# Generation of polarization-entangled photon pairs with arbitrary joint spectrum

Zachary D. Walton,* Alexander V. Sergienko, Bahaa E. A. Saleh, and Malvin C. Teich
*Quantum Imaging Laboratory,† Department of Electrical & Computer Engineering, Boston University, 8 Saint Mary's Street, Boston, Massachusetts 02215-2421, USA*

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We present a scheme for generating polarization-entangled photons pairs with arbitrary joint spectrum. Specifically, we describe a technique for spontaneous parametric down-conversion in which both the center frequencies and the bandwidths of the down-converted photons may be controlled by appropriate manipulation of the pump pulse. The spectral control offered by this technique permits one to choose the operating wavelengths for each photon of a pair based on optimizations of other system parameters (loss in optical fiber, photon counter performance, etc.). The combination of spectral control, polarization control, and lack of group-velocity matching conditions makes this technique particularly well suited for a distributed quantum information processing architecture in which integrated optical circuits are connected by spans of optical fiber.

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## I. INTRODUCTION

Spontaneous parametric down-conversion (SPDC) has proven to be an excellent technology for quantum communication, with SPDC photons functioning as "flying qubits." The discovery by Knill *et al.* [1] that linear optics and single-photon detectors are sufficient for scalable quantum computation has opened the possibility that SPDC may also be useful for quantum computation.

As the proposals for quantum information processing with SPDC become more sophisticated, the technical demands placed on SPDC sources become more stringent. For example, a quantum key distribution experiment based on polarization entanglement requires that two two-photon amplitudes ($|HH\rangle$ and $|VV\rangle$, for example) be made indistinguishable. The spectral properties of the two photons are important only if they are correlated with the polarization degree of freedom. A more stringent form of indistinguishability is typically required for quantum computation with linear optics: it must be impossible to determine which source produced a certain photon after that photon emerges from a beam splitter. This in turn requires that all of the photons' properties be controlled such that it is impossible to learn any information about the identity of a given photon's source. Photon pairs produced by SPDC are often correlated in one or more of their properties (frequency, direction, etc.). These correlations can destroy the requisite indistinguishability by enabling one to learn about one photon by performing measurements on its twin [2].

In this paper, we describe a SPDC source that produces photon pairs that have arbitrary correlation in frequency. The source we propose enables an unusual flexibility in the control of the marginal spectra of the SPDC photons. Specifically, our source can produce a frequency-uncorrelated photon pair in which the center frequency and the bandwidth of each photon is controlled independently, regardless of the nonlinear material's dispersion curves. This makes the source well suited for applications that span quantum communication and computation, such as teleportation [3] and entanglement swapping [4]. In these applications, one photon of a pair (the "communication" photon) is often sent to another party through a long span of optical fiber, while the other photon (the "computation" photon) is analyzed and detected in a localized interferometer. It is desirable that the communication photon be narrowband (such that effects like polarization mode dispersion are mitigated) and have a wavelength in the infrared (where optical fiber is least lossy). Contrariwise, the computation photon should be broadband (such that it can be used in interferometers with small path-length differences) and have a wavelength suited for high-efficiency single-photon counters.

The paper is organized as follows. In Sec. II, we introduce the technique and demonstrate that it permits the generation of photon pairs with arbitrary joint spectrum. In Sec. III, we show how this spectral control can be combined with polarization entanglement by considering a specific example involving a barium bismuthate (BBO) waveguide. In Sec. IV, we discuss the possibility of using this source as part of a distributed quantum information processor based on integrated optics. Finally, we summarize our results in Sec. V.

## II. GENERALIZED AUTO-PHASE-MATCHED SPDC

Our source is a generalization of the design we previously introduced under the name *auto-phase-matched* SPDC [5]; thus, we name this scheme *generalized* auto-phase-matched SPDC. Like the original scheme, this one features counter-propagating SPDC created in a single-mode nonlinear waveguide by a transverse pump pulse. In the original scheme [Fig. 1(a)], the pump pulse is cross-spectrally pure (i.e., the complex envelope factors into separate functions of space and time), and impinges on the waveguide at normal incidence. In the present scheme [Fig. 1(b)], the pump pulse may have cross-spectral correlations, and may approach the waveguide at non-normal incidence. With these constraints on the pump pulse relaxed, the center frequency and bandwidth of

*Electronic address: walton@bu.edu
†http://www.bu.edu/qil

 

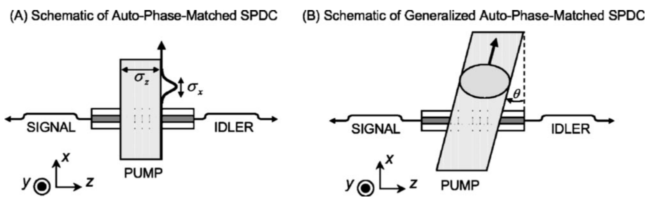


FIG. 1. Schematics of (a) auto-phase-matched SPDC [5] and (b) generalized auto-phase-matched SPDC. In both schemes, a transverse pump pulse stimulates the creation of a pair of counterpropagating photons in a single-mode, nonlinear waveguide. In (a), the pump pulse is cross-spectrally pure and constrained in its direction (normal to the waveguide). This restricts the spectral properties of the SPDC photons. In (b), these constraints on the pump pulse are relaxed. One can exploit this freedom to independently control the center frequency and bandwidth of each SPDC photon, while satisfying the constraint that the photons be uncorrelated in frequency.

each SPDC photon may be controlled independently.

In typical SPDC experiments, a monochromatic pump beam is used. In this situation, the sum of the frequencies of the signal and idler photons is fixed; thus the photons' frequencies are anticorrelated. SPDC with generalized joint spectral properties (i.e., correlated, uncorrelated, anticorrelated) was studied theoretically by Campos *et al.* [6]. Here we review the experimental proposals for generating frequency-uncorrelated SPDC.

A number of techniques have been proposed for creating frequency-uncorrelated SPDC; however, they all impose certain constraints on the center frequencies and/or bandwidths of the SPDC photons. Grice *et al.* proposed a method for creating frequency-uncorrelated SPDC based on a group-velocity matching condition introduced by Keller and Rubin [7]. Their method can be used to create degenerate, frequency-uncorrelated photons; however, the center frequency of down-conversion is fixed by the nonlinear material, and the bandwidth of two SPDC photons must be equal. They also demonstrate that degenerate, frequency-uncorrelated photons with different bandwidths may be generated; however, in this case the bandwidths are fixed and cannot be independently controlled. Giovannetti *et al.* proposed extending the approach of Grice *et al.* by using a periodically poled nonlinear crystal [8]. This allows one to to satisfy the zeroth-order term in the phase-matching relation at an arbitrary pump wavelength, making the group-velocity matching relation easier to satisfy. Even with such an enhancement, this approach does not have sufficient flexibility to allow independent control of the marginal spectra.

A distinct approach for creating frequency-uncorrelated SPDC was independently discovered by U'Ren *et al.* [2] and Walton *et al.* [5]. Instead of relying on the satisfaction of a group-velocity matching condition, these approaches rely on the geometrical symmetry of degenerate, noncollinear type-I SPDC (the previously mentioned techniques worked only with type-II SPDC). The essential difference between the two proposals is that for U'Ren *et al.*, the phase-matching relation in the pump propagation direction is a constraint that must be satisfied, while for our auto-phase-matched technique, the single-mode waveguide ensures that this relation is satisfied, regardless of the system parameters. The relative lack of constraints for these techniques makes them attractive, since it suggests that they remain viable options even if the center frequency of SPDC is constrained by some other factor (optical fiber loss, detector efficiency, etc.). Nonetheless, both techniques suffer a lack of flexibility that is reminiscent of the previously mentioned schemes. The SPDC photons must be degenerate, and the bandwidths of the two photons must be equal. In the next section, we show that by generalizing our auto-phase-matched technique, we can obtain independent spectral control of the SPDC photons.

To begin, we review the relationship between the pump pulse and the SPDC in the geometry of Fig. 1. Following the derivation in Ref. [9], a classical pump pulse described on the free-space side of the waveguide-air interface by

$$E_p(z,t) \propto \int\int dk\, d\omega \tilde{E}_p(k,\omega) e^{-i(kz-\omega t)} \tag{1}$$

stimulates the creation of a pair of photons described by the two-photon wave function

$$|\Psi\rangle \propto \int\int d\omega_s d\omega_i \phi(\omega_i,\omega_s)|\omega_s\rangle_s|\omega_i\rangle_i, \tag{2}$$

where

$$\phi(\omega_i,\omega_s) = \tilde{E}_p\left(\frac{\beta_i(\omega_i)-\beta_s(\omega_s)}{n_p(\omega_i+\omega_s)}, \omega_i+\omega_s\right). \tag{3}$$

$n_p(\omega_i+\omega_s)$ is the refractive index for the pump polarization. Here, and for the rest of the paper, we use the variable $k$ to refer to the component of the pump wave vector along the $z$ axis. The ket $|\omega_s\rangle_s|\omega_i\rangle_i$ represents a signal photon in the frequency mode $\omega_s$ and an idler photon in the frequency mode $\omega_i$ with corresponding propagation constants $\beta_s(\omega_s)$ and $\beta_i(\omega_i)$, respectively. Equation (3) conveys the main result of this paper: Assuming the dispersion properties of the medium are known, it is possible to generate a down-converted photon pair with arbitrary joint spectrum by appropriately engineering the spatial and temporal characteristics of the pump pulse.

In Fig. 1(a), the pump pulse is parametrized by three numbers: the center frequency, the temporal coherence length $\sigma_x$, and the spatial coherence length $\sigma_z$. These parameters may be chosen to produce degenerate SPDC with controllable entanglement, as described in Ref. [5]; however, in order to obtain independent control of the center frequency and bandwidth of each SPDC photon, one must relax the constraints on the pump pulse, as in Fig. 1(b). Using Eqs. (1) and (3), it is straightforward to show that a pump pulse described by

$$\tilde{E}_p(k,\omega) \propto \exp\left[-\left(\frac{(n_pk-k_p)+(\omega-\omega_p)\beta_s'}{2\beta'\sigma_i}\right)^2 - \left(\frac{(n_pk-k_p)-(\omega-\omega_p)\beta_i'}{2\beta'\sigma_s}\right)^2\right] \tag{4}$$

will yield the following frequency-uncorrelated two-photon state:

$$|\Psi\rangle \propto \int \int d\omega_s d\omega_i \exp\left[-\left(\frac{\omega_i-\omega_i^{(\circ)}}{2\sigma_i}\right)^2-\left(\frac{\omega_s-\omega_s^{(\circ)}}{2\sigma_s}\right)^2\right] \times |\omega_s\rangle_s|\omega_i\rangle_i, \tag{5}$$

where $\omega_s^{(\circ)}$ and $\omega_i^{(\circ)}$ are the center frequencies of the signal and idler beams, respectively, and we have used the following definitions and approximations:

$$k_p \equiv \beta_i(\omega_i^{(\circ)}) - \beta_s(\omega_s^{(\circ)}), \tag{6}$$

$$\omega_p \equiv \omega_s^{(\circ)} + \omega_i^{(\circ)}, \tag{7}$$

$$\beta_j(\omega_j) \approx \beta_j(\omega_j^{(\circ)}) + (\omega_j-\omega_j^{(\circ)})\beta_j', \quad j=s,i, \tag{8}$$

$$n_p(\omega_i+\omega_s) \approx n_p(\omega_p) \equiv n_p. \tag{9}$$

These approximations are valid in typical situations; however, if required, more terms may be used at the expense of a more complicated expression for the pump pulse.

Equations (4) and (5) summarize the central result of this work. Taken together, these relations can be thought of as an algorithm for producing frequency-uncorrelated SPDC with arbitrary marginal spectra. The wave function in Eq. (5) describes a frequency-uncorrelated two-photon state in which the signal photon is centered on $\omega_s^{(\circ)}$ with a bandwidth $\sigma_s$, and the idler photon is centered on $\omega_i^{(\circ)}$ with a bandwidth $\sigma_i$. Note that these two photons are not themselves indistinguishable (unless $\omega_s^{(\circ)}=\omega_i^{(\circ)}$ and $\sigma_s=\sigma_i$). As previously mentioned, the indistinguishability arises in a multiphoton experiment when one photon of the pair enters an interferometer with one or more photons that have identical spectra. In this case, the lack of frequency correlations between the signal and idler prevents a loss of interferometric visibility by ensuring that spectral measurements on one photon of the pair will not reveal any spectral information about the other.

Equations (4)–(9) demonstrate that the four numbers $\omega_s^{(\circ)}, \omega_i^{(\circ)}, \sigma_s$, and $\sigma_i$, along with the dispersion properties of the waveguide, are sufficient to determine the form of the pump pulse required to generate the desired wave function. We can simplify the description of the pump pulse by rewriting Eq. (4) as

$$\tilde{E}_p(k,\omega) = \exp\left[-\left(\frac{\omega-\omega_p}{2A}\right)^2-\left(\frac{(k-k_p/n_p)+C(\omega-\omega_p)}{2B}\right)^2\right], \tag{10}$$

where

$$A = \frac{\beta_s'+\beta_i'}{\sqrt{(\beta_s'/\sigma_i)^2+(\beta_i'/\sigma_s)^2-(\beta_s'\sigma_s^2-\beta_i'\sigma_i^2)^2/(\sigma_s\sigma_i)^2(\sigma_s^2+\sigma_i^2)}}, \tag{11}$$

$$B = \frac{\beta_s'+\beta_i'}{n_p\sqrt{1/\sigma_s^2+1/\sigma_i^2}}, \tag{12}$$

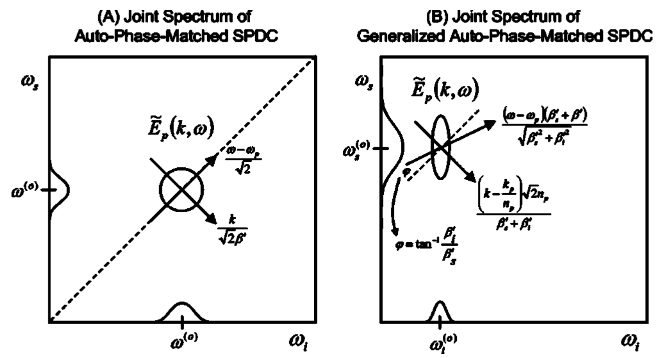


FIG. 2. A graphical technique for associating the spectral/spatial properties of a pump pulse [$\tilde{E}_p(k,\omega)$, plotted on the inner axes], with the joint spectrum of the resulting down-converted photon pair (the outer axes), for auto-phase-matched SPDC (a) and generalized auto-phase-matched SPDC (b). In both (a) and (b), $\tilde{E}_p(k,\omega)$ is plotted on axes that are both rotated and scaled by constants related to the waveguide dispersion properties, such that the resulting plot can be interpreted using the outer axes. The central point of this paper is conveyed in (b). That is, by judicious choice of $\tilde{E}_p(k,\omega)$, one can generated nondegenerate, frequency-uncorrelated SPDC, such that the bandwidth of each photon is independently controllable. The marginal spectrum of each photon is plotted along the appropriate axis in each figure to demonstrate the connection between cross-spectral correlation in the pump and independent control of the photons' bandwidths.

$$C = \frac{\beta_s'\sigma_s^2-\beta_i'\sigma_i^2}{n_p(\sigma_s^2+\sigma_i^2)}. \tag{13}$$

The algorithm for creating the appropriate pump pulse to produce the state in Eq. (5) is as follows. A pulse is created with center frequency $\omega_p$, spectral bandwidth $A$, and spatial bandwidth $B$. Next, a dispersive element such as a wedge of quartz or a diffraction grating is used to correlate $k$ and $\omega$ by effecting the substitution

$$k \to k + C(\omega-\omega_p). \tag{14}$$

Finally, the pulse is directed toward the nonlinear waveguide at incidence angle

$$\theta = \sin^{-1}\frac{k_p c}{n_p\omega_p}, \tag{15}$$

where $\theta$ is measured outside the waveguide [see Fig. 1(b)], and $c$ is the speed of light in vacuum.

In Fig. 2, we present a graphical depiction of the relationship between the pump pulse and the resulting two-photon state, for both auto-phase-matched SPDC [Fig. 2(a)] and generalized auto-phase-matched SPDC [Fig. 2(b)]. In both cases, a plot of $\tilde{E}_p(k,\omega)$ is superposed over the joint spectrum of the signal and idler photons. By plotting $\tilde{E}_p(k,\omega)$ at the correct location and on the correct inner axes, one can immediately infer the joint spectrum of the down-converted photons, simply by interpreting the plot using the outer axes. In Fig. 2(a), the nonzero portion of $\tilde{E}_p(k,\omega)$ is centered on the $\omega_s=\omega_i$ axis, and one of the inner axes is scaled by the factor $\beta'$, which is the first derivative of $\beta(\omega)$ evaluated at $\omega^{(\circ)}$. In Fig. 2(b), the nonzero portion of $\tilde{E}_p(k,\omega)$ is located at a general position, and the inner axes are no longer orthogonal (unless $\beta_s'=\beta_i'$). Using this figure, the two desirable features of the two-photon joint spectrum (nondegeneracy and independently controlled bandwidths) are easily interpreted in terms of the pump pulse. That is, the nondegeneracy of the photon pair derives from the condition $k_p \neq 0$, which in turn derives from the non-normal incidence of the pump pulse. Similarly, the independent control of the two photons' respective bandwidths derives from the cross-spectral correlation in the pump pulse [$\tilde{E}_p(k,\omega)$ does not factor into a function of $k$ times a function of $\omega$].

## III. EXAMPLE: POLARIZATION-ENTANGLED FREQUENCY-UNCORRELATED SPDC FROM A BBO WAVEGUIDE

In the generalized auto-phase-matched technique, one can obtain polarization entanglement by adjusting the polarization state of the pump pulse, without sacrificing the spectral control described above. To illustrate this feature, we present an example involving nondegenerate, polarization-entangled SPDC produced in a single-mode BBO waveguide.

The general idea is to use two of the nonlinear medium's $\chi^{(2)}$ tensor elements at the same time, by preparing a coherent superposition of two polarization modes of the pump pulse. When producing polarization-entangled photon pairs, it is typically desirable that a given photon have the same spectral properties for both two-photon polarization amplitudes. Therefore, in creating the pump pulse, we use the same four numbers $\omega_s^{(\circ)}, \omega_i^{(\circ)}, \sigma_s$, and $\sigma_i$ in calculating the desired pulse shape for both pump polarization modes. However, since the two two-photon amplitudes relate to SPDC processes taking place in distinct polarization modes, the dispersion properties of the waveguide will in general be different. Thus, using the notation of Fig. 1, the pump pulse will be characterized by two functions: $\tilde{E}_p^y(k,\omega)$, which describes the $y$-polarized component of the pump pulse, and $\tilde{E}_p^z(k,\omega)$, which describes the $z$-polarized component of the pump pulse.

In the case of BBO, the relevant tensor elements are $\chi_{yyy}^{(2)}=2.22$ pm/V and $\chi_{zxx}^{(2)}=0.16$ pm/V [10]. Therefore, using the notation of Fig. 1, the pump beam will approach the waveguide in the $x$-$z$ plane, and will be composed of a $y$-polarized pulse and a $z$-polarized pulse. In Table I, we list the calculated values of $A,B,C$, and $\theta$ (defined in Sec. II) that correspond to a frequency-uncorrelated polarization-entangled pair of photons with the signal photon at 0.8 μ with coherence length 1 mm, and the idler photon at 1.5 μ with coherence length 1 cm. These values for center wavelength and coherence length were chosen in order to make the signal photon suitable for long-distance optical fiber transmission, and the idler photon suitable for local processing in an integrated optical circuit. In calculating the values in Table I, we have ignored waveguide dispersion, using instead the Selmeier curves to describe the material dispersion in the BBO single-mode waveguide.

TABLE I. Parameters that describe the pump pulse required to produce nondegenerate, frequency-uncorrelated, polarization-entangled SPDC in a single-mode BBO waveguide, using the technique depicted in Fig. 1(b). The signal photon is at 0.8 μ with coherence length 1 mm, and the idler photon is at 1.5 μ with coherence length 1 cm. The pump pulse is comprised of coherently superposed, independently controlled pulses $\tilde{E}_p^z(k,\omega)$ and $\tilde{E}_p^y(k,\omega)$ in the two polarization modes $z$ and $y$, respectively. The parameters $A,B,C$, and $\theta$ are defined in Sec. II. The negative values of $\theta$ indicate that the projection of the pump wave vector along the waveguide is oriented in the negative $z$ direction [see Fig. 1(b)].

| | $\tilde{E}_p^z(k,\omega)$ | $\tilde{E}_p^y(k,\omega)$ |
|---|---|---|
| $A$ ($10^{12}$ rad/s) | 1.89 | 1.89 |
| $B$ ($10^3$ rad/m) | 1.35 | 1.25 |
| $C$ ($10^{-9}$ s/m) | 3.54 | 3.28 |
| $\theta$ (deg) | −20.1 | −18.6 |

## IV. QUANTUM INFORMATION PROCESSING WITH INTEGRATED OPTICAL CIRCUITS

Generalized auto-phase-matched SPDC is particularly well suited for quantum information processing on an integrated optical circuit (see Fig. 3). Among the advantages of replacing an array of discrete optical elements with an integrated optical circuit are the following: reduced size, reduced loss due to fewer connectors, and "common mode" noise processes because of the close proximity of optical elements. However, there are substantial experimental challenges associated with constructing an integrated optical quantum information processor. Perhaps the most obvious challenge is finding a material that can perform as many of the required functions (photon source, modulation, detection) as possible. A significant advantage of generalized auto-phase-matched SPDC is that the choice of material places essentially no limitation on the spectral and polarization properties of the photon pairs that will be produced. All that is required is that the material's $\chi^{(2)}$ tensor have the appropriate nonzero elements such that, for a given orientation of the optic axis with respect to the waveguide, the desired SPDC process will occur.

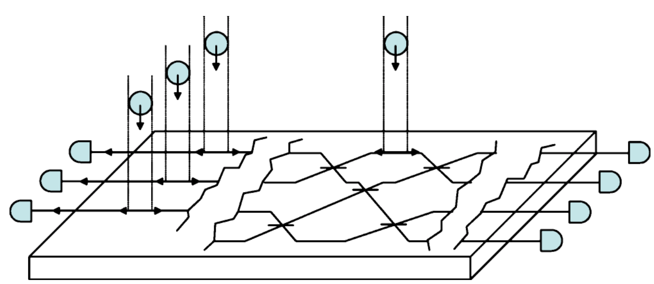

FIG. 3. A conceptual schematic of an integrated optical quantum information processor. The entire integrated circuit is constructed on a nonlinear material such that any stretch of waveguide may be used as a source of counterpropagating photon pairs. In the text, we describe how generalized auto-phase-matched SPDC is particularly well suited for this application.

Figure 3 depicts a conceptual schematic of an integrated optical quantum information processor which employs generalized auto-phase-matched SPDC for generating photons. The figure highlights several of the practical advantages associated with this technology. First, the sources may be placed at the edge of the circuit and combined with single-photon counters to implement conditional single-photon sources. Second, due to the transverse-pump configuration, the photon pairs may be created within the interior of the optical circuit. Finally, since there is no group-velocity matching relation associated with generalized auto-phase-matched SPDC, poling of the nonlinear waveguide at each source is not required.

## V. CONCLUSIONS

We have described a scheme for generating polarization-entangled photon pairs with arbitrary spectrum. By controlling the spatial, temporal, and polarization properties of the pump pulse, it is possible to generate the desired two-photon state, regardless of the dispersion properties of the nonlinear medium. We provided a calculation of the parameters describing the pump pulse required to generate a photon pair with a particular joint spectrum in a single-mode BBO waveguide. Finally, we discussed the role this source technology might play in a distributed quantum information processor based on integrated optics.

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[10] Since $\chi^{(2)}_{yyy}$ and $\chi^{(2)}_{zxx}$ are not equal, we adjust the ratio of the optical powers in each pump polarization mode in order to obtain the maximally entangled state $|HH\rangle + e^{i\phi}|VV\rangle$, where the relative phase $\phi$ is determined by the relative phase between the two pump polarization modes. It is straightforward to produce nonmaximally entangled and/or mixed polarization states with this technique. Nonmaximally entangled states may be produced by adjusting the ratio of the optical powers in each of the pump's polarization modes. Mixed states may be produced by allowing the pump pulse to become partially depolarized. Although in this paper we are concerned with frequency-uncorrelated (and thus frequency-unentangled) photon pairs, analogous generalizations for partial entanglement and mixing, in the frequency degrees of freedom are possible, given the appropriate manipulations of the pump pulse.