PHYSICAL REVIEW A **70**, 041801(R) (2004)

Subwavelength coincidence interference with classical thermal light

Kaige Wang[1,2] and De-Zhong Cao[2]
[1]*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China*
[2]*Department of Physics, Applied Optics Beijing Area Major Laboratory, Beijing Normal University, Beijing 100875, China*
(Received 13 April 2004; revised manuscript received 28 June 2004; published 8 October 2004)

We show that a thermal light source which is random in the transverse direction can produce a sub-wavelength double slit interference in a joint intensity measurement. This is the classical version of quantum lithography, and it can be explained with the correlation of rays instead of the entanglement of photons.

DOI: 10.1103/PhysRevA.70.041801 PACS number(s): 42.50.Dv, 42.50.St, 42.25.Hz

Noise is usually considered harmful to communication and interference because disorder and chaotic fluctuation degrade information and interference visibility. It would thus seem incredible that noise could be used as a signal carrier in communication or as a source for interference. A recent debate in quantum optics has inspired the authors to consider this very question. In the last decade, theoretical and experimental studies have shown that an entangled photon pair generated by spontaneous parametric down-conversion (SPDC) exhibits peculiar effects, such as sub-wavelength lithography [1–8], and coincidence or "ghost" imaging and interference [9–17]. These effects had indeed never been observed in classical optics before. In particular, sub-wavelength interference was regarded as one of the paradoxes related to violations of the quantum mechanical uncertainty principle. Recently, Bennink *et al.* showed in their experiment that coincidence imaging and coincidence interference can also be realized with a classical light source under certain conditions [18]. This demonstrates that classical correlation can play the same (or similar) role as quantum entanglement. A recent theoretical analysis [19] has shown that a thermal or quasi-thermal source can also exhibit such a classical correlation. On the other hand, sub-wavelength lithography is considered to be a form of a nonclassical interference that surpasses the Rayleigh diffraction limit. Physically, this effect is explained to be due to the quantum entanglement of photons [3,4] or the photonic de Broglie wavelength of a multiphoton wavepacket [1,2,5]. A similar question has emerged: can the sub-wavelength interference be produced by a classical beam too? In this work we demonstrate that a thermal light source random in the transverse plane possesses a second-order spatial correlation similar to that of a photon pair perfectly entangled in the transverse wavevector. The difference between them is that the former has the self-correlation of transverse wavevectors while the latter has a correlation between a pair of conjugate transverse wavevectors satisfying momentum conservation. Therefore, a thermal or thermal-like light source can produce not only ghost imaging and interference as shown in Refs. [18,19], but also the sub-wavelength interference. Due to the different correlation relationships, the sub-wavelength interference patterns must be observed by coincidence detection at a pair of symmetric positions. The result can be used to explain two kinds of macroscopic observation of sub-wavelength interference occurring in SPDC from a type I phase matched nonlinear crystal [20,21].

We consider the first- and second-order correlations of the fields for a two-photon quantum system and a classical thermal light source. The fields propagate with a central wavevector $\mathbf{k}_0$ and frequency ω_0. For a general two-photon state, the quantum wavefunction in the Schrödinger picture is written as

$$|\Psi\rangle = \int d\mathbf{q}_1\, d\mathbf{q}_2\, d\omega_1\, d\omega_2 \times C(\mathbf{q}_1,\omega_1;\mathbf{q}_2,\omega_2)\, a_1^\dagger(\mathbf{q}_1,\omega_1)\, a_2^\dagger(\mathbf{q}_2,\omega_2)|0\rangle,$$

where $\mathbf{q}_i$ and ω_i $(i=1,2)$ are the transverse wavevector (or the spatial frequency) and the frequency deviation from the central frequency, respectively. We assume that the two photons are distinguishable by their polarizations. By taking into account the commutation relations of the field operators, we obtain the first- and second-order spectral correlations for the two-photon state $|\Psi\rangle$,

$$\langle a_1^\dagger(\mathbf{q},\omega)a_1(\mathbf{q}',\omega')\rangle = \int d\mathbf{q}_2\, d\omega_2\, C^*(\mathbf{q},\omega;\mathbf{q}_2,\omega_2)C(\mathbf{q}',\omega';\mathbf{q}_2,\omega_2), \quad (1a)$$

$$\langle a_2^\dagger(\mathbf{q},\omega)a_2(\mathbf{q}',\omega')\rangle = \int d\mathbf{q}_1\, d\omega_1\, C^*(\mathbf{q}_1,\omega_1;\mathbf{q},\omega)C(\mathbf{q}_1,\omega_1;\mathbf{q}',\omega'), \quad (1b)$$

and

$$\langle a_1^\dagger(\mathbf{q}_1,\omega_1)a_2^\dagger(\mathbf{q}_2,\omega_2)a_2(\mathbf{q}_2',\omega_2')a_1(\mathbf{q}_1',\omega_1')\rangle = C^*(\mathbf{q}_1,\omega_1;\mathbf{q}_2,\omega_2)C(\mathbf{q}_1',\omega_1';\mathbf{q}_2',\omega_2'), \quad (2)$$

respectively. Equation (2) shows explicitly the separable product of the wavefunction and its conjugate one, reflecting the nature of the quantum wavepacket. This implies that the wavevector and frequency correlation may exist only within the same (positive or negative) frequency component. For the perfectly entangled two-photon state generated in SPDC, we have $C(\mathbf{q}_1,\omega_1;\mathbf{q}_2,\omega_2)=\delta(\mathbf{q}_1+\mathbf{q}_2)\delta(\omega_1+\omega_2)$ as a consequence of the conservation of momentum and energy in the SPDC process. Thus we obtain

$$\langle a_i^\dagger(\mathbf{q},\omega)a_i(\mathbf{q}',\omega')\rangle = \delta(\mathbf{q}-\mathbf{q}')\delta(\omega-\omega') \quad (i=1,2), \quad (3)$$

$$\langle a_1^\dagger(\mathbf{q}_1,\omega_1)a_2^\dagger(\mathbf{q}_2,\omega_2)a_2(\mathbf{q}_2',\omega_2')a_1(\mathbf{q}_1',\omega_1')\rangle = \delta(\mathbf{q}_1+\mathbf{q}_2)\delta(\omega_1+\omega_2)\delta(\mathbf{q}_1'+\mathbf{q}_2')\delta(\omega_1'+\omega_2'). \quad (4)$$

These results are also true for two photons with the same polarization.

Let us now consider classical thermal light. We assume a monochromatic plane wave $E_0\exp[i(k_0z-\omega_0t)]$ illuminating a material containing disordered scattering centers. After scattering, the field is written as $E(\mathbf{x},z,t)=\int E(\mathbf{q})\exp[i(\mathbf{q}\cdot\mathbf{x}+k_zz-\omega_0t)]d\mathbf{q}$ where $\mathbf{q}$ is the transverse wavevector introduced by the random scattering and satisfies $|\mathbf{q}|^2+k_z^2=k_0^2$. Hence, $E(\mathbf{q})$ is a stochastic variable obeying Gaussian statistics. However, the scattered waves with different transverse wavevectors are statistically independent. If $|\mathbf{q}|\ll k_0$, the scattered field can be approximately written as $E(\mathbf{x},z,t)=A(\mathbf{x})\exp[i(k_0z-\omega_0t)]$ where $A(\mathbf{x})=\int E(\mathbf{q})\exp[i\mathbf{q}\cdot\mathbf{x}]d\mathbf{q}$ is the slowly varying envelope. As a result, we have defined a monochromatic thermal light random in both strength and propagation direction. According to the Wiener-Khintchine theorem, the first-order spectral correlation must satisfy

$$\langle E^*(\mathbf{q})E(\mathbf{q}')\rangle = S(\mathbf{q})\delta(\mathbf{q}-\mathbf{q}'), \quad (5)$$

where $S(\mathbf{q})$ is the power spectrum of the spatial frequency. Obviously, Eqs. (3) and (5) indicate the same incoherence of the first-order for an ideal two-photon entangled state and a wide bandwidth thermal light source. Though the beam is monochromatic, the disorder in spatial frequency washes out the interference pattern.

For any field with thermal statistics, all high-order correlations can be expressed in terms of the first-order ones. Hence, the second-order spectral correlation of thermal light can be written as

$$\begin{aligned}\langle E^*(\mathbf{q}_1)E^*(\mathbf{q}_2)E(\mathbf{q}_2')E(\mathbf{q}_1')\rangle &= \langle E^*(\mathbf{q}_1)E(\mathbf{q}_1')\rangle\langle E^*(\mathbf{q}_2)E(\mathbf{q}_2')\rangle \\ &\quad + \langle E^*(\mathbf{q}_1)E(\mathbf{q}_2')\rangle\langle E^*(\mathbf{q}_2)E(\mathbf{q}_1')\rangle \\ &= S(\mathbf{q}_1)S(\mathbf{q}_2)[\delta(\mathbf{q}_1-\mathbf{q}_1')\delta(\mathbf{q}_2-\mathbf{q}_2') \\ &\quad + \delta(\mathbf{q}_1-\mathbf{q}_2')\delta(\mathbf{q}_2-\mathbf{q}_1')]. \end{aligned} \quad (6)$$

Unlike the case of two-photon entanglement in which two photons with opposite wavevectors are correlated, Eq. (6) shows that two fields with the same wavevectors are correlated. When the thermal light is split into two beams at a beam splitter, the correlated output beams are spatially separated. This is the origin of the coincidence imaging and coincidence interference for a classical thermal source [19]. As a matter of fact, in any coherent beam, there is no such correlation so coincidence imaging cannot occur.

We now discuss the double-slit interference in a joint intensity observation. Let a beam illuminate a double slit of slit width b and slit distance d. The double slit and the detection plane are placed at the two focal planes of a lens of focal length f. The Fourier transform of the double-slit function is written as $\widetilde{T}(q)=(2b/\sqrt{2\pi})\mathrm{sinc}(qb/2)\cos(qd/2)$. For simplicity, we omit the time variable. This can be done by using a narrow frequency filter in the beam. According to Ref. [20], the second-order spatial correlation of the field in the detection plane is

$$\begin{aligned}\langle E_d^*(x_1)E_d^*(x_2)E_d(x_2)E_d(x_1)\rangle &= \frac{k_0^2}{(2\pi f)^2}\int \widetilde{T}\left(\frac{k_0x_1}{f}-q_1\right)\widetilde{T}\left(\frac{k_0x_2}{f}-q_2\right) \\ &\quad\times\widetilde{T}\left(\frac{k_0x_2}{f}-q_2'\right)\widetilde{T}\left(\frac{k_0x_1}{f}-q_1'\right) \\ &\quad\times\langle E^*(q_1)E^*(q_2)E(q_2') \\ &\quad\times E(q_1')\rangle dq_1\,dq_2\,dq_2'\,dq_1'. \end{aligned} \quad (7)$$

For an ideal two-photon entangled state, from Eq. (4) we have $\langle E_d^*(x_1)E_d^*(x_2)E_d(x_2)E_d(x_1)\rangle\propto\widetilde{T}^2[(k_0/f)(x_1+x_2)]$. If we use a two-photon detector to scan the position, sub-wavelength interference fringes with perfect visibility can be observed. Since a two-photon detector is not available at present, in the experiment the two-photon detection is carried out by a coincidence measurement (CM) of two orthogonally polarized photons [2,4]. However, for the thermal light described above, we obtain

$$\begin{aligned}\langle E_d^*(x_1)E_d^*(x_2)E_d(x_2)E_d(x_1)\rangle &= \frac{k_0^2}{(2\pi f)^2}\left\{\int \widetilde{T}^2\left(\frac{k_0x_1}{f}-q\right)S(q)dq\right. \\ &\quad\times\int \widetilde{T}^2\left(\frac{k_0x_2}{f}-q\right)S(q)dq+\left[\int \widetilde{T}\left(\frac{k_0x_1}{f}-q\right)\right. \\ &\quad\left.\left.\times\widetilde{T}\left(\frac{k_0x_2}{f}-q\right)S(q)dq\right]^2\right\}. \end{aligned} \quad (8)$$

In the broadband limit, we can set $S(q)\approx S(0)$ and so obtain

$$\langle E_d^*(x_1)E_d^*(x_2)E_d(x_2)E_d(x_1)\rangle = \frac{k_0^2S^2(0)}{2\pi f^2}\left\{\widetilde{T}^2(0)+\widetilde{T}^2\left[\frac{k_0}{f}(x_1-x_2)\right]\right\}. \quad (9)$$

When the two detectors are placed in symmetric positions, i.e., $x_1=-x_2=x$, to perform a joint intensity measurement, sub-wavelength interference fringes with 33.3% visibility can be observed. In the general case we set the Gaussian spectrum to be $S(q)=(\sqrt{2\pi}w)^{-1}\exp[-q^2/(2w^2)]$. Figure 1 shows the interference fringes as a function of the normalized bandwidth $W=wb/(2\pi)$. At very small bandwidth, the fringe pattern is "normal," i.e., the same as that for a coherent beam. As the bandwidth increases, the sub-wavelength interference effect appears and grows.

Sub-wavelength coincidence interference from classical thermal correlation can be illustrated schematically in Fig. 2.

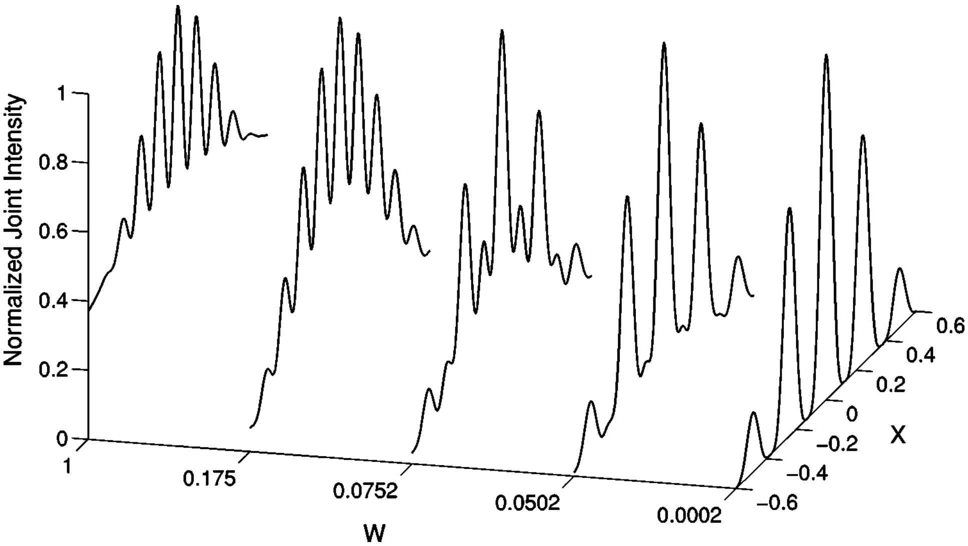

FIG. 1. Coincidence interference patterns for different normalized bandwidths $W=wb/(2\pi)$ of the spatial frequency spectrum of thermal light. The X-axis $X=xk_0b/(2\pi f)$ is the normalized position in the detection plane and the double slit parameter is taken to be $d=4b$.

The four fields $E_A(q)$, $E_A(-q')$, $E_B(q')$, and $E_B(-q)$ are involved in the joint intensity measurement, as shown in Fig. 2(a). For greater clarity, in Fig. 2(b), we fold the two lower beams with respect to the source and replace $E_A(-q')$ and $E_B(-q)$ by $E_A^*(q')$ and $E_B^*(q)$, respectively, since the spatial frequency component satisfies $E(-q)=E^*(q)$. It is the nature of the thermal correlation that causes the diffraction beams $E_A(q)$ and $E_B(q')$ to be correlated with the diffraction beams $E_B^*(q)$ and $E_A^*(q')$, respectively. Therefore, in the joint intensity observation, the pair of correlated beams $E_A(q)$ and $E_B^*(q)$ interfere with the pair of correlated beams $E_B(q')$ and $E_A^*(q')$, resulting in twice the optical path difference of the one-photon interference.

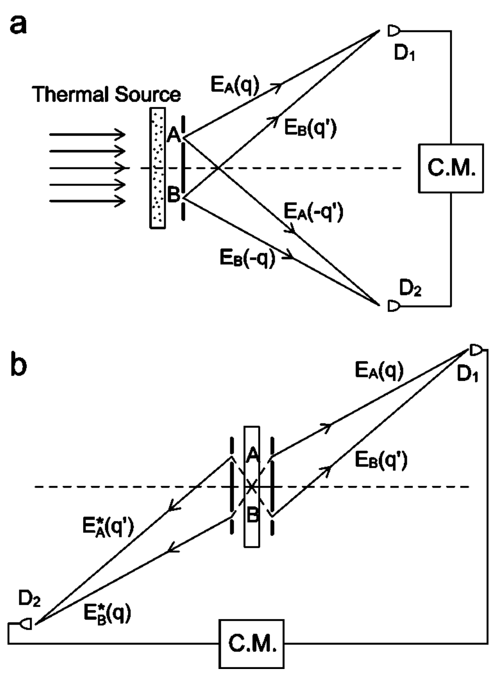

FIG. 2. (a) Schematic ray diagram of sub-wavelength coincidence interference of thermal light. (b) An equivalent ray diagram of the interference of the correlated beams $E_A(q)$ and $E_B^*(q)$ with the correlated beams $E_B(q')$ and $E_A^*(q')$. It can be seen that the optical path difference is doubled.

The beam generated in SPDC may incorporate both the quantum entanglement and the classical thermal correlation. When a plane-wave pump field activates a $\chi^{(2)}$ nonlinear crystal, the basic unitary transformation is described by [16]

$$a_m(\mathbf{q},\omega) = U_m(\mathbf{q},\omega)a_m^{in}(\mathbf{q},\omega) + V_m(\mathbf{q},\omega)a_n^{in\dagger}(-\mathbf{q},-\omega) \quad (m \neq n = s,i), \tag{10}$$

where $a_m(\mathbf{q},\omega)$ and $a_m^{in}(\mathbf{q},\omega)$ are the output and input field operators, respectively. The first-order correlation is obtained to be

$$\langle a_m^\dagger(\mathbf{q},\omega)a_m(\mathbf{q}',\omega')\rangle = |V_m(\mathbf{q},\omega)|^2\delta(\mathbf{q}-\mathbf{q}')\delta(\omega-\omega'). \tag{11}$$

For a type I crystal, we may omit the subscript in Eq. (10) and the second-order correlation is written as

$$\begin{aligned}\langle a^\dagger(\mathbf{q}_1,\omega_1)a^\dagger(\mathbf{q}_2,\omega_2)a(\mathbf{q}_2',\omega_2')a(\mathbf{q}_1',\omega_1')\rangle \\ = V^*(\mathbf{q}_1,\omega_1)V(\mathbf{q}_1',\omega_1')U^*(-\mathbf{q}_1,-\omega_1)U(-\mathbf{q}_1',-\omega_1') \\ \times\delta(\mathbf{q}_1+\mathbf{q}_2)\delta(\omega_1+\omega_2)\delta(\mathbf{q}_1'+\mathbf{q}_2')\delta(\omega_1'+\omega_2') \\ + |V(\mathbf{q}_1,\omega_1)V(\mathbf{q}_2,\omega_2)|^2[\delta(\mathbf{q}_1-\mathbf{q}_1')\delta(\mathbf{q}_2-\mathbf{q}_2') \\ \times\delta(\omega_1-\omega_1')\delta(\omega_2-\omega_2') + \delta(\mathbf{q}_1-\mathbf{q}_2')\delta(\mathbf{q}_2-\mathbf{q}_1') \\ \times\delta(\omega_1-\omega_2')\delta(\omega_2-\omega_1')].\end{aligned} \tag{12}$$

The first term shows the same correlation as the two-photon entangled state, whereas the second term shows the same correlation as the thermal light. Note that this result is valid in general whatever the SPDC coupling gain even when the converted beam contains a large number of photons. When the gain is very small, the second term is negligible in comparison with the first term and the converted field is almost a two-photon entangled state. As the gain increases, two kinds of macroscopic sub-wavelength interference coexist [20]. For a type II crystal, however, the second-order correlation of the two orthogonally polarized beams is obtained to be

$$\begin{aligned}\langle a_m^\dagger(\mathbf{q}_1,\omega_1)a_n^\dagger(\mathbf{q}_2,\omega_2)a_n(\mathbf{q}_2',\omega_2')a_m(\mathbf{q}_1',\omega_1')\rangle \quad (m\neq n=s,i)\\ = V_m^*(\mathbf{q}_1,\omega_1)V_m(\mathbf{q}_1',\omega_1')U_n^*(-\mathbf{q}_1,-\omega_1)U_n(-\mathbf{q}_1',-\omega_1')\delta(\mathbf{q}_1\\ +\mathbf{q}_2)\delta(\omega_1+\omega_2)\delta(\mathbf{q}_1'+\mathbf{q}_2')\delta(\omega_1'+\omega_2')\\ +|V_m(\mathbf{q}_1,\omega_1)V_n(\mathbf{q}_2,\omega_2)|^2\delta(\mathbf{q}_1-\mathbf{q}_1')\delta(\mathbf{q}_2-\mathbf{q}_2')\\ \times\delta(\omega_1-\omega_1')\delta(\omega_2-\omega_2'). \qquad (13)\end{aligned}$$

The first term is the same as that for the type I case, but the second term does not represent thermal correlation. Only quantum sub-wavelength interference can occur [20]. The thermal correlation in a type II crystal can be recovered through extraction of either the signal or the idler beams by a polarization beamsplitter. The second-order correlation for the signal/idler beam reads as

$$\begin{aligned}\langle a_m^\dagger(\mathbf{q}_1,\omega_1)a_m^\dagger(\mathbf{q}_2,\omega_2)a_m(\mathbf{q}_2',\omega_2')a_m(\mathbf{q}_1',\omega_1')\rangle \quad (m=s,i)\\ = |V_m(\mathbf{q}_1,\omega_1)V_m(\mathbf{q}_2,\omega_2)|^2[\delta(\mathbf{q}_1-\mathbf{q}_1')\delta(\mathbf{q}_2-\mathbf{q}_2')\\ \times\delta(\omega_1-\omega_1')\delta(\omega_2-\omega_2')+\delta(\mathbf{q}_1-\mathbf{q}_2')\delta(\mathbf{q}_2-\mathbf{q}_1')\\ \times\delta(\omega_1-\omega_2')\delta(\omega_2-\omega_1')]. \qquad (14)\end{aligned}$$

We have thus obtained the thermal correlation for both signal and idler beams.

In conclusion, we have demonstrated a classical version of quantum sub-wavelength lithography, in which the classical correlation of thermal light plays a similar role to the quantum entanglement of a two-photon state in optical sub-wavelength interference, coincidence imaging and interference.

The authors thank L. A. Wu for helpful discussions. This research was supported by the National Fundamental Research Program of China Project No. 2001CB309310, and the National Natural Science Foundation of China, Project Nos. 60278021 and 10074008.

[1] J. Jacobson, G. Björk, I. Chuang, and Y. Yamamoto, Phys. Rev. Lett. **74**, 4835 (1995).
[2] E. J. S. Fonseca, C. H. Monken, and S. Pádua, Phys. Rev. Lett. **82**, 2868 (1999).
[3] A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. **85**, 2733 (2000).
[4] M. D'Angelo, M. V. Chekhova, and Y. Shih, Phys. Rev. Lett. **87**, 013602 (2001).
[5] E. J. S. Fonseca, Z. Paulini, P. Nussenzveig, C. H. Monken, and S. Pádua, Phys. Rev. A **63**, 043819 (2001).
[6] E. M. Nagasako, S. J. Bentley, R. W. Boyd, and G. S. Agarwal, Phys. Rev. A **64**, 043802 (2001).
[7] K. Edamatsu, R. Shimizu, and T. Itoh, Phys. Rev. Lett. **89**, 213601 (2002).
[8] R. Shimizu, K. Edamatsu, and T. Itoh, Phys. Rev. A **67**, 041805(R) (2003).
[9] D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, Phys. Rev. Lett. **74**, 3600 (1995); T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, Phys. Rev. A **52**, R3429 (1995).
[10] P. H. S. Ribeiro, S. Pádua, J. C. Machado da Silva, and G. A. Barbosa, Phys. Rev. A **49**, 4176 (1994).
[11] P. H. Souto Ribeiro and G. A. Barbosa, Phys. Rev. A **54**, 3489 (1996).
[12] G. A. Barbosa, Phys. Rev. A **54**, 4473 (1996).
[13] E. J. S. Fonseca, P. H. Souto Ribeiro, S. Pádua, and C. H. Monken, Phys. Rev. A **60**, 1530 (1999).
[14] B. E. A. Saleh, A. F. Abouraddy, A. V. Sergienko, and M. C. Teich, Phys. Rev. A **62**, 043816 (2000).
[15] A. F. Abouraddy, B. E. A. Saleh, A. V. Sergienko, and M. C. Teich, Phys. Rev. Lett. **87**, 123602 (2001).
[16] A. Gatti, E. Brambilla, and L. A. Lugiato, Phys. Rev. Lett. **90**, 133603 (2003).
[17] M. D'Angelo, Y.-H Kim, S. P. Kulik, and Y. Shih, Phys. Rev. Lett. **92**, 233601 (2004).
[18] R. S. Bennink, S. J. Bentley and R. W. Boyd, Phys. Rev. Lett. **89**, 113601 (2002); R. S. Bennink, S. J. Bentley, R. W. Boyd, and J. C. Howell, *ibid.* **92**, 033601 (2004).
[19] A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, e-print quant-ph/0307187.
[20] D. Zh. Cao and K. Wang, e-print quant-ph/0310046; D. Zh. Cao, Zh. Li, Y. H. Zhai, and K. Wang, e-print quant-ph/0401109.
[21] A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, e-print quant-ph/0405056. In this paper, the authors also indicate that sub-wavelength interference can be performed by the classical thermal correlation in the SPDC process.