Phase control of the inverse above-threshold-ionization process with few-cycle pulses

S. X. Hu* and L. A. Collins†
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
(Received 21 May 2004; published 24 September 2004)

Intense laser-induced recombination of electrons and ions has been investigated with ultrashort few-cycle pulses. Our numerical simulations demonstrate that the absolute phase of a few-cycle laser field with respect to its pulse envelope plays a crucial role in the inverse above-threshold-ionization processes. By controlling the phase of a few-cycle pulse we can maximize the multiphoton recombination probability for electrons having certain energies.

DOI: 10.1103/PhysRevA.70.035401 PACS number(s): 32.80.Wr, 34.80.Lx, 34.50.Rk

I. INTRODUCTION

Electron-ion recombination processes have been investigated for a few decades due to their fundamental importance [1]. The invention of the laser has added additional possibilities to modify such basic processes [2]. Using weak laser fields, experimentalists have observed significant enhancement for laser-assisted electron-ion recombinations [3]. However, intense laser-induced recombination (ILIR) has not received much attention until recent studies [4], which mostly focused on its resulting radiation. Our most recent investigation [5] showed that the ILIR spectrum exhibits most features of the above-threshold-ionization (ATI) process. For example, we found (1) electron-ion recombination peaks at specific energies of the injected electrons, (2) energy gaps between peaks approximately equal to an integer number of laser photons, and (3) association of the inverse ATI (IATI) phenomena with multiphoton radiative recombination processes.

Techniques of producing intense, ultrashort laser pulses have advanced considerably in the last few years. Few-cycle pulses (FCPs) as short as only a few femtoseconds can be routinely generated in laboratories [6]. Generally, such pulses are so short that they have only a few laser oscillations within their temporal envelopes. The absolute phase of the laser field with respect to the pulse envelope, which determines the actual field configuration inside the pulse, plays a crucial role in the interaction of FCPs with matter. The phase dependence of FCPs interacting with atomic gases [7] and metals [8] has recently been studied for ATI and high-harmonic generation both in theoretical studies and in experiments. Most importantly, technical advances have enabled experimentalists to measure and stabilize the absolute phase of FCPs [9]. Furthermore, using ultrashort FCPs to assist electron-ion recombination has the advantage of avoiding reionization of the formed neutral atoms.

In this Brief Report, we investigated the phase control of electron-proton recombination with intense few-cycle pulses. Our numerical results indicate that by adjusting the absolute phase of FCPs, we can maximize the multiphoton radiative recombination probability into the ground state for electrons with particular energies. We describe the calculational details in Sec. II and present our results and discussions in the subsequent section. Finally, we shall summarize the paper.

II. NUMERICAL METHOD

We consider the interaction of a free electron with a proton in the presence of a linearly polarized (along the x axis), intense, ultrashort laser pulse. The system evolution is governed by a time-dependent Schrödinger equation (TDSE) of the form (in atomic units [10])

$$i\frac{\partial\Psi(x,y|t)}{\partial t} = \left[-\frac{1}{2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) - \frac{1}{\sqrt{q_e+x^2+y^2}} + x\varepsilon(t)\right] \times\Psi(x,y|t). \tag{1}$$

We have applied the dipole approximation and considered linear polarization, which imparts a preferred direction that permits a reduction to a two-dimensional form. An electron wave packet with a momentum p_0 ($E_k \equiv p_0^2/2$) is initially located at a distance R_0 from the proton. The laser pulse has a $\sin^2$ envelope and is described by the following extensively used vector potential [11]:

$$A(t) = A_0 \sin^2(\pi t/T)\cos(\omega t+\phi) \tag{2}$$

with a peak amplitude of A_0, a pulse duration T, a frequency of ω, and an absolute phase ϕ with respect to the pulse envelope. The laser field $\varepsilon(t)$ is given by $\varepsilon(t) = -dA(t)/dt$. A two-dimensional soft-core potential [12] approximately describes the Coulomb attraction of the proton. In the absence of external fields, this potential supports a number of bound states n, designated by the wave function $\Phi_n(x,y)$. Selecting the screening parameter as $q_e = 0.63$ gives a lowest energy level of $\epsilon_1 = -0.5$ a.u., corresponding to the ground state of the hydrogen atom. The initial state consists of a Gaussian electron wave packet at a distance R_0 from the proton with momentum p_0 (which is moving along the x axis toward the proton), specifically,

$$\Psi(x,y|t=0) = N\exp\left[-\frac{(x+R_0)^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right]e^{ip_0x}, \tag{3}$$

where σ_x and σ_y represent the width in the x and y directions, respectively, and N the normalization constant. Typically, we

*Email address: suxing@lanl.gov
†Email address: lac@lanl.gov

choose $\sigma_x = \sigma_y = 5a_B$ to produce a narrow momentum distribution.

We solve the TDSE for the initial condition in Eq. (3) by applying a split-operator algorithm [13] in a spatial box of size $204.8a_B \times 51.2a_B$ with step sizes $[\Delta_x = \Delta_y]$ of $0.1a_B$. The application of absorption edges [14] in each dimension significantly reduces reflection at the boundaries. We performed one TDSE calculation for each electron wave packet characterized by its initial momentum p_0 at a specific laser phase ϕ. The recombination probability $P_1(p_0, \phi | t=T)$ to the ground state Φ_1 is calculated by projecting onto the total wave function at the end of the interaction:

$$P_1(p_0, \phi | t = T) = \int \int \Phi_1^*(x,y)\Psi(x,y|t = T)dx\, dy. \quad (4)$$

We focused on the recombination dependence of the initial electron energies and the laser phases.

III. RESULTS AND DISCUSSION

In this section, we shall discuss two means of controlling the phase between the electron-proton scattering and the external laser field: (1) by varying the initial position of the wave packet with respect to the proton for a fixed pulse form and (2) by varying the absolute phase of a few-cycle pulse for a set starting position of the wave packet.

We investigate the first type of phase control by fixing the laser phase at $\phi = 0$ and calculating the electron-proton recombination probability P_1 for different initial positions of an electron wave packet. As an example, we use a laser pulse having a duration of 5.33 fs (including only two-cycle laser oscillations), and wavelength $\lambda = 800$ nm with a peak intensity of 10^{14} W/cm^2. The electron wavepacket is assumed to have an initial momentum $p_0 = 0.38$ a.u. and starts from different initial positions R_0 as the laser pulse turns on. According to our previous analysis [5], we attribute the electron-proton recombination between this initial energy and the ground state to the $n = 13$ multiphoton process. For each initial position R_0, we solve the TDSE and calculate the recombination probability P_1 at the end of the laser pulse. The recombination probability $P_1(p_0 = 0.38, \phi = 0 | t=T)$, plotted as a function of R_0 in Fig. 1, exhibits three recombination peaks at around initial positions $R_0 = -22.5$, $R_0 = -42$, and $R_0 = -63$ bohr, respectively. Picking the maximum peak located at $R_0 = -42$ bohr, we can calculate the time required for the electron wave packet to reach the proton (i.e., approximately $t_s = |R_0|/p_0 \simeq 2.67$ fs, by neglecting the Coulomb acceleration of the proton). We thus find that t_s is exactly equal to half of the laser pulse duration ($T = 5.33$ fs) at which the pulse envelope $\sin^2(\pi t_s/T)$ has its maximum. Therefore, by starting from $R_0 = -42$ bohr the electron wave packet with $p_0 = 0.38$ a.u. has the best match with the applied laser pulse ($\phi = 0$). The electron-proton collision is thus in phase with the applied pulse, resulting in a peak probability for the laser-induced recombination. Similarly, we determine the collision times $t_s \simeq 1.4$ fs and $t_s \simeq 4.0$ fs for the other two initial conditions $R_0 = -22.5$ and $R_0 = -63$ bohr (corresponding to the other two peaks in Fig. 1), respectively. These two

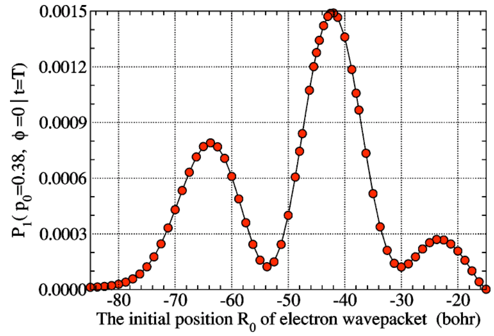

FIG. 1. The recombination probability $P_1(p_0 = 0.38, \phi = 0 | t=T)$ as a function of the initial position R_0 of the electron wave packet. The maxima in P_1 occur for cases in which the electron wave packet arrives at the target when the laser vector potential reaches a local maximum. (See text for further description.)

collision times are either earlier or later by about ~ 1.3 fs compared to the collision time $t_s = 2.67$ fs in the case of $R_0 = -42$ bohr. This time difference of 1.3 fs is approximately equal to half of the laser period; namely, for these two conditions ($R_0 = -22.5$ and $R_0 = -63$ bohr) the electron wave packet "hits" the $\pm\pi$ shifted laser field with respect to the $R_0 = -42$ bohr case. They also result in peaks though their amplitudes become smaller. Since the electron wave packet passes through the scattering center (proton) much earlier, the $R_0 = -22.5$ peak turns out even smaller in magnitude than the $R_0 = -63$ one. Furthermore, we note that the two recombination dips in Fig. 1 at around $R_0 \simeq -30$ and $R_0 \simeq -54$ bohr correspond to the cases in which the phase between the electron-ion collision and the pulse maximum is shifted by a quarter of the laser period with respect to the case of the *in-phase* central peak. Subsequently, letting the electron wave packet start from different initial positions is equivalent to controlling the timing between the electron-proton collision and the applied laser pulse envelope. This can initiate one kind of phase control for electron-ion recombination within an external laser pulse.

We discuss then the second kind of phase control for intense laser-induced recombination with FCPs, namely, directly controlling the absolute phase of an applied few-cycle pulse. As indicated above, we place the electron wave packet at such a distance $R_0 (= -p_0 \times T/2)$ that the collision happens at the maximum of the pulse envelope. We perform TDSE calculations for each chosen initial momentum p_0 with a certain laser field configuration (i.e., a certain absolute phase ϕ). By doing so, we obtain an IATI spectrum for the phase ϕ. Then, repeating the same calculations for different ϕ, we can explore the phase dependence of the IATI spectrum. Our numerical results are shown by Figs. 2(a)–2(c) for FCP phases varying from $\phi = 0°$ to $\phi = 360°$. The corresponding FCP field configurations are plotted in Figs. 3(a)–3(c), respectively.

For the case of $\phi = 0°$ indicated in the upper left-hand corner of Fig. 2(a), the three peaks located at $p_0 \simeq 0.21$, $p_0 \simeq 0.38$, and $p_0 \simeq 0.60$ a.u. correspond to the $n \simeq 12$, $n \simeq 13$, and $n \simeq 15$ multiphoton radiative recombination process, respectively. The peak associated with the $n = 14$ process disappears due to destructive interference [5]. According to our

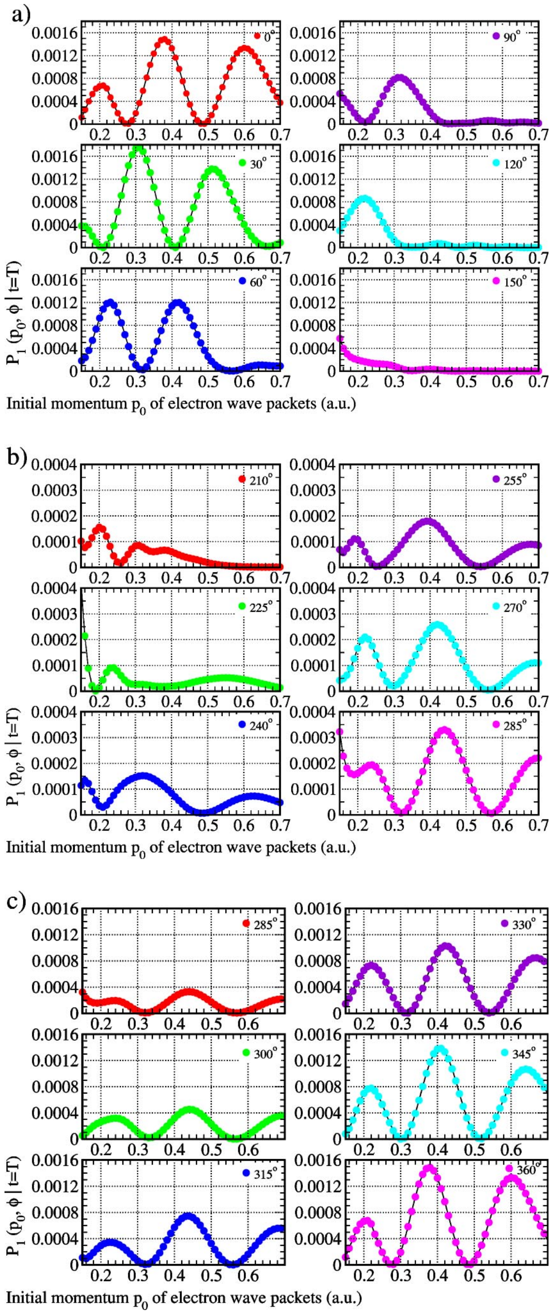

FIG. 2. The IATI spectrum, i.e., the intense laser-induced recombination probability as a function of the injected electron momentum, for different phases of FCPs varying from 0° to 360°. The FCPs corresponding to each panel are plotted in Fig. 3. For a fixed electron momentum, P_1 shows considerable sensitivity to the relative phase of the laser electric field with respect to its time envelope.

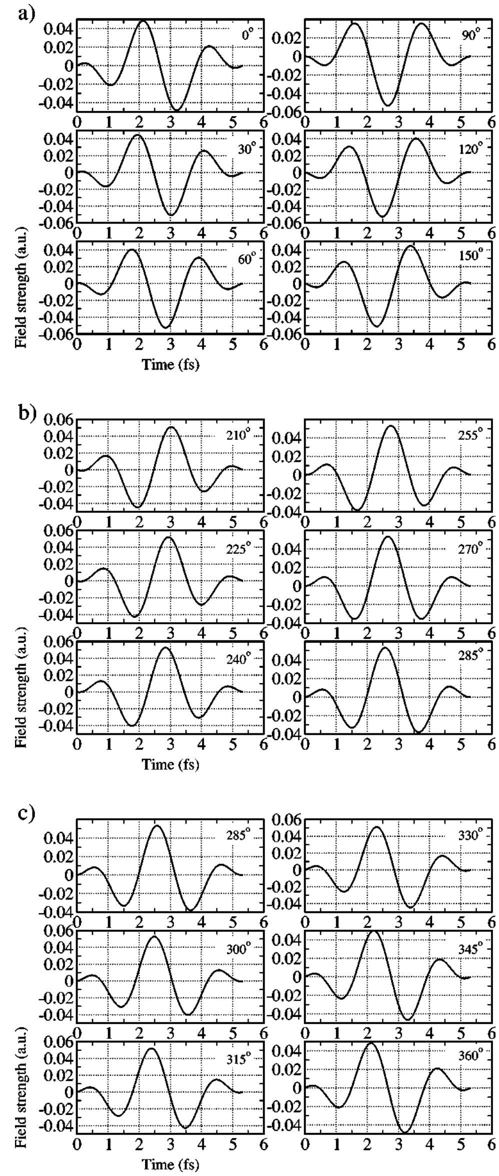

FIG. 3. The field configuration of few-cycle pulses for ϕ varying from 0° to 360°. The laser parameters are the same as used in Fig. 1.

previous analysis [5], once the following condition for a multiphoton resonance is met,

$$E_k + I_p + U_p = n\hbar\omega, \tag{5}$$

the electron, having a kinetic energy E_k, can jump to the recombined ground state of hydrogen, by emitting a high-energy $(n\hbar\omega)$ photon. Here, I_p is the ionization potential of the hydrogen atom, and U_p denotes the ponderomotive potential of a free electron in the laser field. Since the actual maximum field (ε_{max}) inside a FCP depends on the absolute phase ϕ, the "effective ponderomotive potential" U_p (calculated by the actual maximum field of FCP, i.e., $U_p = \varepsilon_{max}^2/4\omega^2$) in Eq. (5) becomes a function of ϕ. Thus, by changing the FCPs' absolute phase ϕ, we can shift the IATI resonance peaks to different electron energies. This was indicated in Fig. 2(a), in which the IATI peaks (associated with different n) move toward the low-energy side due to the increasing U_p as the absolute phase varys from ϕ=0° to ϕ=90°. To make this clearer, we take a closer look at those varying fields shown in Fig. 3(a) as ϕ changing from 0° to 90°. We found that the actual maximum field strength ε_{max} changes from −0.04832, −0.051, −0.05272, to −0.0533 a.u.; and the corresponding "effective ponderomotive potential" is varying, respectively, from U_p=4.888, 5.446, 5.820, to 5.948 eV when ϕ increases from 0°, 30°, 60°, up to 90°. Furthermore, the IATI peak magnitude decreases with increasesing ϕ. The two cases of ϕ=120° and ϕ=150° in Fig. 2(a) show smaller peaks on the high-energy side. For those phases ϕ over 180° shown as Fig. 2(b) the IATI peaks retreat from the low-energy side. For example, we see that the two broad peaks at $p_0 \sim 0.32$ and $p_0 \sim 0.62$ a.u., indicated in the ϕ=240° panel of Fig. 2(b), move back to around $p_0 \sim 0.40$ and $p_0 \sim 0.70$ a.u. in the cases of ϕ=255°, 270°, and 285°, respectively; Also a small peak at $p_0 \sim 0.22$ a.u. gradually forms as ϕ increasing. However, the IATI peak amplitudes shown in Fig. 2(b) are several or ten times smaller than that of the ϕ=0° case. This may be understood in the following way. Since the electron wave packet is assumed to start from the negative x axis and move toward the proton (i.e., with a positive initial momentum), it first encounters a positive half-cycle field (though having small strength) for those FCPs shown in Fig. 3(b). Consequently, this positive half-cycle field acts to decelerate the electron wave packet and makes it spread significantly. Thus, the resulting recombination probability decreases considerably. Figure 2(c) shows the results for FCP phase varying from ϕ=285° to ϕ=360°. We observe that as $\phi > 300°$ the low-energy IATI peaks, located at around $p_0 \sim 0.21$ and $p_0 \sim 0.40$ a.u., shift only a little; but the high-energy one moves down from $p_0 \sim 0.70$ to $p_0 \sim 0.60$ a.u. when ϕ increases. Moreover, Fig. 2(c) indicates that the IATI peak amplitudes dramatically increase by a factor of 5 for ϕ varying from 285° to 360°. Going back to Fig. 3(c), we find the first positive half-cycle field that makes the wave packet spread considerably disappears gradually. This may explain the increasing IATI peak magnitude exhibited in Fig. 2(c).

IV. SUMMARY

We have investigated phase control of the IATI process induced by intense few-cycle pulses. Our results elucidate that by timing the electron-proton collision with the applied FCPs, either by shooting the electron from different initial positions or by controlling the absolute phase of FCPs, one can maximize the electron-ion recombination probability of certain injected electron energies. This may have implications for synthesizing exotic species.

ACKNOWLEDGMENTS

This work was performed under the auspices of the U.S. Department of Energy through the Los Alamos National Laboratory under Contract No. W-7405-ENG-36.

[1] Y. Hahn, Rep. Prog. Phys. **60**, 691 (1997).
[2] E. E. Fill, Phys. Rev. Lett. **56**, 1687 (1986); A. Wolf, Hyperfine Interact. **76**, 189 (1993); F. Robicheaux and M. S. Pindzola, Phys. Rev. Lett. **79**, 2237 (1997); C. Wesdorp, F. Robicheaux, and L. D. Noordam, *ibid.* **84**, 3799 (2000); L. B. Madsen, J. P. Hansen, and L. Kocbach, *ibid.* **89**, 093202 (2002).
[3] U. Schramm *et al.*, Phys. Rev. Lett. **67**, 22 (1991); F. B. Yousif *et al.*, *ibid.* **67**, 26 (1991).
[4] M. Yu. Kuchiev and V. N. Ostrovsky, Phys. Rev. A **61**, 033414 (2000); J. Phys. B **34**, 405 (2001); D. B. Milošević and F. Ehlotzky, Phys. Rev. A **65**, 042504 (2002).
[5] S. X. Hu and L. A. Collins, Phys. Rev. A **70**, 013407 (2004).
[6] M. Nisoli *et al.*, Opt. Lett. **22**, 522 (1997); T. Brabec and F. Krausz, Rev. Mod. Phys. **72**, 545 (2000), and references therein.
[7] A. de Bohan *et al.*, Phys. Rev. Lett. **81**, 1837 (1998); G. G. Paulus *et al.*, Nature (London) **414**, 182 (2001); D. B. Milošević, G. G. Paulus, and W. Becker, Phys. Rev. Lett. **89**, 153001 (2002); S. X. Hu and A. F. Starace, Phys. Rev. A **68**, 043407 (2003).
[8] A. Apolonski *et al.*, Phys. Rev. Lett. **92**, 073902 (2004).
[9] A. Baltuska, T. Fuji, and T. Kobayashi, Phys. Rev. Lett. **88**, 133901 (2002).
[10] Energy in units of hartrees (27.21 eV), length in units of bohrs (a_B=0.052 918 nm), and time in units of 0.0242 fs.
[11] R. M. Potvliege, N. J. Kylstra, and C. J. Joachain, J. Phys. B **33**, L743 (2000).
[12] J. H. Eberly, Q. Su, and J. Javanainen, Phys. Rev. Lett. **62**, 881 (1989).
[13] M. D. Feit, J. A. Fleck, Jr., and A. Steiger, J. Comput. Phys. **47**, 412 (1982).
[14] J. L. Krause, K. J. Schafer, and K. C. Kulander, Phys. Rev. A **45**, 4998 (1992); S. X. Hu and Z. Z. Xu, *ibid.* **56**, 3916 (1997).