PHYSICAL REVIEW A **70**, 050302(R) (2004)

Quantum entanglement for acoustic spintronics

Godfrey Gumbs and Yonatan Abranyos
Department of Physics and Astronomy, Hunter College of the City University of New York, 695 Park Avenue, New York, New York 10021, USA
(Received 15 February 2004; published 23 November 2004)

We consider the entanglement of spins for two electrons contributing to the acoustoelectric current driven by a surface acoustic wave (SAW) in two adjacent narrow channels by calculating their exchange energy (J). The channels belong to an acoustic nanocircuit which comprises a network of quasi-one-dimensional pinched-off channels serving as wires along which SAW quantum dots transport electrons. This is motivated by possible practical applications involving quantum information processing and quantum computers. We calculate J as a function of time as the electrons travel side-by-side in the adjacent channels and as a function of the distance between the centers of the channels. The leakage from the state in which the system is prepared, is calculated. The oscillations in the leakage indicate the probability for the electron system to make transitions between the ground and excited states, or for an electron to hop back and forth between channels.

DOI: 10.1103/PhysRevA.70.050302 PACS number(s): 03.67.Mn, 71.70.−d, 77.65.Dq

Recently, there has been a considerable amount of interest in the interaction of a surface acoustic wave (SAW) with a two-dimensional electron gas (2DEG) in a GaAs/AlGaAS heterostructure. A SAW propagating in this piezoelectric material interacts with the 2DEG through the electric field accompanying the elastic waves [1–4]. A consequence of the interaction between the SAW and 2DEG is an acoustoelectric current in the 2DEG due to the drag of the 2D electrons by the SAW [5–10]. Recent measurements of the SAW-induced acoustoelectric current were carried out in a narrow channel formed in GaAs/AlGaAs heterostructures with the use of split gates [11–14].

Measurements done at high SAW power and with gate voltages beyond pinch off revealed a remarkable feature of the acoustoelectric effect in quasi-one-dimensional (quasi-1D) ballistic channels, i.e., the quantization of the acoustoelectric current in the channel at SAW wavelengths comparable with the channel length ($\sim 1\ \mu$). The current-gate voltage curves have steps with the current plateaus equal to $I=nef$, where e is the electron charge, f is the SAW frequency, and n is an integer [13,14]. These values of I correspond to the transfer of n electrons through the channel per SAW cycle and can be explained as the result of the trapping of electrons in the moving SAW-induced quantum potential wells and the transfer of electrons, residing in these wells, through the channel [13–17].

A natural extension of the use of the SAW single electron pump is an acoustic nanocircuit which comprises a network of the quasi-1D undoped channels serving as wires along which the SAW quantum dots transport electrons. In this paper, we will consider the entanglement of spins for two electrons contributing to the acoustoelectric current in two adjacent narrow channels by calculating their exchange energy. This is motivated by possible practical applications involving quantum information processing and quantum computers [18]. If one could control the spin degree of freedom of the electrons in the SAW quantum dots, then this would allow one to use SAW circuits for quantum computation. This was described in the work of Barnes, Shilton, and Robinson [18]. Our paper is motivated by Ref. [18] and our aim is to support their suggestion by doing numerical calculations of the entanglement and leakage. Both these quantities contribute to the accuracy of the two-bit gate and together they show that the scheme described in Ref. [18] is feasible.

We now consider when the SAW beam transports electrons along two adjacent channels, shown schematically in Fig. 1. When the channels are close to each other, two adjacent SAW dots in different channels form a pair of coupled quantum dots. If the electron spin state in one channel is the same or opposite as the spin in the other channel, then two electrons in coupled SAW quantum dots will form either triplet or singlet states. We shall estimate the exchange energy J by calculating the difference in the energies of the singlet and triplet states. For this, we must first determine the two lowest energy states for a single electron. The coupled dots are modeled by the Hamiltonian $H=\Sigma_{i=1,2}H_0(i)+H_{12}$ where $H_0(i)=(1/2m^*)\mathbf{p}_i^2+V(\mathbf{r}_i)$ and $H_{12}=e^2/\varepsilon_s r_{12}$ with $H_0(i)$ the Hamiltonian for noninteracting electrons having coordinates $\mathbf{r}_i=(x_i,y_i)$ $(i=1,2)$ and separation r_{12} in the

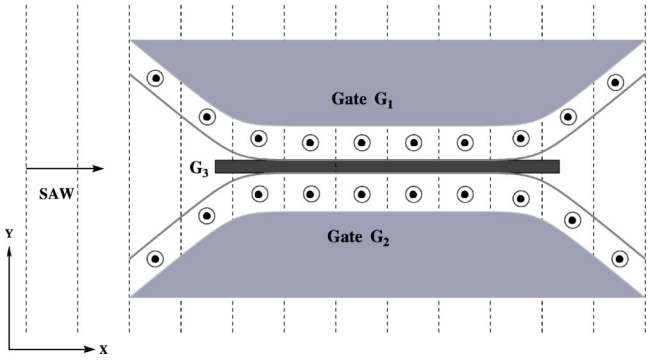

FIG. 1. Schematic plot of the SAW beam transporting electrons along a pair of channels. Two adjacent SAW dots in different channels form a pair of coupled quantum dots. The vertical lines in the figure represent maxima of the SAW potential which carries electrons (depicted as full circles) along the channels. The split gates defining the channels are denoted by G_1, G_2, and G_3.

quantum well; we denote their interaction by H_{12}. Also, $\varepsilon_s = 4\pi\epsilon_0\varepsilon_b$, where ε_b is the background dielectric constant. The confining potential $V(\mathbf{r}_i)$ is separable in the x and y directions, and we denote each term by V_i $(i=1,2)$. The coupling of the dots in the y direction includes tunneling. To simulate this bound-state tunneling, we choose the following 1D quartic potential with degenerate minima, i.e.,

$$V_2(y) = \frac{1}{8a^2} m^* \Omega^2 (y^2 - a^2)^2, \tag{1}$$

in terms of an oscillator frequency Ω and separation distance $2a$ between the centers of the dots. Here, both Ω and a are assumed to be independent of the x coordinate. However, if we allow the barrier height between the channels to vary as the quantum dots travel along the channels, then we could control the exchange of electrons between channels over the SAW period. Each dot is moving side-by-side in the x direction within the channel in a potential that arises from the gate voltage with a negative bias and the SAW potential, which we model by

$$V_1(x,t) = \frac{V_0}{\cosh^2(x/\ell)} + V_S \cos(kx - \omega t), \tag{2}$$

where ω and k are the SAW frequency and wave number, respectively, and V_S is the SAW amplitude. The height of the electrostatic potential barrier in the channel is denoted by $V_0 = \hbar^2/(2m^* l_0^2)$, where l_0 is a parameter and 2ℓ is the effective length of the channel. When the split-gate voltage is sufficiently negative, the channel between the gates is pinched off and the conductance in the channel is zero, indicating that there is a potential barrier in the channel between the source and drain [11]. This potential barrier is represented by the first term in Eq. (2). The model Hamiltonian does not contain any spin-dependent term and is therefore diagonal in the spin variables of both electrons.

We have obtained the exchange interaction in the *s*-wave Heitler-London approach [19] by using symmetric and antisymmetric combinations of single-dot ground-state orbital wave functions in the two adjacent channels. This method gives contributions from the direct and exchange terms with

$$\begin{aligned} J(t) = \frac{2S_{+-}^2}{1 - S_{+-}^4} \Bigg\{ & \langle \varphi_+(\mathbf{r}_1,t)\varphi_-(\mathbf{r}_2,t)| \\ & \times \left[\frac{e^2}{\epsilon_s r_{12}} + \Delta V_+ + \Delta V_- \right] |\varphi_+(\mathbf{r}_+,t)\varphi_-(\mathbf{r}_2,t)\rangle \\ & - \frac{1}{S_{+-}^2} \langle \varphi_+(\mathbf{r}_1,t)\varphi_-(\mathbf{r}_2,t)| \left[\frac{e^2}{\epsilon_s r_{12}} + \Delta V_+ + \Delta V_- \right] \\ & \times |\varphi_+(\mathbf{r}_2,t)\varphi_-(\mathbf{r}_1,t)\rangle \Bigg\}, \end{aligned} \tag{3}$$

where $\varphi_\pm(\mathbf{r},t) = \phi(x,t)\psi_\pm(y)$ are single-dot wave functions within the channels centered at $y = \pm a$; $\phi(x,t)$ is the lowest eigensolution of $h_0 = p_x^2/2m^* + V_1$, and $\psi_\pm(y) = \psi(y \mp a)$ are the single-particle harmonic-oscillator orbitals shifted to $(0, \pm a)$ with $\psi(y) = e^{-y^2/2\lambda_0^2}/\pi^{1/4}\lambda_0^{1/2}$ and $\lambda_0 = \sqrt{\hbar/m^*\Omega}$. Our assumption that the electrons are in the ground state $\phi(x,t)$ is valid when the acoustoelectric current is on (or below) the first plateau. Also, the overlap $S_{+-} = \int_{-\infty}^{\infty} dy\, \psi_+(y)\psi_-(y) = e^{-a^2/\lambda_0^2}$ and $\Delta V_\pm = V_2(y) - m^*\Omega^2(y \mp a)^2/2$. Evaluation of the matrix elements of $\Delta V_\pm$ in our model yields

$$\begin{aligned} & \langle \varphi_+(\mathbf{r}_1,t)\varphi_-(\mathbf{r}_2,t)|\Delta V_\pm|\varphi_+(\mathbf{r}_1,t)\varphi_-(\mathbf{r}_2,t)\rangle \\ & \quad = (3/32)(\lambda_0^4/a^2) m^* \Omega^2, \end{aligned}$$

$$\begin{aligned} & \langle \varphi_+(\mathbf{r}_1,t)\varphi_-(\mathbf{r}_2,t)|\Delta V_\pm|\varphi_+(\mathbf{r}_2,t)\varphi_-(\mathbf{r}_1,t)\rangle \\ & \quad = 3/32a^2[(\lambda_0^2 + 2a^2)(\lambda_0^2 - 2a^2) - 4\lambda_0^2 a^2] m^* \Omega^2 S_{+-}^2, \end{aligned}$$

which are independent of time. However, the matrix elements of the Coulomb energy are time dependent and must be evaluated numerically.

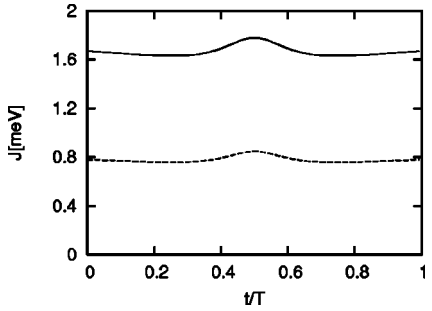

FIG. 2. The exchange interaction energy J as a function of time for $\beta = 0.5$ and separation $a = 0.7\lambda_0$ between a pair of narrow channels. J is larger when we take the difference between the energies for the singlet and triplet states, obtained by finding the eigenvalues of the Slater determinant numerically (solid line) compared to the Heitler-London approximation for separated harmonic wells (dashed line). The parameters used in the calculation are given in the text.

In Fig. 2, we present results for the exchange energy as a function of time for a fixed separation a between the centers of the channels. We chose $\hbar\Omega = 3.0$ meV, $m^* = 0.067 m_e$, appropriate for a GaAs/AlGaAs heterostructure, where m_e is the free-electron mass and $a/\lambda_0 = 0.7$. Also, we used $\ell = 1500$ Å, $l_0 = 100$ Å, and the parameter $\beta = 0.5$; this dimensionless parameter is defined as the ratio of the SAW potential amplitude to the height of the electrostatic potential barrier in the channel and its value corresponds to an acoustoelectric current on the first plateau. Also, the SAW velocity $v \approx 3.0 \times 10^3$ m/s and $k \approx 2\pi \times 10^{-3}$ nm. We observe that the exchange energy obtained by taking the difference $E_t - E_s$ between the triplet and singlet states which are determined by solving the interacting Schrödinger equation numerically is larger than the result calculated in the Heitler-London model. However, both results exhibit similar behavior such as positive exchange energy $J > 0$ (antiferromagnetism) as well as time-reversal symmetry. Figure 2 shows that the variation of J is a few percent over the period of a SAW cycle. We found that this remains true over a wide range of values for the parameters a, l_0, ℓ, and β. As the electrons are transmitted through the channels, it is required that, for two-qubit operation, the transformation associated with the exchange interaction is unitary. As a matter of fact,

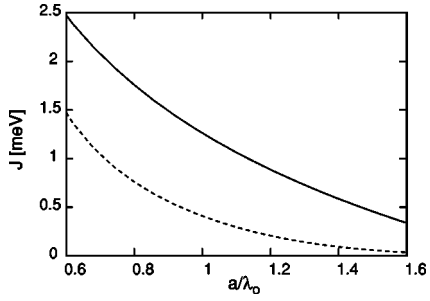

FIG. 3. The exchange interaction at $t=0$ as a function of the separation between a pair of parallel channels. The solid line is the difference in energies between the singlet and triplet states obtained by numerically solving the Schrödinger equation for a pair of interacting electrons. The dashed line is the result in the Heitler-London model. The parameters used in the calculation are given in the text.

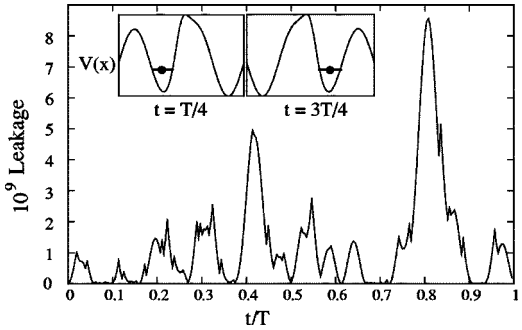

FIG. 4. The leakage $1-|c_0(t)|^2$ as a function of time. Here, $a/\lambda_o=0.7, \beta=0.5$. The oscillations in the leakage are due to the phase factors for the levels involved in our calculations. The inset shows the position of the electron in the potential of Eq. (2) at $t=T/4$ and $t=3T/4$.

the exchange Hamiltonian behaves like $J(t)\hat{\mathbf{S}}_i\cdot\hat{\mathbf{S}}_j$, which is Hermitian for real J and gives rise to a unitary transformation. The fact that the variation of J is small illustrates the practical nature of the gate operation with a SAW.

In Fig. 3, we plot the exchange energy as a function of the separation between channels for a fixed time. We see that $J>0$ at all separations due to antiferromagnetic spin-spin coupling. There is an exponential decay of the exchange energy as the separation between the channels increases, due to the factor S_{+-}^2 that represents the overlap of the wave functions in the y direction. The numerical plot also decays to zero as the separation increases. Furthermore, as β is increased from zero, J increases monotonically through a SAW cycle. In the simplest Heitler-London method, these results show how the exchange energy behaves in the adiabatic approximation as a function of the parameters used in the model. The exchange energy has been evaluated in extended basis such as the Hund-Mulliken approximation, using as basis four two-particle wave functions constructed from the single-electron wave functions $\varphi_{\pm}(\mathbf{r},t)$ within the channels centered at $y=\pm a$ [19,20]. The results obtained for a Hamiltonian with these four basis functions only differ by a few percent from the results in Figs. 2 and 3 for the parameters chosen [19].

We now discuss imperfections arising during the transport of the pair of electrons through the channels. We calculate the entanglement because it plays a key role in quantum information processing (QIP) and quantum computing (QC), and the preservation of entanglement is a necessary condition for the implementation of QIP and QC. The confining potential of the two-electron system changes over time due to the presence of the SAW in the x direction and the pinched-off potential arising from the split-gate potential. The electrons may not remain in the lowest spin singlet entangled state but may undergo spontaneous transitions to excited states, or an electron may tunnel back and forth between channels. The resulting gate error can be calculated by making use of the solutions of the time-dependent Schrödinger equation for the two-particle system. As a matter of fact, we have

$$\frac{\partial c_\alpha(t)}{\partial t}=\sum_{\beta\neq\alpha}\frac{c_\beta(t)}{E_\alpha(t)-E_\beta(t)}\left\langle u_\alpha\left|\frac{\partial H(t)}{\partial t}\right|u_\beta(t)\right\rangle \times e^{(-i/\hbar)\int_{t_0}^{t}d\tau(E_\beta(\tau)-E_\alpha(\tau))}+\left\langle u_\alpha(t)\left|\frac{\partial u_\alpha(t)}{\partial t}\right.\right\rangle. \quad (4)$$

where t_0 is an initial time, $u_\alpha(t)$ are instantaneous eigenstates of the time-dependent Hamiltonian $H(t)$ with eigenvalue $E_\alpha(t)$, i.e., $H(t)u_\alpha(t)=E_\alpha(t)u_\alpha(t)$, and $c_\alpha(t)$ are the coefficients in the expansion of the wave function $\Psi(t)$ in the eigenstates $u_\alpha(t)$, i.e., $\Psi(t)=\Sigma_\alpha c_\alpha(t)u_\alpha(t)$. If we note that the Berry phase $\gamma_\alpha(t)$ is given by [21]

$$\frac{d\gamma_\alpha(t)}{dt}=i\left\langle u_\alpha(t)\left|\frac{\partial u_\alpha(t)}{\partial t}\right.\right\rangle, \quad (5)$$

then it follows that the coefficients $c_\alpha(t)$ are naturally determined by a geometric phase factor $\exp[i\gamma_\alpha(t)]$. However, since the Hamiltonian depends on a single time parameter, the contribution from the geometric phase over the period of the time-dependent Hamiltonian is zero [21]. Consequently, the leakage from the dots, i.e., $1-|c_{m=0}|^2$, where $m=0$ denotes the state in which the system is prepared at $t=t_0$, will be obtained by solving Eq. (4) without the geometric phase factor.

In order for us to solve Eq. (4), we must first obtain the eigenstates for the interacting pair of electrons. Let $H_0(i)\Phi_i(\mathbf{r}_i)=\mathcal{E}_i\Phi_i(\mathbf{r}_i)$ for each electron moving in the channel independent of the presence of the other. Here, we have $\Phi_\alpha(\mathbf{r}_i)=\phi_{xi}(x)\phi_{yj}(y)$, where $\phi_{xi}(x)$ and $\phi_{yi}(y)$ are the eigenfunctions for electron motion in the x and y directions, respectively. From these, we can form a spin triplet (Slater determinant) state $\Phi_{ij}^{(A)}(\mathbf{r}_1,\mathbf{r}_2;t)$ and a spin singlet state $\Phi_{ij}^{(S)}(\mathbf{r}_1,\mathbf{r}_2;t)$. We expand the eigenstates $u_\alpha(t)$ in terms of the two-particle states in the form $u_\alpha(t)=\Sigma_{i,j}a_{ij}\Phi_{ij}^{(m)}(\mathbf{r}_1,\mathbf{r}_2;t)$, where $m=S,A$ for symmetric or antisymmetric states. It is a simple matter to show that the coefficients a_{ij}, as well as the energy eigenvalues E_α, are determined from the set of equations

$$(\mathcal{E}_l + \mathcal{E}_m)a_{lm} + \sum_{i<j} \langle \Phi_{lm}^{(m)} | H_{12} | \Phi_{ij}^{(m)} \rangle a_{ij} = E_\alpha a_{lm}, \qquad (6)$$

which require numerical evaluation of the Coulomb matrix elements. The single-particle states we used are $\phi_{x1}(x)\phi_{y1}(y)$, $\phi_{x2}(x)\phi_{y1}(y)$, $\phi_{x1}(x)\phi_{y2}(y)$, and $\phi_{x2}(x)\phi_{y2}(y)$ involving the two lowest eigenstates in the x and y directions. With $u_\alpha(t)$ expressed as a linear combination of $\Phi_{ij}^{(m)}(\mathbf{r}_1, \mathbf{r}_2; t)$, we solved the set of equations (4) numerically for the time-dependent Hamiltonian and then determine the leakage, which we plot in Fig. 4. The oscillations in the leakage are an interesting feature of our calculations involving the pair of channels. To better understand our results, we calculated the leakage for a single electron launched by a SAW in one of the channels but with the electron allowed to go back and forth between a pair of adjacent channels. The electron was assumed to be initially in its ground state. The calculated leakage is several orders of magnitude larger than in Fig. 4 and oscillates with time, indicating that it is more likely for the electron to go back and forth between energy levels. Furthermore, when we considered two independent electrons, i.e., we neglected the Coulomb interaction but appropriately antisymmetrize the wave functions, we found that the leakage oscillates with time but its magnitude is about the same as Fig. 4. This means that the Pauli principle dominates for the parameters chosen in our calculations. The oscillations are due to the energy transfer in the phase factors integrated over time, as given in Eq. (4). These oscillations contribute to the accuracy and the design of the two-bit gate and play as important a role as J to gate errors.

In summary, motivated by the suggestion in Ref. [18], we used a simple model to calculate the entanglement of spins transported by a SAW. Our approaches demonstrate that the qualitative nature of our results is not affected by the approximations. This indicates that the entanglement remains stable. Also, an advantage of the SAW technique is that the number of identical computations that could be performed per second is equal to the SAW frequency, which is in the GHz range. This should facilitate implementation of the readout. The SAW quantum computer has the advantage that information is distributed quickly along the nanocircuit when decoherence times are short. Some of the effects on the SAW decoherence times also arise from interactions of the qubit with other electrons in the 2DEG, as well as with impurities and phonons. The model we have presented only considers two contributions to the gate error which dominate at low temperature and in sufficiently pure samples.

We acknowledge partial support from the National Science Foundation under Grants No. DMR-0303574 and CREST 0206162, PSC-CUNY Grant No. 65485-00-34, as well as Grant No. 4137308-04 from the NIH.

[1] A. R. Hutson and D. L. White, J. Appl. Phys. **33**, 40 (1962).
[2] P. Bierbaum, Appl. Phys. Lett. **21**, 595 (1972).
[3] A. Wixforth, J. P. Kotthaus, and G. Weimann, Phys. Rev. Lett. **56**, 2104 (1986).
[4] A. Wixforth, J. Scriba, M. Wassermeier, J. P. Kotthaus, G. Weimann, and W. Schlapp, Phys. Rev. B **40**, 7874 (1989).
[5] A. Esslinger, A. Wixforth, R. W. Winkler, J. P. Kotthaus, H. Nickel, W. Schlapp, and R. Losch, Solid State Commun. **84**, 939 (1992).
[6] A. Esslinger, R. W. Winkler, C. Rocke, A. Wixforth, J. P. Kotthaus, H. Nickel, W. Schlapp, and R. Losch, Surf. Sci. **305**, 83 (1994).
[7] J. M. Shilton, D. R. Mace, V. I. Talyanskii, M. Pepper, M. Y. Simmons, A. C. Churchill, and D. A. Ritchie, Phys. Rev. B **51**, 14770 (1995).
[8] J. M. Shilton, D. R. Mace, V. I. Talyanskii, M. Y. Simmons, M. Pepper, A. C. Churchill, and D. A. Ritchie, J. Phys.: Condens. Matter **7**, 7675 (1995).
[9] A. L. Efros and Yu. M. Galperin, Phys. Rev. Lett. **64**, 1959 (1990).
[10] V. I. Fal'ko, S. V. Meshkov, and S. V. Iordanskii, Phys. Rev. B **47**, 9910 (1993).
[11] J. M. Shilton, V. I. Talyanskii, M. Pepper, D. A. Ritchie, J. E. F. Frost, C. J. B. Ford, C. G. Smith, and G. A. C. Jones, J. Phys.: Condens. Matter **8**, L531 (1996).
[12] C. L. Foden, V. I. Talyanskii, G. J. Milburn, M. L. Leadbeater, and M. Pepper, Phys. Rev. A **62**, 011803(R) (2000).
[13] H. Totland and Y. M. Galperin, Phys. Rev. B **54**, 8814 (1996).
[14] V. I. Talyanskii, J. M. Shilton, M. Pepper, C. G. Smith, C. J. B. Ford, E. H. Linfield, D. A. Ritchie, and G. A. C. Jones, Phys. Rev. B **56**, 15180 (1997).
[15] G. R. Aizin, G. Gumbs, and M. Pepper, Phys. Rev. B **58**, 10589 (1998); G. Gumbs, G. R. Aizin, and M. Pepper, *ibid.* **60**, R13954 (1999).
[16] V. I. Talyanskii, J. M. Shilton, J. Cunningham, M. Pepper, C. J. B. Ford, C. G. Smith, E. H. Linfield, D. A. Ritchie, and G. A. C. Jones, Physica B **249**, 140 (1998).
[17] N. A. Zimbovskaya and G. Gumbs, J. Phys.: Condens. Matter **13**, L409 (2001).
[18] C. H. W. Barnes, J. M. Shilton, and A. M. Robinson, Phys. Rev. B **62**, 8410 (2000); C. H. W. Barnes, Philos. Trans. R. Soc. London, Ser. A **361**, 1487 (2003).
[19] G. Burkard, D. Loss, and D. P. DiVincenzo Phys. Rev. B **59**, 2070 (1999).
[20] D. C. Mattis, in *The Theory of Magnetism*, Springer Series in Solid State Sciences Vol. 1, (Springer, New York, 1982), Sec. 4.5.
[21] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).