaffian state, the $1/2$-Laughlin quasiparticle, and the $1/2$-Laughlin state. Exact knowledge of the spectrum has allowed us to design adiabatic paths to these states, with all times and parameters well within the reach of current experimental setups. We further discuss the detection of these states by measuring different properties as their density profile, angular momentum, or correlation functions.

DOI: 10.1103/PhysRevA.70.053612 PACS number(s): 03.75.Lm, 73.43.−f

I. INTRODUCTION

The creation of highly entangled multiparticle states is one of the most challenging goals of modern experimental quantum mechanics. In this respect atomic systems offer a very promising arena in which entangled states can be created and manipulated with a high degree of control. The experimental difficulty increases, however, with the number of particles that are to be entangled, since the system becomes then more sensitive to decoherence. Starting with a small number of particles as a first step, important achievements have been already obtained in the creation of atomic entangled states. For example, in recent experiments with trapped ions, entangled states of up to four ions have been demonstrated [1]. Moreover, in experiments with neutral bosonic atoms in optical lattices Bell-type states have been created by accurately controlling the interactions between neighboring atoms [2]. As a typical feature of most of the experimentally realized entangled states, atoms get entangled through their internal degrees of freedom, keeping separable their motional part.

In this article we develop a scheme to create *motional* entangled states of a small number of atoms in an actual experimental setup with an optical lattice [3–5]. These states are a sequence of fractional quantum Hall (FQH) states, analogous to the ones that appear in the context of the fractional quantum Hall effect [6]. In contrast to typical atomic entangled states, the particles are here entangled in real space, and not in internal space. This peculiarity makes them specially interesting, for it represents an unusual nature of atomic entanglement.

The possibility of creating FQH atomic states like the Laughlin state by rapidly rotating the trap confining the atoms has been discussed in several theoretical works [7,8]. However, experiments dealing with typically large number of particles have not yet succeeded in reaching these states. Here, we fully address the case of a small number of particles and design a realistic way of entangling them into FQH states. The experimental setup that we have in mind corresponds to a situation in which a Bose-Einstein condensate is loaded in a deep optical lattice. When the lattice depth is very large tunneling between different sites is strongly suppressed and the system can be treated as a lattice of independent wells, each of them with a small number of particles. By independently rotating each of these three-dimensional (3D) wells [9] the lowest Landau level (LLL) regime can be achieved for each copy. We have studied the problem exactly within the LLL for $N=2$ and 4 particles per well. We have identified a sequence of highly entangled stable ground states, which are the Pfaffian state [10], the $1/2$-Laughlin quasiparticle [11], and the $1/2$-Laughlin state [12]. The $1/2$-Laughlin quasiparticle state (which had never been identified before in an atomic system) is particularly interesting. It is the counterpart of the $1/2$-Laughlin quasihole found in [8] and contains a $1/2$-anyon. Driving the system into these strongly correlated states is, however, not trivial. By simply increasing the frequency of rotation the system will stay in a trivial nonentangled state with angular momentum zero. Exact knowledge of the spectrum of the system has allowed us to design adiabiatic paths to these states by simultaneously rotating and deforming each of the wells. All parameters and evolution times lie well within the reach of present experimental setups. We further discuss how to detect these entangled states by measuring different properties as their density profile, angular momentum, or correlation functions. In particular, we propose a technique to measure the density-density correlation function of these strongly correlated states. Even though the number of atoms per well is small, the lattice setup allows one to have multiple copies of the system, so that the experimental signal is highly enhanced.

We point out that our findings also show that adiabatically achieving FQH states for rapidly rotating traps with a large number of particles turns out to be very challenging, since the relevant experimental parameters scale linearly with the number of particles. Nevertheless, we hope that our results can shed some light on the problems that these current experiments are dealing with, and even may pave the way to additional methods of achieving FQH multiparticle entangled states.

II. IDENTIFICATION OF ENTANGLED STATES

We consider a system of bosonic atoms loaded in a 3D optical lattice. We assume a commensurate filling of N atoms per lattice site [13], and a large value of the lattice depth, $V_0/E_R \gg 1$, where $E_R=\hbar^2k^2/2M$ is the recoil energy, k is the wave vector of the laser light, and M is the atomic mass. In this limit the lattice can be treated as a system of independent 3D harmonic wells, each of them having N atoms and a trapping frequency $\omega \approx \sqrt{V_0 E_R}$.

Let us rotate each of these 3D harmonic wells around the direction x_3 with frequency Ω. We will identify a sequence of motional entangled ground states of the N atoms that appear as the frequency Ω is increased. We will assume the limit of rapid rotation [8]. In this case the motion in the x_3 direction is frozen, and the motion in the plane of rotation x_1, x_2 is restricted to the LLL. Note that in order to project the system onto the LLL we do not need to start with a 2D configuration (as is the case in previous proposals [7]), since the fast rotation itself restricts the motion in the direction of the rotation to zero-point oscillations. The system is then governed by a two-dimensional effective Hamiltonian, which written in units of $\hbar\omega$ has the form

$$H = (1-\Omega/\omega)L + 2\pi\,\eta V, \tag{1}$$

where $L=\Sigma_{m=0} m a_m^\dagger a_m$ is the angular momentum operator in the x_3 direction, and $V=\Sigma_{m_1,m_2,m_3,m_4} V_{m_1,m_2}^{m_3,m_4} a_{m_1}^\dagger a_{m_2}^\dagger a_{m_3} a_{m_4}$ is the interaction operator. Here the bosonic operator $a_m^\dagger$ (a_m) create (anihilate) an atom in the state $|m\rangle$ of the LLL with well defined x_3 component of the angular momentum m. The wave functions of the LLL in complex coordinates read

$$\varphi_m(z) = \langle z|m\rangle = \frac{1}{\sqrt{\pi m!}\,\ell} z^m e^{-|z|^2/2}, \tag{2}$$

where, $z=(x_1+ix_2)/\ell$, $\ell=\sqrt{\hbar/M\omega}$, and $m=0,1,\ldots,\infty$. Assuming contact interactions between the atoms the interaction coefficients are

$$V_{m_1,m_2}^{m_3,m_4} = \frac{(m_1+m_2)!}{2^{m_1+m_2}\sqrt{m_1!m_2!m_3!m_4!}}. \tag{3}$$

In Hamiltonian (1) we have introduced the important interaction parameter $\eta=\sqrt{2/\pi}a_s/\ell$, with a_s the 3D scattering length. Analytical calculations for scattering potentials of finite size a_0 have confirmed that the pseudopotential approximation is also valid for tight traps with $a_s \ll \ell$ as long as $a_0 \ll \ell$ is satisfied [14].

A. $N=2$

First we consider the case of two particles per lattice well, which can be solved analytically. The Hamiltonian (1) is diagonal in the states $|m_r, m_{\text{c.m.}}\rangle$ of well defined relative (m_r) and center of mass ($m_{\text{c.m.}}$) angular momentum:

$$H = \sum_{m_r,m_{\text{c.m.}}} E_{m_r,m_{\text{c.m.}}} |m_r,m_{\text{c.m.}}\rangle\langle m_r,m_{\text{c.m.}}|, \tag{4}$$

with $E_{m_r,m_{\text{c.m.}}} = \delta_{m_r,0}\eta + (1-\Omega/\omega)(m_r+m_{\text{c.m.}})$. We note that due to the restriction to s-wave scattering, only particles with zero relative angular momentum feel the interaction energy. It follows that for $\Omega/\omega < 1-\eta/2$ the ground state of the system is $|0,0\rangle$ (with total angular momentum $L=0$), which is not entangled, whereas for $\Omega/\omega > 1-\eta/2$ the state $|2,0\rangle$ (with $L=2$) becomes energetically favorable. This state, $\langle z_1,z_2|2,0\rangle \propto (z_1-z_2)^2 e^{-|z_1|^2/2} e^{-|z_2|^2/2}$, is clearly entangled since it cannot be written as a product of two single-particle wave functions. It is the Laughlin state $|\psi_L\rangle$ for two particles at filling factor $\nu=1/2$ [11]. In order to quantify the entanglement of this state we write it in the basis of states $|m_1 m_2\rangle$ with well defined single-particle angular momentum. Then the Laughlin state takes the form of a pure two-qutrit state: $|\psi_L\rangle = \frac{1}{2}(|02\rangle+|20\rangle) - (1/\sqrt{2})|11\rangle$. This is already the Schmidt decomposition of the state, and the entropy of entanglement [16] can immediately be calculated to be $E(|\psi_L\rangle)=1.5$. This value is close to $\log_2 3$, corresponding to a maximally entangled pure two qutrit state.

B. $N=3$

The case of three particles per lattice well is very similiar to the situation for $N=2$. The 1/2-Laughlin state ($L=6$) emerges as the ground state after an intermediate state with odd angular momentum $L=3$. As we will explain in the next section, ground states with odd angular momentum cannot be reached using our proposal. Hence we now focus on a setup with four particles per lattice well, for which an interesting sequence of prominent FQH states arises.

C. $N=4$

In order to obtain the multiparticle energy spectrum, we have exactly diagonalized the Hamiltonian (1) numerically. As the frequency of rotation Ω increases the ground state of the system passes through a sequence of states with increasing and well defined total angular momentum $L=0,4,8,12$ (see Fig. 1).

These states can be identified as follows. The state with $L=0$ is a trivial nonentangled state in which all the atoms are condensed in the single-particle Gaussian state with angular momentum $m=0$. The first nontrivial ground state is the $L=4$ state. This state is not, as one might expect, a single-vortex state, in which all the particles would be condensed in the single-particle state $m=1$. In contrast, this state is highly entangled and is very close (fidelity 0.95) to the well-known Pfaffian state

$$\psi_{Pf}([z]) = \prod_{i<j}^{4} (z_i - z_j)\,\text{Pf}\left(\frac{1}{z_i - z_j}\right). \tag{5}$$

This state is especially interesting, also in the context of quantum information theory, because its elementary excitations are known to exhibit non-Abelian statistics [15]. The next stable state in row ($L=8$) can be very well characterized (fidelity 0.98) by a Laughlin quasiparticle state

$$\psi_{QP}([z]) = \frac{\partial}{\partial z_1}\cdots\frac{\partial}{\partial z_4}\psi_L. \tag{6}$$

This state is the counterpart of the quasihole excitation, which has previously been studied in the context of

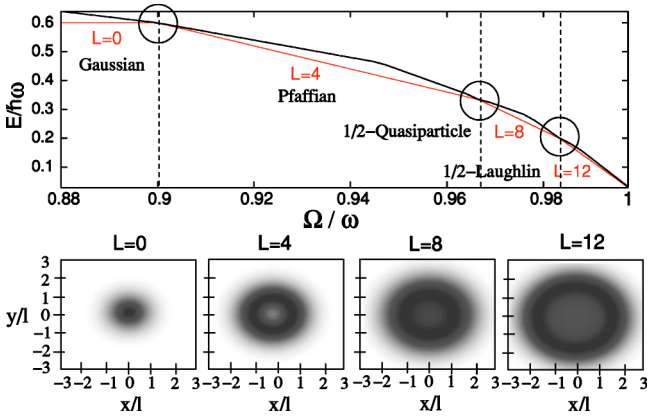

FIG. 1. (Color online) Lowest two eigenenergies (in units of $\hbar\omega$) of the Hamiltonian (1) for four particles and $\eta=0.1$ as a function of the trap rotation frequency Ω/ω. The circles mark the level crossings and L denotes the total angular momentum of the ground state. The ground state sequence can be identified as follows (with fidelity given in brackets): $L=0$ Gaussian ground state (exact), $L=4$ Pfaffian state (0.95), $L=8$ quasiparticle state (0.98), $L=12$ Laughlin state (exact).The change of angular momentum can readily be obtained from the increasing width of the density distribution depicted below.

1/2-anyons in rotating Bose-Einstein condensates [8]. Finally, the last stable state is identical to the $1/2$-Laughlin state, which we have already encountered in the case of two particles per well:

$$\psi_L([z]) = \prod_{i<j}^{4} (z_i - z_j)^2 \prod_{k}^{4} e^{|z_k|^2/2}. \tag{7}$$

This state is an exact eigenstate of (1) with zero interaction energy. In Fig. 1 we have plotted the density distribution in the x_1, x_2 plane of the different stable ground states. As the frequency of rotation Ω/ω increases the wave function spreads, and the interaction between the atoms decreases.

III. ADIABATIC PATHS TO ENTANGLED STATES

The sequence of entangled states we have described above cannot be obtained by simply adiabatically increasing the frequency of rotation Ω. The reason is that the rotational symmetry leads to level crossings between different angular momentum states (Fig. 1). In order to pass adiabatically from the zero angular momentum ground state to higher angular momentum states the spherical symmetry of the trapping potential has to be broken. For our optical lattice setup this can be achieved, for example, by deforming the formerly isotropic trapping potential on each well and letting the deformation rotate with frequency Ω [9]. In the rotating frame the new trapping potential has the form $V_p \propto (\omega+\Delta\omega)^2 x_1^2 + \omega^2 x_2^2$, and the new Hamiltonian is $H+H_\epsilon$, with

$$H_\epsilon = \frac{\epsilon}{4}\sum_m \beta_m a_{m+2}^\dagger a_m + (m+1) a_m^\dagger a_m + \text{H.c.}, \tag{8}$$

where $\beta_m = \sqrt{(m+2)(m+1)}$ and $\epsilon = \Delta\omega/\omega$ is a small parameter. The perturbation (8) leads to quadrupole excitations, so

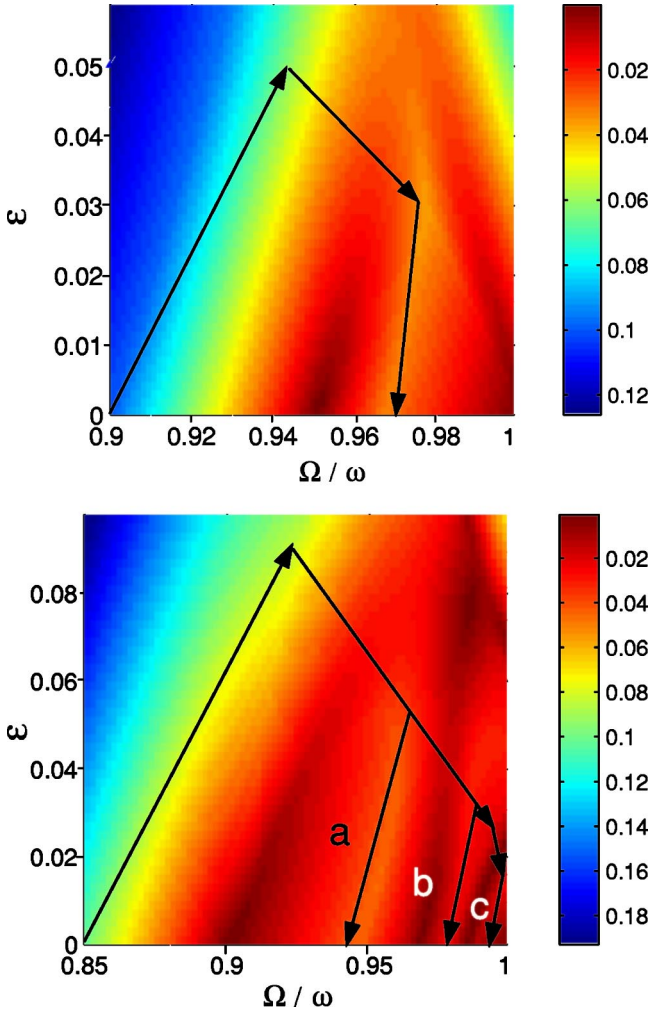

FIG. 2. (Color online) Energy gap in units of $\hbar\omega$ between the ground and first excited states as a function of the rotation frequency Ω/ω and the trap deformation ϵ for an interaction strength $\eta=0.1$. The black lines mark appropriate paths in parameter space for adiabatic ground state evolution starting from the $L=0$ state. The adiabatic evolution times have been calculated for a typical trapping frequency $\omega \simeq (2\pi)30$ kHz. Top ($N=2$): For a final fidelity $\mathcal{F}=|\langle\psi(T)|\psi_L\rangle|^2=0.99$ the Laughlin state ($L=2$) can be reached within $T=6.5$ ms. Bottom ($N=4$): Adiabatic path, evolution time T and fidelity $\mathcal{F}$ for the following final states (see Fig. 1): (a) Pfaffian state, $T=8$ ms, $\mathcal{F}=0.99$; (b) quasiparticle state, $T=12$ ms, $\mathcal{F}=0.99$; (c) Laughlin state, $T=215$ ms, $\mathcal{F}=0.97$.

that states whose total angular momenta differ by two are coupled.

In order to design appropriate adiabatic paths to the entangled states described above, we have computed numerically the energy gap between the ground and first excited states as a function of the parameters Ω/ω and ϵ for $N=2$ and $N=4$ (Fig. 2).

We first note that the isolines of constant energy gap show an approximately linear behavior. This feature can be easily understood from a perturbative treatment of the Hamiltonian (8). To first order, the energy of states with angular momentum L is shifted by an amount $\epsilon L/4$. Therefore the gap profile for a given ϵ is very similar to the one for $\epsilon=0$ but

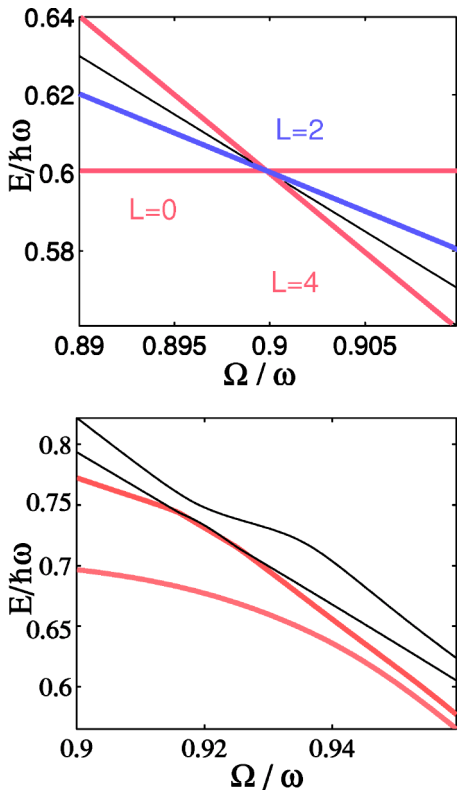

FIG. 3. (Color online) Left side: energy spectrum (in units $\hbar\omega$) for $N=4$ and $\eta=0.1$ in the vicinity of the first level crossing from the $L=0$ to the $L=4$ state (see Fig. 1, left circle). Using quadrupole excitations ($|\Delta L|=2$) coupling between these states is provided by the intermediate state $L=2$. Right side: emergence of an avoided level crossing for a trap deformation $\epsilon=0.06$.

shifted an amount $\sim\epsilon$ to larger rotation frequencies. As expected, we find that for $\epsilon\neq 0$ avoided crossings emerge (see Fig. 3). The energy gap of the avoided crossings does, however, not in general increase monotonically with the deformation ϵ. Due to the interplay with other excited states, "saddle points" appear in the gap profile, which makes the design of appropriate adiabatic paths a nontrivial task. For the stable entangled states of $N=2,4$ identified above these paths are depicted in Fig. 2. The actual time needed for the adiabatic path depends on the number of particles as well as on the state we want to achieve. For a typical trapping frequency $\omega\simeq(2\pi)30$ kHz and an interaction coupling $\eta=0.1$, the evolution times for the $N=2$ Laughlin state as well as for the $L=4$ and $L=8$ states for $N=4$ are of the order of 10 ms. In contrast, the evolution time for the $N=4$ Laughlin state is one order of magnitude larger. We can understand this result in the following way. For the case of $N=2$ direct coupling of the $L=0$ state to the $L=2$ Laughlin state is mediated by (8). For the case of $N=4$ there is no direct coupling between the ground states, since their angular momenta differ by 4. But, as one can see from the spectrum in the vicinity of the crossing to the state $L=4$ (Fig. 3), there is a state with $L=2$ near the crossing that mediates the coupling between the $L=0$ and the $L=4$ states. A similar situation occurs for the crossing to the $L=8$ state. However, there is no such intermediate state in direct proximity of the crossing to the $N=4$ Laughlin state, which leads to a decrease of the energy gap by one order of magnitude.

Let us also comment on the situation $N=3$. Here a ground state with odd angular momentum ($L=3$) arises. From the nature of the perturbation (8) it is clear that ground state evolution is not possible. However, we have shown [14] that the 1/2-Laughlin state can be reached by designing appropriate adiabatic paths via excited levels.

IV. FEASIBILITY

Let us now discuss the experimental feasibility of our proposal for a small number of particles N. The crucial assumption in our scheme is the absence of tunneling between wells, resulting in independent 3D harmonic wells. This requires the overlap between Wannier functions on neighboring sites to be small, which can be achieved by increasing the laser intensity. For a single occupied band (small rotation frequency) the assumption of independent wells (Mott regime) is well justified for a laser intensity of $V_0\approx 20E_r$ [3]. With increasing rotation frequency higher angular momentum states of the LLL manifold can be occupied. In the laboratory frame of the lattice this corresponds to the occupation of higher bands. In order to obtain a bound on the required laser intensity for the setup [3] we consider the limiting case of the Laughlin state ($\Omega\approx\omega$). As a rough estimate we require for a given N that the radius of the highest occupied angular momentum single-particle state ($\approx\sqrt{2N-1\ell}$) is much smaller than the separation between lattice sites ($a=\pi/k$). In terms of the laser intensity this translates to the the condition

$$(V_0/E_R)^{1/4} \gg \sqrt{2N-1}/\pi. \tag{9}$$

Numerical calculations of hopping and on-site interaction matrix elements have confirmed that indeed for $N=2$ (4) and $V_0/E_R\gtrsim 30$ (50) hopping becomes negligible and wells can be treated independently. We further note that these lower bounds for the laser intensity, which can very well be achieved experimentally, also guarantee the validity of the harmonic approximation.

A second important assumption of our proposal is the projection to the LLL manifold. This implies that the typical energies per particle have to be much smaller than the energy gap to the next Landau level $\hbar\omega$. For the limiting cases of the $L=0$ state and the Laughlin state, this leads to the conditions $(N-1)\eta/2$, $(N-1)(1-\Omega/\omega)\ll 1$, which are easily satisfied for typical interaction strengths ($\eta\sim 0.1$) and small N.

Finally, in order to adiabatically achieve the entangled states identified above further conditions are required. We analyze the most restrictive case, which corresponds to the Laughlin state. First of all the frequency of rotation has to be very close to the centrifugal limit. Let us find a lower bound to the critical rotation frequency at which the crossing to the Laughlin state appears. This can be done by calculating the rotation frequency at which the Laughlin quasiparticle state, $\psi_{QP}([z])=(\partial/\partial z_1\ldots\partial/\partial z_N)\psi_L$, becomes equal in energy to the Laughlin state. Since the quasiparticle state has N units of angular momentum less than the Laughlin state and an inter-

action energy $\lesssim\eta$, it follows that $\Omega_c/\omega\geqslant 1-\eta/N$. For the cases of $N=2$ (4) this condition is in agreement with the exact values found above. Second, the evolution time required for the adiabatic path has to be much smaller than the typical decoherence time. We can estimate this time in the following way. Given the critical frequency above and that the position of the avoided crossing is displaced to larger rotation frequencies an amount proportional to ϵ, it follows that the maximum ϵ we can have is $\sim\eta/N$, corresponding to a rotation frequency $\Omega/\omega=1$. Assuming an energy gap $\approx\epsilon$ it follows that the typical evolution time scales as $N\eta$. For the case of $N=2$ (4) and typical η and ω these times are of the order of tens of milliseconds as exactly found above, which is much smaller than the typical lifetime of the lattice states. Finally, a high degree of control of the parameters Ω/ω and ϵ is required to perform the appropriate adiabatic paths. The required precision scales again as η/N, which for the case of $N=4$ means a control of the parameter space up to the second digit.

From our analysis it follows that the adiabatic creation of the Laughlin state by means of low angular momentum excitations, as quadrupole excitations, becomes very difficult in samples with large number of particles [17,18]. Even if the centrifugal limit is possible to achieve, as it happens when including an additional r^4 trapping potential [17], the adiabatic creation of the Laughlin state is still very demanding. One reason is that the rotation frequency and the trap deformation have to be controlled within a precision that also scales linearly with N. Furthermore, we point out that only the exact knowledge of the multi-particle energy spectrum allows one to design adiabatic paths that minimize the evolution time.

V. DETECTION

In this section we consider the important issue of experimental detection by measuring different characteristic properties of the entangled states identified above. As an important feature of our lattice setup of independent wells, we note that any signal will be highly enhanced by a factor equal to the number of occupied lattice sites ($\sim$150 000 [3]).

(i) *Density profiles*. A very characteristic feature of our entangled states is that due to their large angular momentum they exhibit a strongly extended spatial density distribution compared to the nonentangled $L=0$ state. For the 1/2-Laughlin state the typical radius is given by $\overline{r}\approx\sqrt{2N-1}\ell$. In the case of $N=2$ (4) this results in a radius that is $\sim$2 (3) times larger than in the case of the condensate. As proposed in [19] the density profile of states within the LLL can be measured in a time of flight (TOF) image of the atomic system, since the momentum distribution coincides with the density profile for LLL states. In our case of independent 3D wells, a TOF absorption picture after expansion time t will exhibit a broad central peak of the form

$$\rho(\mathbf{r},t)\approx\frac{N_s}{(\omega t)^3}|\rho_0(-iz/(\omega t),x_3/(\omega t))|^2. \tag{10}$$

Here, $\rho_0(z,x_3)$ is the initial density distribution on a single well. In the TOF image it is enhanced by a factor proportional to the number of lattice sites N_s and rescaled by a factor $\omega t\gg 1$. The $\pi/2$ rotation $z\rightarrow -iz$ leaves isotropic states, like the FQH states described above, unaffected. The underlying assumption of free (interactionless) expansion is justified, since the interaction energy is small compared to the kinetic energy (in the stationary frame).

(ii) *Angular momentum*. For any state within the LLL integration over the density distribution gives $\int d\mathbf{r}\, r^2\rho(\mathbf{r})=L+N$. Thus in the limit of weak interaction the total angular momentum can be extracted directly from the TOF picture.

(iii) *Correlation functions*. Here we propose a technique that makes directly use of the rich possibilities offered by the optical lattice setup and which allows us to measure both the $g_1=\langle\psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle$ and $g_2=\langle\psi^\dagger(\mathbf{r})\psi^\dagger(\mathbf{r}')\psi(\mathbf{r})\psi(\mathbf{r}')\rangle$ correlation functions. The g_2 correlation function is, for instance, very characteristic for a Laughlin state. Since particles can only be at least in relative angular momentum $m_r=2$ it follows that $g_2\propto|r-r'|^4$. This behavior reveals the $1/2$ fractional nature of this Laughlin state.

We consider two species a and b (hyperfine levels) of bosonic atoms, which can be coupled via Raman transitions. We start with atoms in level a and create the entangled state of interest $|\Psi_i\rangle$ with the method described above. Next we apply a $\pi/2$ pulse with the laser and create an equal superposition of a and b states. Finally, we shift the lattice potential trapping atoms of type b (as proposed in [20] and realized in [2]) by a distance $\mathbf{r}_0$ small compared to the lattice spacing and perform another $\pi/2$ pulse. In the Heisenberg picture this procedure corresponds to the following transformation of the field operator for species a:

$$\psi_a(\mathbf{r})\rightarrow\psi_a(\mathbf{r})+\psi_a(\mathbf{r}+\mathbf{r}_0). \tag{11}$$

Thus the density distribution of atoms of type a in this new state $|\Psi_f\rangle$ contains information about the g_1 correlation function of the original state:

$$\langle\Psi_f|\psi_a^\dagger(\mathbf{r})\psi_a(\mathbf{r})|\Psi_f\rangle=\langle\Psi_i|[\psi_a^\dagger(\mathbf{r})+\psi_a^\dagger(\mathbf{r}+\mathbf{r}_0)][\psi_a(\mathbf{r})+\psi_a(\mathbf{r}+\mathbf{r}_0)]|\Psi_i\rangle. \tag{12}$$

Using this procedure we can also measure higher order correlation functions like g_2. In this case measuring the interaction energy of the final state will allow us to calculate the g_2 of the initial state. For instance, for the Laughlin state we have

$$E_{int}(\mathbf{r}_0)=\frac{\pi\eta}{4}\int d\mathbf{r}\langle\Psi_i|\psi_a^\dagger(\mathbf{r})\psi_a^\dagger(\mathbf{r}+\mathbf{r}_0)\psi_a(\mathbf{r})\psi_a(\mathbf{r}+\mathbf{r}_0)|\Psi_i\rangle. \tag{13}$$

The interaction energy is, unfortunately, not directly accessible experimentally. Instead, one can obtain the total energy of the final state from integrating over the TOF absorption picture, since energy is conserved during the time of flight. For small coupling η, however, the measurable effect due to interactions will be small compared to the kinetic part of the energy. In addition, the kinetic energy itself shows a significant dependence on the shifting r_0, which has to be distinguished from the interaction. Hence, we propose to tune the

scattering length a_s (e.g., via a photoassociation-induced Feshbach resonance [21]) and to measure the interaction energy in both the weak and strong scattering limits. The difference would then reveal the characteristic behavior of the g_2 correlation function.

We finally note that, as a further way of detection for the $N=4$ Laughlin state, a strong reduction of the three-body losses should be observed.

VI. CONCLUSION

In conclusion, we have shown how to motionally entangle a small number of particles into a sequence of interesting FQH states. We have fully addressed the adiabatic creation of these states and proposed techniques for their experimental detection.

ACKNOWLEDGMENTS

We acknowledge helpful discussions with I. Bloch, J.-J. García-Ripoll, and M. Greiner. This work was supported in part by the EU IST projects (RESQ and QUPRODIS), the DFG (SFB 631), and the Kompetenznetzwerk "Quanteninformationsverarbeitung" der Bayerischen Staatsregierung.

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