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Dynamical suppression of telegraph and $1/f$ noise due to quantum bistable fluctuators

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We study dynamical decoupling of a qubit from non-Gaussian quantum noise due to discrete sources, as bistable fluctuators and $1/f$ noise. We obtain analytic and numerical results for generic operating points. For very large pulse frequency, where dynamic decoupling compensates decoherence, we found universal behavior. At intermediate frequencies noise can be compensated or enhanced, depending on the nature of the fluctuators and on the operating point. Our technique can be applied to a larger class of non-Gaussian environments.

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Controlling the dynamics of a complex quantum system is at the heart of quantum information [1]. However, in any real device the computational variables entangle with the environment leading to decoherence [2]. Bang-bang (BB) control techniques have been proposed as a way to achieve an effective decoupling from the environment [3,4]. They may be operated by a sequence of strong external pulses separated by a time Δt [3]. For $\Delta t \to 0$ full decoupling [3,4] of the unwanted interactions is achieved. The physics in this limit is a manifestation of the quantum Zeno effect [3,5].

In practice Δt is finite especially when full-power pulses are used. This imperfect decoupling is still well described by the Zeno limit if $\Delta t \ll \gamma^{-1}$, the typical time scale of the environment [3,4]. If γ is large one may argue that BB chops noise and frequencies $\omega < 1/\Delta t < \gamma$ are averaged out. This optimistic scenario could foresee applications to solid state coherent devices, where low-frequency noise [6] is the major problem for quantum state processing [7–9]. Investigation of this point is one of the topics of this communication, where we study environments of dissipative quantum bistable fluctuators [7].

Recently, decoupling from classical random telegraph noise (RTN) was studied in the Zeno limit, $\Delta t \ll \gamma^{-1}$ [10]. Gaussian noise with $1/f$ spectrum has also been studied [11] and decoupling for decreasing Δt was found. On the other hand, in echo protocols, details of the structure and of the dynamics of a solid-state discrete environment [12] may become important if the condition $\Delta t \ll \gamma^{-1}$ is not met.

We consider a qubit $[\mathcal{H}_Q = -(\varepsilon/2)\sigma_z - (\Delta/2)\sigma_x]$ coupled to an impurity. The Hamiltonian is

$$\mathcal{H} = \mathcal{H}_Q - \frac{1}{2}\sigma_z \hat{E} + \mathcal{H}_E + \mathcal{V}(t). \qquad (1)$$

The environment Hamiltonian $\mathcal{H}_E = \mathcal{H}_d + \mathcal{H}_T + \mathcal{H}_B$ describes an impurity level occupied by a localized electron, $\mathcal{H}_d = \varepsilon_c d^\dagger d$, tunneling with amplitudes T_k ($\mathcal{H}_T = \sum_k T_k c_k^\dagger d + \mathrm{H.c.}$) to a fermionic band, described by $\mathcal{H}_B = \sum_k \varepsilon_k c_k^\dagger c_k$. The charge in the impurity is coupled to the qubit, $\hat{E} = v\, d^\dagger d$. Control is operated as in Ref. [3], the external field $\mathcal{V}(t)$ being a sequence of π pulses about $\hat{x}$. This model may describe charge noise due to impurities close to a solid-state qubit [7,8,14,15]. The characteristic scale of the impurity is the switching rate $\gamma = 2\pi \mathcal{N}(\epsilon_c)|T|^2$ ($\mathcal{N}$ is the density of states of the fermionic band $|T_k|^2 \approx |T|^2$).

This environment is non-Gaussian [6], a key feature to explain recent observations in Josephson qubits (splitting of spectroscopic peaks, beats in the coherent oscillations [16]) due to individual impurities close to the device. The observed $1/f$ noise is due to a set of such impurities [9]. We find that decoupling of this environment is sensitive to details of its dynamics. If pulses are not very frequent it shows a rich variety of behaviors, suggesting that BB may also be used for spectroscopy.

We operate with instantaneous pulses, which do not modify the environment, the corresponding evolution operator being $\mathcal{S}_P \approx i\sigma_x \otimes \mathbb{1}_E$. The evolution operator of the Hamiltonian (1) is $[\mathcal{S}_P \mathcal{S}]^{2N}$, where $\mathcal{S} = \exp(-i\mathcal{H}\Delta t)$with $\mathcal{V}(t)=0$ is the evolution between pulses. Echo corresponds to $N=1$. The reduced density matrix (RDM) of the qubit is obtained by tracing out the environment

$$\rho(t) = \mathrm{Tr}_E\{[\mathcal{S}_P\mathcal{S}]^{2N} W(0)[\mathcal{S}^\dagger \mathcal{S}_P^\dagger]^{2N}\} = \mathcal{E}_t[\rho(0)], \qquad (2)$$

where $W(t)$ is the full density matrix and $\mathcal{E}_t[\cdot]$ is the quantum map [1] associated with the reduced dynamics starting from a factorized state, $W(0) = \rho(0) \otimes w_E$ [17].

We may try to approximate Eq. (2) by a Bloch-Redfield master equation [18]. In this framework the environment remains in equilibrium and the map for the RDM in the first Δt has the Lindblad form $\mathcal{E}_{\Delta t}[\rho(0)] \approx \exp(\mathcal{L}\Delta t)\rho(0)$. The factorized structure of $W(t)$ is preserved if we apply pulses, so subsequent Δt can be treated in the same way. After $t = 2N\Delta t$ we get

$$\rho(t) \approx [\mathcal{P} e^{\mathcal{L}\Delta t}]^{2N} \rho(0), \qquad (3)$$

where $\mathcal{P}$ is the superoperator of the pulses. This approximation, which is correct for a weakly coupled and fast environment, yields that BB does not affect the decay of the coherence. Of course, BB decoupling is effective only in situations where memory effects are paramount, and the trace in Eq. (2) *must* be taken at the end of the protocol. In these cases we should go beyond the approximation Eq. (3). The possibility we explore is to treat part of the environment on the same footing of the system [7]. We denote with $\rho(t)$ the RDM of the *qubit plus localized level*. The system is now described by $\mathcal{H}_Q-(1/2)\sigma_z\hat{E}+\mathcal{H}_d$ and we use the same steps leading to Eq. (3). The map $\rho(t+\Delta t)=e^{\mathcal{L}\Delta t}\rho(t)$ is evaluated by a master equation [7], with $\mathcal{H}_T$ being the interaction and $\mathcal{H}_B$ the bath. The RDM of the qubit $\rho^Q=\mathrm{Tr}_d[\rho(t)]$ is obtained by tracing out the localized level *at the end* of the protocol.

We express $\rho(t)$ in the basis $|\theta_n^{\pm}\rangle=|\theta_n\pm\rangle|n\rangle$, where $|n\rangle$ ($n=0,1$) are eigenstates of $d^\dagger d$ and $|\theta_n\pm\rangle$ are the two eigenstates of $\mathcal{H}_Q-(v/2)n\sigma_z$, their energy splitting being $\Omega_n=\sqrt{(\varepsilon+nv)^2+\Delta^2}$. We denote $|a\rangle\equiv|\theta_0^+\rangle$, $|b\rangle\equiv|\theta_0^-\rangle$, $|c\rangle\equiv|\theta_1^+\rangle$ and $|d\rangle\equiv|\theta_1^-\rangle$. In Ref. [7] it was found that the impurity remains in an unpolarized state, $\mathrm{Tr}_Q\{\rho(t)\}=\Sigma_{n=0,1}p_n(t)|n\rangle\langle n|$, if initially this was the case. This simplifies the dynamics of ρ_{ij}: the only nonvanishing entries are the four populations and the coherences $\rho_{ab}(t)$ and $\rho_{cd}(t)$ (with the conjugates). Thus, we should diagonalize an 8×8 submatrix of $\mathcal{L}$. Using the representation of $\mathcal{P}$, this is enough to find the approximate map Eq. (3) for a BB protocol, at all times.

If $\Delta=0$, the calculation can be carried out analytically. In the absence of pulses $[\mathcal{H},\sigma_z]=0$, the populations of the qubit do not relax while its coherences are given by $\langle\theta_0+|\mathrm{Tr}_d[\rho(t)]|\theta_0-\rangle=\rho_{ab}(t)+\rho_{cd}(t)$. This holds true also for an *even* number of pulses. This symmetry further simplifies $\mathcal{L}$ leading to independent evolutions of populations (subscript p) and coherences (ϕ)

$$e^{\mathcal{L}t}\equiv\begin{pmatrix}e^{\mathcal{L}_p t} & 0\\ 0 & e^{\mathcal{L}_\phi t}\end{pmatrix},\quad e^{\mathcal{L}_\phi t}\equiv\begin{pmatrix}e^{\Gamma_\phi t} & 0\\ 0 & e^{\Gamma_\phi^* t}\end{pmatrix},\qquad(4)$$

where $\mathcal{L}_{p/\phi}$ are 4×4 matrices, whereas Γ_ϕ is a 2×2 matrix acting on the vector $\rho_\phi\equiv(\rho_{ab},\rho_{cd})$. The pulse $\mathcal{P}$ is also diagonal in the p-ϕ indexes. In the ϕ subspace it is given by $I\otimes\sigma_x$, which allows one to obtain the map for coherences $(\rho_\phi,\rho_\phi{}^*)$ in an echo procedure

$$[\mathcal{P}e^{\mathcal{L}_\phi\Delta t}]^2\equiv\begin{pmatrix}e^{\Gamma_\phi^*\Delta t}e^{\Gamma_\phi\Delta t} & 0\\ 0 & e^{\Gamma_\phi\Delta t}e^{\Gamma_\phi^*\Delta t}\end{pmatrix}.$$

The "diagonal form" implies that the game reduces to the two-component map $\rho_\phi(t)=[e^{\Gamma_\phi^*\Delta t}e^{\Gamma_\phi\Delta t}]^N\rho_\phi(0)$. This can be cast in a convenient form if the map $\exp(\Gamma_\phi t)$ found in Refs. [7] is represented in SU(2)

$$\rho_\phi(t)=[D/|\alpha|^2]^N e^{-\gamma N\Delta t+N\chi\sigma_{\hat{\mathcal{D}}}}\rho_\phi(0).\qquad(5)$$

Here $\alpha=[(1-iw)^2-2i\delta p_{eq}g-g^2]^{1/2}$ determines the rates of the multiexponential reduced dynamics of the qubit, the parameter $g=(\Omega_1-\Omega_0)/\gamma$ quantifies non-Gaussianity [7], δp_{eq} is the equilibrium population difference of the fluctuator, and w is related to the energy shifts produced by the band. Finally, $\vec{\mathcal{D}}(\Delta t)\equiv(\mathcal{D}_x,\mathcal{D}_y,\mathcal{D}_z)$ and the quantities $\mathcal{D}_i(\Delta t)$ are easily found from the results in Refs. [7] [e.g., $\mathcal{D}_0(\Delta t)=|\alpha|^2|\cosh(\gamma\alpha\Delta t/2)|^2+(1+g^2+w^2)|\sinh(\gamma\alpha\Delta t/2)|^2$] and determine $D(\Delta t)=[\mathcal{D}_0-|\vec{\mathcal{D}}|^2]^{1/2}$, $\chi(\Delta t)=\mathrm{arctanh}(|\vec{\mathcal{D}}|/\mathcal{D}_0)$, and $\sigma_{\hat{\mathcal{D}}}=\vec{\sigma}\cdot\vec{\mathcal{D}}/|\mathcal{D}|$.

Equation (5) allows one to discuss the dynamic decoupling of a quantum bistable fluctuator. We expect a rich physics, since this environment has distinctive features depending critically on g [7]. Fast impurities ($g<1$) behave as an equivalent environment of harmonic oscillators in dephasing the qubit, whereas for $g>1$ a different physics emerges, dominated by memory effects, and decoherence depends strongly on details of the protocols. We present (Fig. 1) the decay of the qubit coherences $\Gamma_N(t)=\ln|[\rho_{ab}(t)+\rho_{cd}(t)]/[\rho_{ab}(0)+\rho_{cd}(0)]|$ in the limit of no back action of the qubit on the impurity. This is obtained by letting $w=0$ [7]. At any fixed $\overline{t}=2N\Delta t$, $|\Gamma_N(\overline{t})|$ decreases monotonically when the pulse frequency $1/\Delta t$ increases, which shows that BB effectively suppresses RTN. For large frequencies, $1/\Delta t\gg\gamma$ (or $2N\gg\gamma t$), $|\Gamma_N(\overline{t})|$ shows universal behavior, scaling with g^2. On the other hand, for $2N\ll\gamma t$ qualitative differences in the behavior are apparent for $g<1$ and $g>1$. Notice that for intermediate frequencies $1/\Delta t\lesssim\gamma$, the regime of experimental interest, BB is still able to cancel part of the noise due to a fast fluctuator ($g<1$). For a slow fluctuator ($g>1$) BB cancels the beats [minima in $\Gamma_0(t)$, inset of Fig. 1] in the coherent dynamics [16], but besides this, it is weakly effective against slow RTN, despite of the semiclassical arguments, because there is not much to cancel. Classical RTN causes also a systematic phase error which BB does not cancel [19], but can be compensated otherwise. Notice that the limit we discuss is the exact result for classical RTN, but Eq. (5) contains also the *quantum* dynamics of the fluctuators, including the back action of the qubit. These results will be presented elsewhere [19].

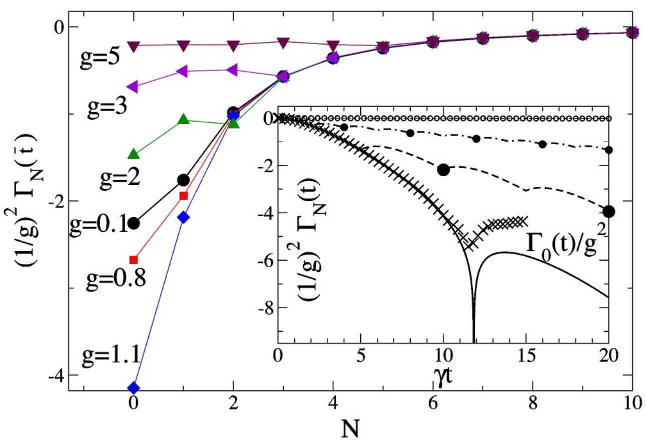

FIG. 1. For a fixed $\overline{t}=10\gamma^{-1}$ we plot $\Gamma_N(\overline{t})/g^2$ for BB procedures with N echo pair of pulses. The parameter is $g\equiv v/\gamma$. $N=0$ corresponds to free-induction decay (FID) [8,9]. A Gaussian environment with the same power spectrum would give, for arbitrary g, the curve here labeled with $g=0.1$, since $\Gamma_N(t)\propto g^2$ [3]. Inset: $\Gamma_N(t)$ for $g=1.1$ for different Δt (lines with dots, $\gamma\Delta t=5,2,0.2$) are compared with the FID $\Gamma_0(t)$ (thick line) and with results obtained by a stochastic Schrödinger equation (crosses) simulated with a very efficient piecewise deterministic algorithm [2,19]. Simulations are not accurate at relatively long times and in general they require large statistics for the process (we used 10^6 realizations).

The physics for $\Delta\neq 0$ is even richer. We study the purity $S=\ln\mathrm{Tr}(\rho^Q)^2$, which gives deviations from unitary dynamics of the qubit [1]. Efficient decoupling, $S=0$, corresponds to localization in a "Zeno subspace" [5], which is a pure state. We study BB for *generic* t and Δt by diagonalizing $\exp(\mathcal{L}t)$. The results (Fig. 2) show that for frequent pulses decoupling

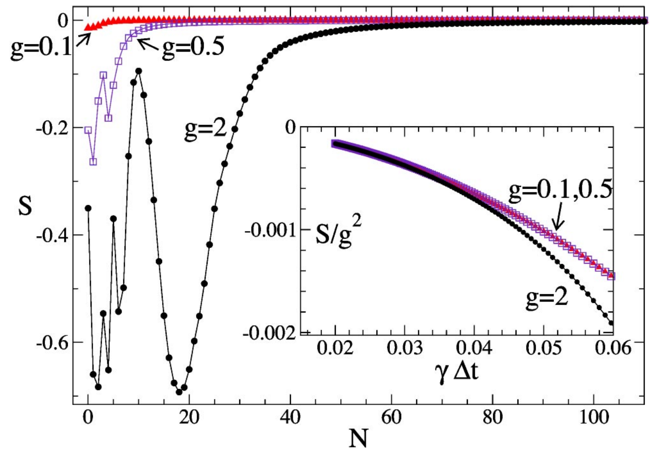

FIG. 2. The purity $S=\ln[\mathrm{Tr}(\rho^Q)^2]$ at fixed $\overline{t}=8\gamma^{-1}$ for protocols with N echo pair of pulses. The parameter is $g=(\Omega_1-\Omega_0)/\gamma$. We take $\varepsilon=\Delta$, $v/\Omega=0.2$ and start from an eigenstate of σ_x. For $g=2$ (slow fluctuator), S is nonmonotonic with N. Fast fluctuators ($g=0.5$ and $g=0.1$) show a more regular behavior. Inset: S scales as g^2 for $N\gg 1$. This regime of efficient decoupling is not easily met for slow fluctuators ($g=2$ requires $\Delta t \lesssim \gamma^{-1}/25$).

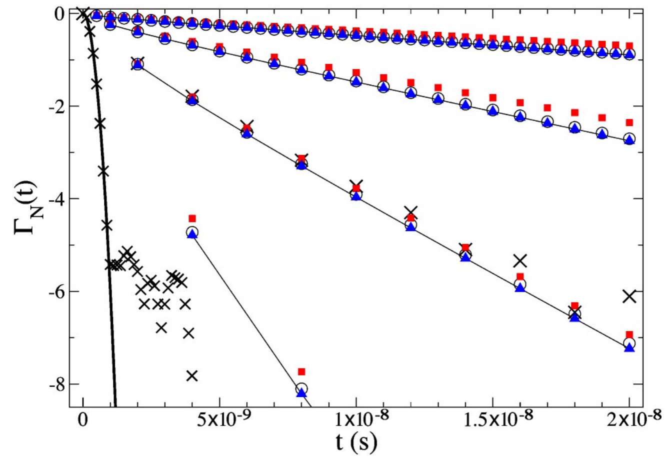

FIG. 3. BB control of $1/f$ noise for $\Delta=0$. The analytic $\Gamma_N(t)$ at $t=2N\Delta t$ (symbols-lines are guides for the eye) is compared with the evolution with no pulses (thick solid line). Noise is generated with N_{fl} fluctuators with rates γ_i distributed from 10^4 to 10^{10} Hz. Slower fluctuators are ineffective [7]. Noise level $\propto v^2 N_{fl}$ is *fixed* at a value typical of experiments in charge qubits: it is realized with coupling $v=9.23\times 10^q$ Hz, for $q=6$ (full triangles), 7 (circles), and 8 (squares), with $N_{fl}=6\times 10^{17-2q}$ scaled accordingly. Points for $q=6$ coincide with results for Gaussian noise with $1/f$ power spectrum. Crosses are stochastic Schrödinger simulations with 10^5 realizations of the $1/f$ process, for $q=7$.

is achieved, $S\approx 0$. This agrees with the results of Ref. [10] for a $g<1$ impurity. However decoupling slower impurities $g>1$, requires comparatively large N. Universal behavior, $S\sim g^2$ is again found. Instead, for a smaller N it may happen, especially for $g>1$ that S is not monotonic with N, including the possibility that the qubit decays *faster* than in the absence of pulses [3]. This is reminiscent of the *inverse* Zeno effect [13], and it is due to the complex coupled dynamics of qubit and impurity for $g>1$.

In order to treat $1/f$ noise we now extend our formalism to a "multimode" environment. We generalize the results at $\Delta=0$ of Ref. [3], to an arbitrary (non-Gaussian) environment. The Hamiltonian is of the general form Eq. (1). For the evolution between two pulses at t_1 and t_2 we use $[\mathcal{H},\sigma_z]=0$ and following the steps of Ref. [3] we obtain the evolution operator at $t=2N\Delta t$ for a BB protocol, $\mathcal{S}_{2N}(t)=[e^{-i(\mathcal{H}_E+1/2\sigma_z\hat{E})\Delta t}e^{-i(\mathcal{H}_E-1/2\sigma_z\hat{E})\Delta t}]^N$. In the overall BB procedure σ_z is conserved, so we need only off-diagonal entries of the RDM of the qubit, in the σ_z basis

$$\rho^Q_{\sigma\sigma'}(t)=\rho^Q_{\sigma\sigma'}(0)\mathrm{Tr}_E\{\mathcal{S}_{2N}(t|\sigma)w_E\mathcal{S}^\dagger_{2N}(t|\sigma')\},\qquad (6)$$

where we assumed factorized initial conditions. Here $\mathcal{S}_{2N}(t|\sigma)=\langle\sigma|\mathcal{S}_{2N}(t)|\sigma\rangle$ expresses the conditional evolution of the environment under a well defined sequence of $\sigma=\pm 1$. The trace in Eq. (6) factorizes if the environment is made of noninteracting "modes," if $\hat{E}$ are additive and if the initial w_E is factorized. If modes are oscillators, one obtains the result of Ref. [3], which has been applied to a *Gaussian* environment with $1/f$ spectral density [11]. This model may have limitations [7,10] in describing discrete noise sources of the solid state, so we study a more realistic model, the multimode version of the Hamiltonian Eq. (1), $\mathcal{H}_E\to\Sigma_\eta\mathcal{H}_{E\eta}$ and $vd^\dagger d\to\Sigma_\eta v_\eta d^\dagger_\eta d_\eta$ [7]. Each "mode" is now a single impurity and we take a distribution $\propto\gamma^{-1}$ of the individual switching rates to produce $1/f$ noise [6,7].

The contribution of each impurity to the suppression factor in Eq. (6) is calculated using the map Eq. (2). The decay of the coherences is $\Gamma_N(t)=\Sigma_\eta \ln|[\rho^{(\eta)}_{ab}(t)+\rho^{(\eta)}_{cd}(t)]/[\rho_{ab}(0)+\rho_{cd}(0)]|$, where each $\rho^{(\eta)}_{ij}(t)$ is given by Eq. (5). The results in Fig. 3 show that frequent pulses (curves with many symbols) suppress decoherence. Under pulsed control $\Gamma_N(t)$ changes from $\propto t^2$ to $\propto t$, i.e., it is described by a rate depending on Δt, as in the Zeno effect. For noise levels typical of experiments with charge qubits (Fig. 3) the pulse rate for substantial recovery is practically independent on v. Thus the criterion for efficient decoupling proposed in Ref. [20] is not effective in this regime. The situation may change if a broad distribution of couplings is considered [12]. The physics is richer for $\Delta\neq 0$ [19,20] and compensation of $1/f$ noise is nonmonotonic for decreasing Δt, as for a single impurity.

BB suppression of noise (RTN and $1/f$) due to quantum fluctuators is an example of general situations where a "structured" environment is involved. Indeed the qubit interacts mainly with the impurity, which is a "quantum filter" modulating the noise from the band. We treat this filter on the same footing of the qubit.

Universal behavior in terms of the scaling parameter g for very frequent pulses ($\Delta t\ll\gamma^{-1}$) indicates that when decoupling is effective, details of the environment are unimportant. Instead, in the experimentally relevant case of finite N ($\Delta t\gtrsim\gamma^{-1}$), the different physics of slow ($g>1$) and fast ($g<1$) fluctuators manifests itself, and may give rise to decoupling and/or to enhancement of decoherence. This picture, unexpected in the naive description of BB, is reminiscent of the inverse Zeno effect [13]. The BB scheme we discussed prevents decoherence but freezes part of the dynamics. More complicated schemes may also allow one to perform computation [21]. The rich physics we find suggests that BB may be used to extract informations on the environment, e.g., for $1/f$ noise at otherwise inaccessible frequencies. Results discussed here are exact for classical RTN and $1/f$ noise, but the formulas we presented have a broader validity: we also studied [19] the back action of the qubit on the fluctuator and $1/f$ noise at general bias point, confirming the qualitative picture of this work. We finally stress that our results apply to other sources of discrete noise, as flux or critical current noise in flux qubits.

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