Anomalous electromagnetically induced transparency in photonic-band-gap materials

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The phenomenon of electromagnetically induced transparency has been studied when a four-level atom is located in a photonic band gap material. Quantum interference is introduced by driving the two upper levels of the atom with a strong pump laser field. The top level and one of the ground levels are coupled by a weak probe laser field and absorption takes place between these two states. The susceptibility due to the absorption for this transition has been calculated by using the master equation method in linear response theory. Numerical simulations are performed for the real and imaginary parts of the susceptibility for a photonic band gap material whose gap-midgap ratio is 21%. It is found that when resonance frequencies lie within the band, the medium becomes transparent under the action of the strong pump laser field. More interesting results are found when one of the resonance frequencies lies at the band edge and within the band gap. When the resonance frequency lies at the band edge, the medium becomes nontransparent even under a strong pump laser field. On the other hand, when the resonance frequency lies within the band gap, the medium becomes transparent even under a weak pump laser field. In summary, we found that the medium can be transformed from the transparent state to the nontransparent state just by changing the location of the resonance frequency. We call these two effects anomalous electromagnetically induced transparency.

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I. INTRODUCTION

Recently, there has been considerable interest in studying the phenomenon of quantum interference in atomic systems [1–9]. The interest stems from the early work of Agarwal [2] who showed that the ordinary spontaneous decay of an excited degenerate V-type three-level atom can be modified due to the interference between the two atomic transitions. Since then, many interesting results have been found such as electromagnetically induced transparency (EIT) [3], high-contrast resonance [4], lasing without inversion [5], amplification without population inversion [5,6], enhancement of the index of refraction without absorption [7], ultraslow velocities of light [8], and coherent population trapping [9].

The aim of the present paper is to study EIT in photonic band gap (PBG) and dispersive polaritonic band gap (DPBG) materials due to their unusual optical properties and potential applications [10–18]. There exists an energy gap in their photon (polariton) dispersion relations. In PBG materials, the existence of the energy gap is due to the multiple photon scattering with spatially correlated scatterers [10], while in DPBG materials such as semiconductors and dielectrics, the energy gap is caused by photons coupling with elementary excitations (excitons, optical phonons, etc.) of the media [17].

Recently, Paspalakis *et al.* and Angelakis *et al.* have studied EIT in Λ-type three-level atoms placed in a modified vacuum interacting with a laser probe field and a reservoir [14]. They assumed that the reservoir has an inverse square-root singularity in its density of states and can be a PBG material. They found that transparency can occur in steady state absorption when one of the atomic transitions is coupled to the reservoir. Here, the transparency occurs due to the singularity in the density of states. Generally, EIT occurs through the application of a pair of strong and weak laser fields [1].

John and Quang [15] have studied the self-induced transparency and the dielectric response of the impurity two-level atoms placed within an imperfect linear dielectric PBG material interacting with a probe laser field. They considered that the response dipole-dipole interaction (RDDI) is mediated by the exchange of high-energy virtual photons between the atoms. The atoms are randomly distributed and their atomic positions are modeled by means of a Gaussian distribution of the RDDI's. When the Rabi frequency associated with the probe laser field exceeds the RDDI energy, the imaginary part of the susceptibility saturates, whereas the real part of the nonlinear susceptibility remains large. Thus PBG materials may act as nearly lossless, but highly nonlinear dielectrics. For a broad range of parameters, the glassy behavior of the atomic system is dominant. They suggested that the resonant, lossless, nonlinearity associated with the glass state may lead to self-induced transparency in a PBG material.

Rostovtsev *et al.* [16] have investigated the EIT effect due to the nonlinear propagation of light waves in the forbidden region of nonlinear PBG materials. Their model PBG material consists of one-dimensional hetrostructures formed by a spatially modulated density of optically active Λ-type three-level atoms. An intense probe laser field interacts with atoms. This produces a periodic nonlinear dielectric structure which produces an energy gap in the system. The gap depends on the spatial modulation of the density of atoms and the nonlinear dielectric constant produced by these atoms. These structures are called nonlinear PBG materials. They calculated the dispersion relation of this model structure and found the EIT effect for waves with frequency lying in the forbidden range. Here EIT occurs due to the periodic nonlinear dielectric constant produced by the doped atoms.

In this paper, we study the EIT effect when noninteracting four-level atoms are located in linear dielectric PBG and DPBG materials. Quantum interference is introduced by

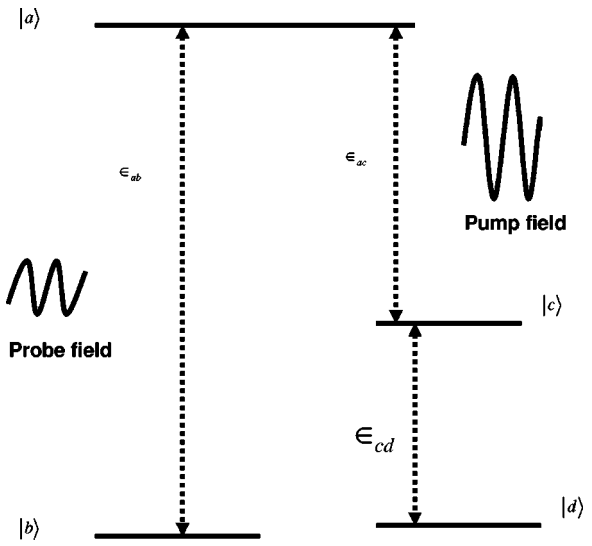

FIG. 1. Level scheme of a four-level atom. Quantum interference is introduced by driving levels $|a\rangle$ and $|c\rangle$ with a strong pump laser field and levels $|a\rangle$ and $|b\rangle$ by a probe laser field. The levels $|a\rangle$ and $|c\rangle$ decay to levels $|c\rangle$ and $|d\rangle$, respectively.

driving the atoms with a strong pump and a weak probe laser fields. The atoms are interacting with the reservoir. The susceptibility due to absorption has been calculated by using the master equation method in the linear response theory. Numerical simulations are performed for the susceptibility in PBG materials with a gap-midgap ratio of 21%. The real and imaginary parts of the susceptibility have been calculated as a function of the detuning and the Rabi frequency associated with the pump laser field. The EIT effect has been found when the resonance frequencies lie within the bands for certain decay rates and at large Rabi frequencies.

Interesting results are found when one of the resonance frequencies lies close to or at the band edge. Our numerical simulations show that in this case, the imaginary part of the susceptibility is not zero even at a large Rabi frequency associated with the pump laser field. Thus, the medium becomes nontransparent even at a large Rabi frequency. On the other hand, when the resonance frequency lies within the band gap, the imaginary part of the susceptibility becomes zero even at a low Rabi frequency and the medium becomes transparent. These two effects are called anomalous electromagnetically induced transparency (AEIT). These findings suggest that a PBG material can be transformed from a transparent to a nontransparent state just by shifting the resonance frequency from the band edge to a different location. It is important to note that the above findings are due to the presence of four-level atoms in the materials. This formulation can also be applied to DPBG materials with minor modifications stated in the theory section.

II. THEORY

We consider a four-level atom located in a PBG material and the levels of the atom are denoted by $|a\rangle$, $|b\rangle$, $|c\rangle$, and $|d\rangle$ (see Fig. 1). Quantum interference is introduced by driving the upper two levels $|a\rangle$ and $|c\rangle$ with a strong pump laser field of energy ϵ_s and Rabi frequency Ω_s. The subscript s is used for the strong pump field. Levels $|a\rangle$ and $|b\rangle$ are coupled by a probe laser field of amplitude E_p and energy ϵ_p. Here, the subscript p is used for the probe field. In Fig. 1, ϵ_{ab}, ϵ_{ac}, and ϵ_{cd} are the energy differences between levels $|a\rangle$ and $|b\rangle$, $|a\rangle$ and $|c\rangle$, and $|c\rangle$ and $|d\rangle$, respectively. Note that the quantities ϵ_{ab}, ϵ_{ac}, and ϵ_{cd} are variables and their values can be adjusted according to experiments.

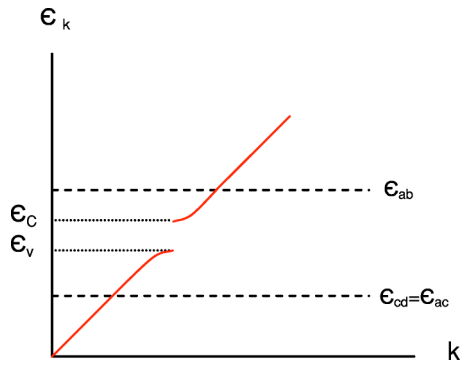

FIG. 2. Plot of the photon energy ϵ_k as a function of the wave vector k. Here, ϵ_c is the minimum energy of the upper energy band and ϵ_v is the maximum energy of the lower energy band. The resonance energies lie within the bands.

We consider a simple isotropic model for the photon dispersion relation in a three-dimensional PBG material [12]. According to the model, the photon dispersion relation is written as

$$\cos\left(\frac{4na\epsilon_k}{\hbar c}\right) = \frac{4n\cos(kL) + (1-n)^2}{(1+n)^2} \tag{1}$$

where ϵ_k and k are the photon energy and the photon wave vector, respectively. Here, n is the refractive index, a is the the radius of the scatterer, and c is the speed of light. The lattice constant is defined as $L=2a+b$, where $b=2na$. The limitations of this model have been discussed in Ref. [12]. A typical photon energy dispersion relation is shown in Fig. 2.

The minimum energy ϵ_c of the upper energy band and the maximum energy ϵ_v of the lower energy band can be obtained from Eq. (1) as

$$\epsilon_v = \frac{c\hbar}{4na}\left[\cos^{-1}\left(\frac{n^2-6n+1}{1+2n+n^2}\right)\right], \tag{2}$$

$$\epsilon_c = \frac{c\hbar}{4na}\left[2\pi - \cos^{-1}\left(\frac{n^2-6n+1}{1+2n+n^2}\right)\right]. \tag{3}$$

The energy gap lies between the energies ϵ_v and ϵ_c (see Fig. 2). Sometimes, the lower band is called the valence band and the upper band is called the conduction band.

Initially, the atoms are prepared in level $|b\rangle$. The main aim of the paper is to study the absorption and the dispersion for the transition $|b\rangle \rightarrow |a\rangle$. The other transitions are dipole forbidden. We consider that the atom is interacting with the reservoir and levels $|a\rangle$ and $|c\rangle$ decay to levels $|c\rangle$ and $|d\rangle$,

respectively. The model Hamiltonian of the system in the presence of the strong pump field, the weak probe field, and the damping is written in energy space as

$$H_S = H_A + H_{AF} + H_R + H_{AR} \tag{4}$$

where H_A corresponds to the Hamiltonian of the atom and is written as

$$H_A = \epsilon_{ab}(\sigma_{ab}^z + 1/2) + \epsilon_{ac}(\sigma_{ac}^z + 1/2) + \epsilon_{cd}(\sigma_{cd}^z + 1/2). \tag{5}$$

The σ_{ij}^z operators appearing in the equation are given as

$$\sigma_{ab}^z = |a\rangle\langle a| - |b\rangle\langle b|, \quad \sigma_{ac}^z = |a\rangle\langle a| - |c\rangle\langle c|,$$
$$\sigma_{cd}^z = |c\rangle\langle c| - |d\rangle\langle d|.$$

H_{AF} in Eq. (4) has the following form:

$$H_{AF} = -\frac{1}{2}P_{ab}E_p Z_{ab}(\epsilon_p)[\sigma_{ab}^+ e^{-i(\epsilon_p t/\hbar)} + \sigma_{ab}^- e^{i(\epsilon_p t/\hbar)}]$$
$$-\frac{\hbar}{2}\Omega_s Z_{ac}(\epsilon_s)[\sigma_{ac}^+ e^{-i(\epsilon_s t/\hbar)} + \sigma_{ac}^- e^{i(\epsilon_s t/\hbar)}]. \tag{6}$$

The first and second terms correspond to the interaction Hamiltonian between the atom and the weak probe field and the atom and the strong pump field, respectively. Here, P_{ij} is the matrix element of the polarization operator between levels $|i\rangle$ and $|j\rangle$. The Rabi angular frequency Ω_s is associated with the coupling of the field mode of energy ϵ_s to the atomic transition $|a\rangle \rightarrow |c\rangle$. The above equation is obtained in the dipole and rotating-wave approximations [17,19]. The $\sigma_{ij}^{\pm}$ operators appearing in the equation are given as

$$\sigma_{ab}^+ = |a\rangle\langle b|, \quad \sigma_{ab}^- = |b\rangle\langle a|,$$
$$\sigma_{ac}^+ = |a\rangle\langle c|, \quad \sigma_{ac}^- = |c\rangle\langle a|. \tag{7}$$

H_R in Eq. (4) is the Hamiltonian of the photon field (i.e., reservoir) and is written as

$$H_R = \int_C \frac{d\epsilon_k}{2\pi}\epsilon_k p^\dagger(\epsilon_k)p(\epsilon_k) \tag{8}$$

where $p(\epsilon_k)$ and $p^\ddagger(\epsilon_k)$ are the photon annihilation and creation operators in the energy space, respectively. The integration contour C consists of two intervals: $-\infty < \epsilon_k \leqslant \epsilon_v$ and $\epsilon_c \leqslant \epsilon_k < \infty$ [17].

Finally, the last term H_{AR} has the following form:

$$H_{AR} = -\int_C \frac{d\epsilon_k}{2\pi}\sqrt{\gamma_a}Z_{ac}(\epsilon_k)[p(\epsilon_k)\sigma_{ac}^+ e^{-i(\epsilon_k t/\hbar)}$$
$$+ p^\ddagger(\epsilon_k)\sigma_{ac}^- e^{i(\epsilon_k t/\hbar)}] - \int_C \frac{d\epsilon_q}{2\pi}\sqrt{\gamma_c}Z_{cd}(\epsilon_q)$$
$$\times[p(\epsilon_q)\sigma_{cd}^+ e^{-i(\epsilon_q t/\hbar)} + p^\ddagger(\epsilon_q)\sigma_{cd}^- e^{i(\epsilon_q t/\hbar)}] \tag{9}$$

where $\sigma_{cd}^+ = |c\rangle\langle d|$ and $\sigma_{cd}^- = |d\rangle\langle c|$. The first and second terms in the above equation are the interaction Hamiltonians between the atom and the reservoir and are responsible for the decays of $|a\rangle$ to $|c\rangle$ and $|c\rangle$ to $|d\rangle$, respectively. The above equation is obtained in the dipole and rotating-wave approximations [17,19]. Here, γ_a and γ_c are defined as [17]

$$\gamma_a = \left(\frac{4}{3\hbar^3 c^3}\epsilon_{ac}^3 P_{ac}^2\right),$$

$$\gamma_c = \left(\frac{4}{3\hbar^3 c^3}\epsilon_{cd}^3 P_{cd}^2\right). \tag{10}$$

The form factor $Z_{ij}(\epsilon)$ for PBG materials, appearing in Eqs. (6) and (9), is written as [17]

$$Z_{ij}(\epsilon_k) = \sqrt{\frac{2\pi^2\hbar c}{k^2(\epsilon_k)}\rho(\epsilon_k)} \tag{11}$$

where $\rho(\epsilon_k)$ is the photon density of states. Using Eq. (1), the density of states has been calculated. Substituting it into the above equation, the form factor brcomes

$$Z_{ij}(\epsilon_k) = \left[\frac{(a/L)(1+n)^2\sin(4na\epsilon_k/\hbar c)}{[1-\varsigma^2(\epsilon_k)]^{1/2}}\right]^{1/2} \tag{12}$$

where $\varsigma(\epsilon_k)$ is defined as

$$\varsigma(\epsilon_k) = \left[\frac{(1+n)^2\cos(4na\epsilon_k/\hbar c) - (1-n)^2}{4n}\right] \tag{13}$$

We now calculate the absorption of a photon for the transition $|b\rangle \rightarrow |a\rangle$. The absorption is related to the imaginary part of the susceptibility. Let us calculate the susceptibility of the system using linear response theory. The susceptibility, due to the polarization induced by the above transition, is expressed in terms of the density matrix element $\rho_{ab}(t)$ as [7]

$$\chi(\epsilon_p, t) = \chi' + i\chi'' = -\frac{P_{ab}\overline{\rho}_{ab}(\epsilon_p, t)}{\epsilon_0 E_p} \tag{14}$$

where $\overline{\rho}_{ab} = \rho_{ab}e^{i\epsilon_p t/\hbar}$ and ϵ_0 is the dielectric constant of the medium. Here, χ' and χ'' are the real and imaginary parts of the susceptibility, respectively. To calculate the susceptibility in Eq. (14), we need to calculate the density matrix element $\overline{\rho}_{ab}(t)$ to the lowest order in E_p. In other words, $\overline{\rho}_{ab}(t)$ is calculated using linear response theory.

The atom is prepared initially in the ground level $|b\rangle$ and this gives $\overline{\rho}_{bb}(0) = 1$. The method of the master equation is used to calculate the time-dependent density matrix elements [1,19]. The equations of motion for the density matrix elements that are required to calculate the susceptibility are written as

$$\frac{d\overline{\rho}_{ab}}{dt} = -\frac{i}{\hbar}(\epsilon_{ab} - \epsilon_p + \Xi_a)\overline{\rho}_{ab} + \left(\frac{iP_{ab}E_p}{2\hbar}\right)Z_{ab} + \frac{i}{2}\Omega_s Z_{ac}\overline{\rho}_{cb}, \tag{15}$$

$$\frac{d\overline{\rho}_{cb}}{dt} = -\frac{i}{\hbar}(\epsilon_{ab} - \epsilon_p + \Xi_c)\overline{\rho}_{cb} + \frac{i}{2}\Omega_s^* Z_{ac}\overline{\rho}_{ab}, \tag{16}$$

where $\epsilon_{ac} = \epsilon_s$ is considered. The coupling between levels $|a\rangle$ and $|c\rangle$ is large due to the strong pump field and we treat this term exactly. Therefore, we keep all orders of Ω_s in our calculations.

In Eqs. (15) and (16), Ξ_a and Ξ_c are the self-energies for the decays of $|a\rangle$ to $|c\rangle$ and $|c\rangle$ to $|d\rangle$, respectively, and are found as

$$\Xi_a = \lim_{o\to 0}\int_C \frac{\gamma_a Z_{ac}^2(\epsilon_k)}{(\epsilon_k - \epsilon_{ac}) + io}\frac{d\epsilon_k}{2\pi}, \tag{17}$$

$$\Xi_c = \lim_{o\to 0}\int_C \frac{\gamma_c Z_{cd}^2(\epsilon_q)}{(\epsilon_q - \epsilon_{cd}) + io}\frac{d\epsilon_q}{2\pi}. \tag{18}$$

Generally, the self-energy is a complex quantity [17] and is expressed as $\Xi_i = \Delta_i - i\Gamma_i$. Here, Δ_i and Γ_i are called the energy shift and the energy linewidth, respectively. One can see from Eqs. (17) and (18) that when the resonance energies lie outside the energy gap, the self-energies are complex quantities and are written as

$$\Delta_a = \mathrm{P}\int_C \frac{\gamma_a Z_{ac}^2(\epsilon_k)}{(\epsilon_k - \epsilon_{ac})}\frac{d\epsilon_k}{2\pi},$$

$$\Gamma_a = \frac{1}{2}\gamma_a Z_{ac}^2(\epsilon_{ac}), \tag{19}$$

$$\Delta_c = \mathrm{P}\int_C \frac{\gamma_c Z_{cd}^2(\epsilon_q)}{(\epsilon_q - \epsilon_{cd})}\frac{d\epsilon_q}{2\pi},$$

$$\Gamma_c = \frac{1}{2}\gamma_c Z_{cd}^2(\epsilon_{cd}), \tag{20}$$

where P stands for the principal value. On the other hand, when the resonance energies lie within the energy gap, the self-energies are real quantities and are written as

$$\Xi_a = \Delta_a = \int_C \frac{\gamma_a Z_{ac}^2(\epsilon_k)}{(\epsilon_k - \epsilon_{ac})}\frac{d\epsilon_k}{2\pi},$$

$$\Gamma_a = 0, \tag{21}$$

$$\Xi_c = \Delta_c = \int_C \frac{\gamma_c Z_{cd}^2(\epsilon_q)}{(\epsilon_q - \epsilon_{cd})}\frac{d\epsilon_q}{2\pi},$$

$$\Gamma_c = 0. \tag{22}$$

Note that for this case the linewidths of levels $|a\rangle$ and $|c\rangle$ are zero.

We use the Laplace transform method to find the density matrix element $\overline{\rho}_{ab}$ from Eqs. (15) and (16). Substituting the expression of $\overline{\rho}_{ab}$ into Eq. (14) and after some mathematical manipulation, we get the following expression for the real and the imaginary susceptibilities:

$$\chi'(\epsilon_p) = \left(\frac{\Gamma_a P_{ab}^2}{2\epsilon_0\hbar^2}\right)\left(\frac{Z_{ab}^2(\epsilon_p)\{(\Gamma_c/\Gamma_a)B - [(\delta+\Delta_c)/\Gamma_a]A\}}{[A^2+B^2]}\right), \tag{23}$$

$$\chi''(\epsilon_p) = \left(\frac{\Gamma_a P_{ab}^2}{2\epsilon_0\hbar^2}\right)\left(\frac{Z_{ab}^2(\epsilon_p)\{[(\delta+\Delta_c)/\Gamma_a]B + (\Gamma_c/\Gamma_a)A\}}{[A^2+B^2]}\right), \tag{24}$$

where A and B are given as

$$A = \left(\frac{\Gamma_a^2}{4\hbar^2}\right)\left[\left(\frac{\hbar\Omega_\mu}{\Gamma_a}\right)^2 Z_{ac}^2 + \left(\frac{4\Gamma_c}{\Gamma_a}\right) + \left(\frac{\Delta_a - \Delta_c}{\Gamma_a}\right)^2\right] - \left(\frac{\Gamma_a^2}{4\hbar^2}\right) \times\left(\frac{(2\delta + \Delta_a + \Delta_c)}{\Gamma_a}\right)^2 \tag{25}$$

$$B = \left(\frac{\Gamma_a^2}{4\hbar^2}\right)\left(\frac{\Gamma_a + \Gamma_c}{\Gamma_a}\right)\left(\frac{(2\delta + \Delta_a + \Delta_c)}{\Gamma_a}\right) - \left(\frac{\Gamma_a^2}{4\hbar^2}\right)\left(\frac{\Gamma_a - \Gamma_c}{\Gamma_a}\right) \times\left(\frac{\Delta_a - \Delta_c}{\Gamma_a}\right). \tag{26}$$

To make calculations simple without compromising any physics, we neglect the real part of the self-energy. We know that the real part the self-energy shifts the energy of a state and its values are very small compared to the energy of the state. In our calculations, we are interested in the resonance frequencies which are nothing but the difference between the two energy states. Hence, neglecting the real part of the self-energy is not going to affect the main finding of the paper. The real and imaginary parts of the susceptibility expression reduce to

$$\chi'(\epsilon_p) = \left(\frac{\Gamma_a P_{ab}^2}{2\epsilon_0\hbar}\right)\left(\frac{Z_{ab}^2(\epsilon_p)[(\Gamma_c/\Gamma_a)B - (\delta/\Gamma_a)A]}{[A^2+B^2]}\right), \tag{27}$$

$$\chi''(\epsilon_p) = \left(\frac{\Gamma_a P_{ab}^2}{2\epsilon_0\hbar}\right)\left(\frac{Z_{ab}^2(\epsilon_p)[(\delta/\Gamma_a)B + (\Gamma_c/\Gamma_a)A]}{[A^2+B^2]}\right), \tag{28}$$

where A and B are given as

$$A = \left(\frac{\Gamma_a^2}{4\hbar^2}\right)\left[\left(\frac{\hbar\Omega_\mu}{\Gamma_a}\right)^2 Z_{ac}^2 + \left(\frac{4\Gamma_c}{\Gamma_a}\right) - \left(\frac{2\delta}{\Gamma_a}\right)^2\right], \tag{29}$$

$$B = \left(\frac{\Gamma_a^2}{4\hbar^2}\right)\left[\left(\frac{\Gamma_a + \Gamma_c}{\Gamma_a}\right)\left(\frac{2\delta}{\Gamma_a}\right)\right]. \tag{30}$$

The other parameters such as α and β take the simple forms

$$\alpha = \frac{1}{2\hbar}(\Gamma_a + \Gamma_c + 2i\delta), \tag{31}$$

$$\beta = \frac{1}{2\hbar}\sqrt{Z_{ac}^2\Omega_\mu^2\hbar^2 - (\Gamma_a - \Gamma_c)^2}. \tag{32}$$

We will perform numerical simulations of these equations in the next section.

The above equations, derived for PBG materials, are also valid for DPBG materials. For the latter, the polariton dispersion relation is written as [17,20]

$$\epsilon_k^{\pm} = \frac{1}{2}\{(\epsilon_L + \hbar ck) \pm [(\epsilon_L - \hbar ck)^2 + 4\hbar ck(\epsilon_L - \epsilon_T)]^{1/2}\}, \tag{33}$$

where ϵ_L is the minimum energy of the upper polariton branch (ϵ_k^+) and ϵ_T is the maximum energy of the lower polariton branch (ϵ_k^-). The form factor $Z_{ij}(\epsilon)$, appearing in Eq. (11), is written as [17]

$$Z_{ij}(\epsilon_k) = \frac{(\epsilon_L - \epsilon_k)^2}{(\epsilon_T - \epsilon_k)^2 + \kappa^2}, \tag{34}$$

where a constant κ is introduced to account for the relaxation processes in the medium.

III. RESULTS AND DISCUSSION

There are an infinite number of possible geometries for making three-dimensional photonic crystals [13]. The following two geometries are most popular. The first type is created by taking a three-dimensional lattice and placing a sphere at each lattice point. Crystals of this type are characterized by a lattice constant, the dielectric constant of the spheres and the embedding material, and the radius of the spheres. The second type results from taking a lattice and connecting the lattice points with cylindrical columns. Structures of this type have been made by drilling a regular pattern of holes into a solid block of a dielectric. This type of crystal is characterized by the dielectric constants of the different regions, the pattern and angles of the drilling, and the radius of the holes.

A complete band gap exists, whether one embeds dielectric spheres in air or air spheres in a dielectric medium, as long as the radius of the spheres is chosen appropriately [13]. Yablonovite crystals, named after their discoverer, have been built on the microwave length scale and have the distinction of being the first three-dimensional photonic crystals with a complete band gap. Drilling holes with radius $a/L=0.23$ results in a structure whose photonic band gap has a gap-midgap ratio of 18%. The band gap lies approximately between $\epsilon_v L/2\pi\hbar c=0.5$ and $\epsilon_c L/2\pi\hbar c=0.6$.

For present calculations, we choose a PBG crystal with $n=1.2$ and $(a/L)=0.21$. From Eqs. (2) and (3), we find that the band gap lies between $\epsilon_v L/2\pi\hbar c=0.45$ and $\epsilon_c L/2\pi\hbar c=0.55$ and the gap-midgap ratio is 20%. It is important to note that the main findings of this section do not depend on the choice of crystal structure.

We define the normalized susceptibility as $\chi_n = \chi/(\epsilon_0\hbar/2\gamma_a P_{ab}^2)$, the normalized detuning energy as $\delta_n = \delta/\gamma_a$, and the normalized Rabi energy as $\Omega_n = \hbar\Omega_s/\gamma_a$. Note that the real and the imaginary parts of the susceptibility depend on the quantities γ_c/γ_a, Ω_n, and Z_{ij}. In the literature, the quantities Ω_n and γ_c/γ_a have been taken as parameters [7]. Therefore, in our calculations, we will also consider them as parameters. The form factors depend on the resonance energies. Hence, to calculate the susceptibilities, we have to choose the locations of the resonance energies within the photon dispersion relation. We found that these locations of the resonance energies in a PBG material play a very important role.

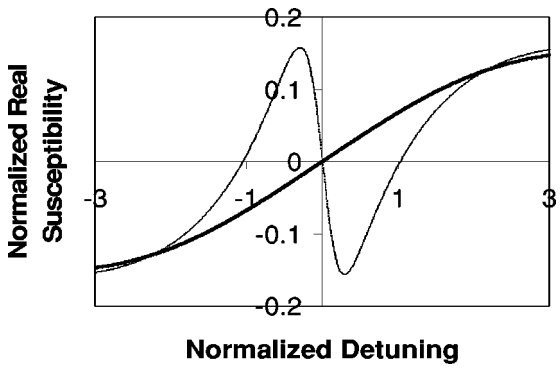

FIG. 3. Plot of the normalized real susceptibility χ_n' as a function of the normalized detuning δ_n for $(\epsilon_v-\epsilon_{cd})/\epsilon_v=(\epsilon_v-\epsilon_{ac})/\epsilon_v=0.5$, $(\epsilon_{ab}-\epsilon_c)/\epsilon_c=0.2$, and $\Omega_n=1.8$. The thick and thin solid lines correspond to $\gamma_c/\gamma_a=1$ and $\gamma_c/\gamma_a=10^{-3}$, respectively.

Let us calculate the susceptibility when the resonance energies lie within the bands. Numerical calculations for χ_n' and χ_n'' are plotted as a function of δ_n in Figs. 3 and 4, respectively for $(\epsilon_v-\epsilon_{ac})/\epsilon_v=(\epsilon_v-\epsilon_{cd})/\epsilon_v=0.5$ and $(\epsilon_{ab}-\epsilon_c)/\epsilon_c=0.2$. The thick and thin solid curves represent $\gamma_c/\gamma_a=1$ and $\gamma_c/\gamma_a=10^{-3}$, respectively. For both figures, we have taken $\Omega_n=1.8$. In Fig. 3, the real part of the susceptibility becomes zero at zero detuning for both curves. In Fig. 4, one can see that χ_n'' decreases with decreasing γ_c/γ_a and it finally becomes almost zero at $\gamma_c/\gamma_a=10^{-3}$. Thus, we find that the absorption coefficient is zero and the index of refraction is unity when χ_n' and χ_n'' have zero values. It means that the medium becomes transparent under the action of the strong pump laser field. This is known as EIT in quantum optics.

For the above case, a four-level atom gives similar results as a three-level atom. It is found that when the value of γ_c is close to γ_a, EIT occurs at a very large value of Ω_n. The value of Ω_n at which the EIT effect occurs also depends on the positions of the resonance frequencies. In other words, the EIT effect in PBG materials depends on the locations of the resonance frequencies, the Rabi frequency and ratio of γ_c and γ_a.

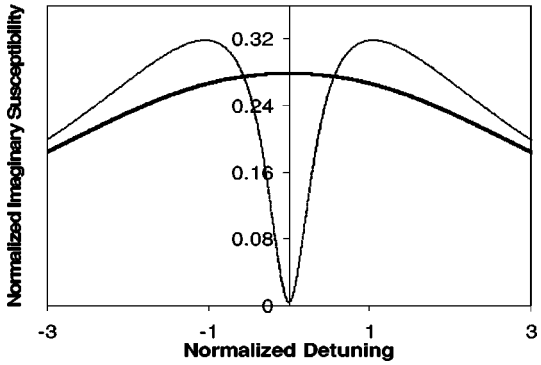

FIG. 4. Plot of the normalized imaginary susceptibility χ_n'' as a function of the normalized detuning δ_n, when $(\epsilon_v-\epsilon_{cd})/\epsilon_v=(\epsilon_v-\epsilon_{ac})/\epsilon_v=0.5$, $(\epsilon_{ab}-\epsilon_c)/\epsilon_c=0.2$, and $\Omega_n=1.8$. The thick and thin solid lines correspond to $\gamma_c/\gamma_a=1$ and $\gamma_c/\gamma_a=10^{-3}$, respectively.

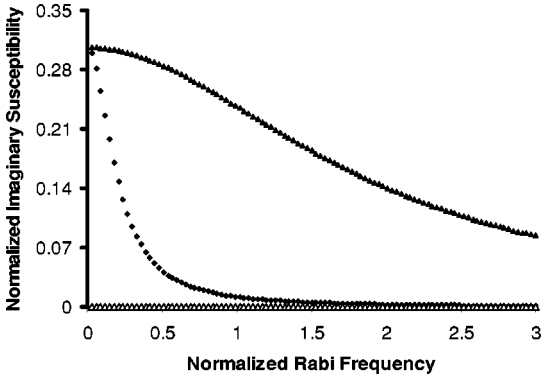

FIG. 5. Plot of the normalized imaginary susceptibility χ_n'' as a function of the normalized Rabi frequency Ω_n. The solid squares and triangles correspond to $(\epsilon_v-\epsilon_{cd})/\epsilon_v=0.5$ and $(\epsilon_v-\epsilon_{cd})/\epsilon_v=10^{-3}$, respectively. Other parameters are taken as $(\epsilon_v-\epsilon_{cd})/\epsilon_v=0.5$, $(\epsilon_{ab}-\epsilon_c)/\epsilon_c=0.2$, and $\gamma_c/\gamma_a=10^{-3}$. The curve with open triangles is found when the resonance energy ϵ_{cd} lies within the energy gap.

An interesting effect is found when the resonance frequency ϵ_{cd} lies near or at either of the band edges and the other resonance energies remain within the bands. The normalized real and imaginary susceptibilities are calculated at zero normalized detuning. It is found that the real susceptibility χ_n' is zero for all values of Ω_n. It is due to the fact that χ_n' is directly proportional to the detuning.

In Fig. 5, χ_n'' is plotted as a function of Ω_n for $(\epsilon_v-\epsilon_{cd})/\epsilon_v=0.5$ (solid squares) and $(\epsilon_v-\epsilon_{cd})/\epsilon_v=10^{-3}$ (solid triangles). Note that the former value lies in the middle of the band and the latter value lies very close to the band edge of the lower band. Other parameters are taken as $(\epsilon_v-\epsilon_{ac})/\epsilon_v=0.5$, $(\epsilon_{ab}-\epsilon_c)/\epsilon_c=0.2$ and $\gamma_c/\gamma_a=10^{-3}$. The curve with solid square decreases with increasing Ω_n and finally becomes zero for $\Omega_n>1.5$. This is the EIT effect, as discussed previously. But for the solid triangle curve, the imaginary part of the susceptibility does not become zero even at high values of Ω_n. Thus, the absorption coefficient is not zero and the index of refraction is also not unity. Similar results are also found when the resonance energy lies near the upper band edge. This means that when the resonance energy lies at either of the band edges, the medium is nontransparent even under the action of the strong pump field. This has an interesting consequence: the medium can be transformed from the transparent state to the nontransparent state just by shifting the resonance frequency from the band edge to a different location in the band.

The form factor, which includes the effect of the PBG material, plays an important role in understanding the above effect. The form factor is directly related to the density of states [see Eq. (11)]. The density of states for PBG materials has a very large value near the band edges. Hence, the form factor also has a very large value near the band edges and it has a dominating effect compared to the intensity of the pump field (i.e., Ω_n). This is why the effect of the pump field is negligible and the susceptibility does not become zero.

In principle, there must be a threshold value of the pump field intensity above which its effect is dominant again. Let us first define a threshold value of the pump field intensity. It is defined as the value of the Rabi frequency at which the imaginary susceptibility becomes almost zero. We have calculated the threshold Rabi frequency for the solid triangle curve in Fig. 5. It is found that the normalized threshold Rabi frequency is equal to about 20. The threshold value depends on the location of the resonance frequency and the ratio of γ_c and γ_a.

Another interesting effect is also found when the resonance energy ϵ_{cd} lies within the band gap. According to Eq. (22), the imaginary part of the self-energy is zero (i.e., $\Gamma_c=0$). Physically, this means that there is a photon-atom bound state for this energy [17]. For this case, we have plotted a third curve in Fig. 5 denoted by open triangles. One can see that χ_n'' is zero for all values of Ω_n. This is because χ_n'' is proportional to Γ_c. Thus, the absorption coefficient is zero and the index of refraction is unity. This means that when the resonance energy lies within the band gap, the medium becomes transparent even at a weak pump laser field. We call these two effects anomalous electromagnetically induced transparency. It is important to note that these effects are due to the presence of the fourth level in the doped atoms. According to our theory, if the pump field is absent, there will be no AEIT effect.

Finally, let us consider a case when the pump field is absent (i.e., $\Omega_n=0$). We have calculated the real and the imaginary parts of the susceptibility when the resonance energy ϵ_{ac} lies near the band edge and the other resonance energies lie within the band. At zero detuning, we found that both susceptibilities have zero values. This means that the system becomes transparent for this case too. Similar results are found in Ref. [14] where the authors used a Λ-type three-level atom.

Paspalakis *et al.* and Angelakis *et al.* have studied the EIT effect when a Λ-type three-level atom placed in a modified vacuum is interacting with a reservoir and a laser probe field [14]. They assumed that the density of states of the reservoir has a square-root singularity. They considered that the transition $|b\rangle\leftrightarrow|a\rangle$ is coupled by a probe laser field and level $|a\rangle$ is decaying spontaneously to level $|c\rangle$ through atom-reservoir interaction. The transition energy ϵ_{ac} for $|a\rangle\leftrightarrow|c\rangle$ is taken to be near the band edge. They assumed that the transition $|a\rangle\leftrightarrow|b\rangle$ is far away from the gap and treated as occurring in free space. They found EIT occurring due to the presence of the square-root singularity in the density of states.

John and Quang [15] and Rostovtsev *et al.* [16] have also studied the EIT effect in PBG materials. John and Quang considered dipole-dipole interacting two-level atoms are placed randomly in a *linear* dielectric PBG material, interacting with a probe laser field. They found that the atomic system has a glassy behavior due to the dipole-dipole interaction and that this state may lead to self-induced transparency in a PBG material. On the other hand, Rostovtsev *et al.* considered *nonlinear* PBG materials formed by the spatially modulated density of optically active Λ-type three-level atoms in a high intense probe laser field. They found EIT due to the nonlinearity in the dielectric constant.

In conclusion, we found that PBG materials can be transformed from the transparent state to the nontransparent state

just by shifting the resonance frequency from the band edge to a different location. We have also found that when the resonance frequency lies within the band gap, the medium becomes transparent even under a weak pump field.

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