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Nonlinear atom-optical δ-kicked harmonic oscillator using a Bose-Einstein condensate

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We experimentally investigate the atom-optical δ-kicked harmonic oscillator for the case of nonlinearity due to collisional interactions present in a Bose-Einstein condensate. A Bose condensate of rubidium atoms tightly confined in a static harmonic magnetic trap is exposed to a one-dimensional optical standing-wave potential that is pulsed on periodically. We focus on the quantum antiresonance case for which the classical periodic behavior is simple and well understood. We show that after a small number of kicks the dynamics are dominated by dephasing of matter wave interference due to the finite width of the condensate's initial momentum distribution. In addition, we demonstrate that the nonlinear mean-field interaction in a typical harmonically confined Bose condensate is not sufficient to give rise to chaotic behavior.

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The δ-kicked rotor is an extensively investigated system in the field of classical chaos theory. During the last decade great progress has been achieved in understanding quantum dynamics of a classically chaotic system using atom-optical techniques and cold atoms. From an experimental point of view, cold atoms in optical potentials [1–5] provide an ideal environment to explore quantum dynamics. To date, all experimental work has focused on linear atomic systems (see, for example [6–9], and references therein) where the quantum dynamics are stable due to the linearity of the Schrödinger equation. In stark contrast to the chaotic behavior of classical dynamics, the linear quantum systems exhibit antiresonance (periodic motion), dynamical localization (quasiperiodic motion), or resonant dynamics [10,11].

Recently, theoretical investigations have considered how the nonlinearity due to many-body (collisional) interactions in a Bose-Einstein condensate (BEC) modifies the behavior of the atom-optical kicked rotor system, providing a route to chaotic dynamics. Gardiner *et al.* developed a theoretical formalism to treat the one-dimensional nonlinear kicked harmonic oscillator (a particular manifestation of the generic δ-kicked rotor) using Gross-Pitaevskii and Liouville-type equations to describe the dynamics of a Bose-Einstein condensate, and estimated the growth rate in the number of noncondensate particles [12]. Zhang *et al.* investigated the generalized quantum kicked rotor by considering a periodically kicked Bose condensate confined in a ring potential for the case of quantum antiresonance [13]. As opposed to the familiar periodic behavior exhibited by a corresponding linear system, they predicted quasiperiodic variation in energy for a weak-interaction strength and chaotic behavior for strong interactions.

In this work we investigate the nonlinear δ-kicked harmonic oscillator by performing experiments on Bose-Einstein condensates in a harmonic potential. A Bose condensate of rubidium atoms tightly confined in a static harmonic magnetic trap is exposed to a periodically pulsed one-dimensional optical standing-wave potential. Our focus is on the particular case of quantum antiresonance for which the linear behavior is simple and well understood [14]. The finite width of the initial condensate momentum distribution is shown to have a profound effect on the dynamics. After a small number of kicks the behavior is dominated by dephasing of matter wave interference. We present numerical solutions of the Gross-Pitaevskii equation which match the observed behavior and confirm our interpretation.

In the atom-optical kicked harmonic oscillator, the effective Planck's constant $\bar{k}$ can be adjusted, in a sense, to make the system "more" or "less" quantum mechanical. At specific values of $\bar{k}$—in particular, where $\bar{k}$ is a rational multiple of 2π—quite remarkable phenomena can occur in the form of so-called quantum resonances and antiresonances [6,15–20]. In this work we focus our attention on the case of the $\bar{k}=2\pi$ antiresonance at which the energy of a linear system exhibits simple periodic behavior. This antiresonance requires a particular initial momentum state which evolves in such a way that during the period of free evolution in between kicks, the different components of the state vector of the system experience a phase shift that alternates in sign from one momentum component to the next, so that the system returns identically to its initial state after every second kick. The underlying physics of linear atom-optical kicking at antiresonance has already been neatly described, albeit in a different context [14]. In the short pulse (thin grating) limit the first kick imprints a sinusoidal phase profile onto the plane matter wave, thereby populating a number of momentum states (diffraction orders), and the phase evolution of the n^{th} state is proportional to n^2 so that after free evolution (between kicks) corresponding to half the Talbot time [$T_T = h/4E_r$, where the recoil energy $E_r=(\hbar k)^2/2m$, k is the wave vector and m is the atomic mass] the second pulse cancels the spatial variation induced by the first. For multiple pulses this process repeats so that the initial plane-wave state is reconstructed after every second pulse.

Bose condensate evolution in an optical standing wave, or lattice, has previously been well described by the Gross-Pitaevskii equation (GPE) (see, for example [21]), and condensate behavior in a kicked harmonic potential can be described in this formalism using the one-dimensional GPE along the direction of the kicking beams,

$$i\frac{\partial \psi(x,t)}{\partial t} = \left\{ -\frac{\partial^2}{\partial x^2} - \frac{\kappa}{k\tau_p}\cos\left(\sqrt{\frac{k}{2T}}x\right) \times \sum_{n=0}^{N} f(t-nT) + \frac{1}{4}x^2 + C|\psi(x,t)|^2 \right\}\psi(x,t), \quad (1)$$

where $\psi(x,t)$ is the condensate wave function and $\kappa = E_r \Omega T \tau_p / \hbar$ is the classical stochasticity parameter (or kick strength) for the effective Rabi frequency Ω. Here $f(t-nT)$ represents a square pulse, such that $f(t-nT)=1$ for $0 < t - nT < \tau_p$, where τ_p is the pulse length. The length scale is the characteristic harmonic oscillator length $\sqrt{\hbar/2m\omega_t}$, and the temporal scale is the effective trapping frequency ω_t along the axis of the kicking beams. The nonlinear strength $C = (8\mu/3)^{3/2}$ is calculated such that the one-dimensional chemical potential μ is equal to the chemical potential of the three-dimensional condensate, in the Thomas–Fermi approximation. Optimization techniques developed by Blakie and Ballagh [21] are used to calculate the condensate ground state and the GPE is evolved using a Runge-Kutta fourth-order interaction picture algorithm [22].

The energy of the system is calculated using

$$E = \int \psi^*(x,t)\left\{ -\frac{\partial^2}{\partial x^2} - \frac{\kappa}{k\tau_p}\cos\left(\sqrt{\frac{k}{2T}}x\right) \times \sum_{n=0}^{N} f(t-nT) + \frac{1}{4}x^2 + \frac{1}{2}C|\psi(x,t)|^2 \right\}\psi(x,t)dx, \quad (2)$$

which is evaluated after each kick to make a direct comparison with experiment.

Note that in this formalism any noncondensate particles are not accounted for. Previous theoretical papers [12,13] have investigated the proliferation of noncondensate particles, and for our nonlinearity, kicking strength, and number of kicks, this is predicted to be negligible. Starting with a pure Bose condensate, we do not observe any formation of noncondensate particles.

Bose-Einstein condensates with up to 10^5 ^{87}Rb atoms are created in the $F=2, m_F=2$ hyperfine state with no discernible thermal component. A description of the BEC apparatus was given previously [23], but there have been some modifications. We now use injection-seeded diode lasers to drive the two magneto-optical traps, and atoms are transferred continuously between the traps using a focused resonant laser beam. Condensates are formed in the static harmonic potential of a quadrupole-Ioffe-configuration trap [24], characterized by radial and axial oscillation frequencies of $\omega_r/2\pi = 164$ Hz and $\omega_z/2\pi = 14$ Hz, respectively. A condensate, while confined in the magnetic trap, is then exposed to a pulsed optical standing wave generated by two counterpropagating laser beams with parallel linear polarizations derived from a single beam which is detuned 1.48 GHz from the $5S_{1/2}, F=2 \rightarrow 5P_{3/2}, F'=3$ transition. Each beam has an intensity of 1052 $\mathrm{W/m^2}$ and intercepts the condensate at an angle of 27° to the radial direction. A double-pass acoustic-optic modulator is used in each beam for switching the optical potential on and off. The pulse length is 796 ns, which is much less than the minimum classical oscillation period of 130 μs, so that the kicking potential is well described as a thin phase grating [25]. Following the kicks, the momentum distribution is determined from a time-of-flight absorption image after a free expansion period of 29 ms, by which time the momentum components have separated. The energy of the atomic sample is determined by calculating $(\int p^2 dp)/2m$, then dividing by the total number of atoms. The kicking period is 33.16 μs to match the condition for the quantum antiresonance at $k = 8E_r T/\hbar\omega_t = 2\pi$ (corresponding to half the Talbot time), where T is the pulse period in units of $1/\omega_t$. The beam detuning and intensity were chosen to give a relatively strong kicking strength while maintaining a negligible spontaneous emission rate ($<34\ \mathrm{s}^{-1}$). Up to 25 kicks were delivered to the condensate for each experimental run. For each number of kicks, the energy measurement was repeated six times, and the mean value is plotted in Fig. 1.

Figure 1 shows an experimental and theoretical plot of energy versus kick number. The theoretical calculation uses $\tau_p = 6.9686 \times 10^{-4}$, $T = 0.029$, and $C = 50$ in correspondence with the experimental conditions. The height of the optical potential has been adjusted ($\kappa = 8.25$) so that the energy after the first kick is consistent with the experimental value. Initially, periodic behavior is observed, but after several kicks the oscillation in the energy of the system damps away to an average value (to within our experimental uncertainty). The theoretical points indicate that this average value gradually increases, but no further significant oscillation is expected. This steady increase occurs because in the time between kicks, atoms moving in the harmonic potential gain a small amount of potential energy.

The damping in the oscillation of the energy is due to dephasing associated with the finite width of the condensate's initial momentum distribution. The initial momentum state is not perfectly reconstructed after each free evolution period, because different momentum components of the initial distribution have a slightly different Talbot time (associated with their slightly different phase evolution). This is illustrated in Fig. 2. The rate of coupling between momentum states is not uniform across the momentum distribution of the condensate. The central (zero momentum) region of the initial condensate momentum distribution couples to the higher-order momentum states at a slower rate than the nonzero wings of the condensate wave function. This causes, for example, the development of the double-peaked structure in the first-order diffraction components. As time evolves, the cycling between momentum states for different components of the initial distribution become progressively out of phase.

This process of dephasing occurs even in the absence of collisions. Figure 3 illustrates the results of theoretical simulations, showing the energy dynamics with and without in-

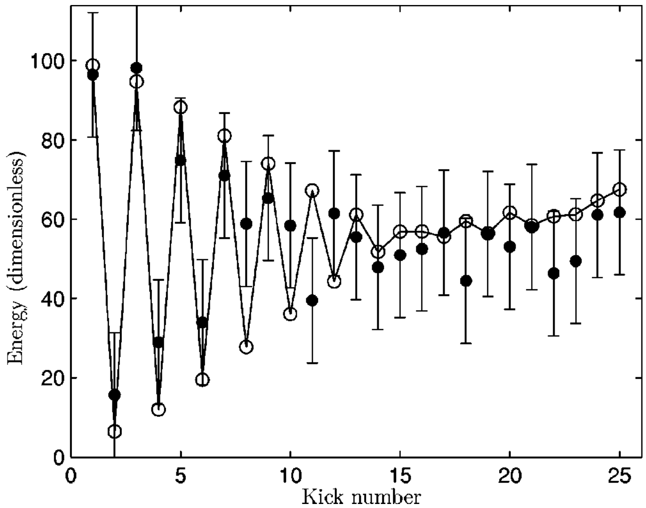

FIG. 1. Energy plotted vs kick number for the quantum antiresonance condition, $\bar{k}=2\pi$ with mean-field interactions. The solid circles are the measured mean energies and the error bars include shot-to-shot variation and systematic uncertainty in the calculation of the energy. The open circles are the corresponding theoretical values computed using Eq. (2) and the solid line is to guide the eye.

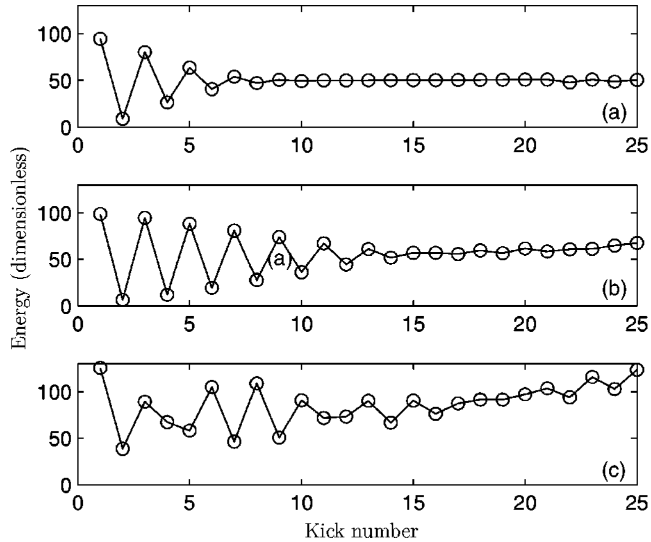

FIG. 3. Numerical simulations of energy vs kick number for the quantum antiresonance condition, $\bar{k}=2\pi$. (a) No collisional interaction ($C=0$), (b) collisional interaction corresponding to the experiment ($C=50$), and (c) strong collisional interaction ($C=1000$).

teractions. For the experimental condition of $C=50$ [Fig. 3(b)], the behavior is essentially the same as the collisionless case [Fig. 3(a)], although dephasing occurs on a longer time scale. This is due to the fact that the initial condensate momentum width decreases with increasing collisional interactions [26].

After a significant time one might expect a rephasing of the condensate wave function, leading to quasiperiodic dynamics. While we have not observed this, our numerical simulations indicate that some rephasing is possible for the collisionless case, but this is highly sensitive to noise and we predict that rephasing will not occur in the nonlinear regime.

In Fig. 3(c) the nonlinear term C in our calculation is a factor of 20 times larger than that corresponding to our experiment, and the behavior is no longer dominated by the

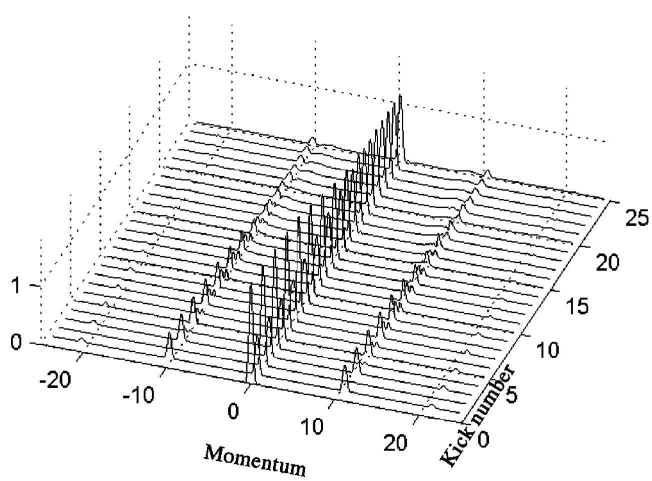

FIG. 2. Momentum distribution vs kick number for the numerical simulation in Fig. 1

damped oscillations. We begin to predict what appears to be unstable behavior similar to that predicted by Zhang *et al.* [13], and this is consistent with our values of kick strength and nonlinearity. Experimentally, this is attainable with condensates of $>10^7$ atoms (which is beyond our reach), or by increasing the *s*-wave scattering length via a Feshbach resonance [27]. Although Feshbach resonances have not been observed in the magnetically trapped states of ^{87}Rb, and one would therefore need to use other spin states confined in an optical dipole trap, this is a particularly appealing method for controlling the strength of the nonlinearity. Another possibility for reaching the chaotic regime is by using a much lower kick strength. We repeated our measurements for a kick strength $\kappa=4.125$ and observed similar features to those presented in Fig. 1, with the main difference being a smaller amplitude of the energy oscillations. We estimate that we would have to reduce our kick strength by a factor of 100 to enter the chaotic regime predicted by Zhang *et al.* [13]. While it may seem straightforward to simply further reduce the intensity of the kicking beams, this reduces the energy of the system to the point where shot-to-shot variations exceed the predicted signal. For a kick strength lower than $\kappa\approx 4$ the signal to noise is compromised and the energy of our system becomes immeasurable.

In summary, we experimentally investigated the possibility of using nonlinear collisional interactions in a typical Bose-Einstein condensate to observe chaotic dynamics in the quantum-kicked harmonic oscillator system. We applied a pulsed, far-detuned, optical standing wave to a rubidium Bose condensate, and measured the system energy as a function of kick number for the case of the quantum antiresonance condition at $\bar{k}=2\pi$. We found that, even in the presence of nonlinear interactions, our system exhibits the well-known periodic behavior associated with the linear system. Using numerical solutions to the Gross-Pitaevskii equation, we showed that observed dephasing of the oscillations is due to the finite width of the condensate's initial momentum distribution. This severely limits the possibility of observing an extended period of chaotic behavior in the energy of the system.

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