Large linewidth-enhancement factor in a microchip laser

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We evidence experimentally that the linewidth-enhancement factor α can take a rather large value ($\alpha \approx 1$) for a nonsemiconductor laser, here a Nd^{3+}: YAG microchip laser. This measure is performed using an original and simple method adapted to this kind of laser and based on the variations of the laser relaxation frequency when the laser is subjected to an optical feedback.

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The linewidth-enhancement factor α is one of the key parameters to describe the behaviors of semiconductor lasers. Indeed, this factor characterizes the coupling between phase and amplitude that, for example, induces linewidth broadening in semiconductor lasers [1]. This factor plays also a crucial role in the dynamics of semiconductor lasers with external feedback [2–4] as well as of injected lasers [3,5]. Most of the studies about the α factor concern semiconductor lasers. In contrast, little is known about the case of solid-state lasers such as microchip lasers. However, recent studies evidence a large four-wave-mixing effect in a solid-state laser [6] that implies a non-negligible α factor for such lasers. Moreover, the dynamics of microchip lasers is better described by taking the α factor into account [7]. Since microchip lasers are widely used for practical applications such as imaging [8,9], laser Doppler velocimetry [10], or vibrometry [11], it is crucial to have a complete characterization of their behavior including the α factor.

In this paper, we evidence experimentally that the linewidth-enhancement factor of a microchip solid-state laser can take a non-negligible value ($\alpha \approx 1$). We performed this measure using a simple and original method. The principle of the method is to measure the phase shift between the variation of the relaxation frequency and the variation of the output intensity when the laser is subjected to a modulated round-trip time feedback. This phase shift is directly related to the factor α. Standard techniques used to determine the linewidth-enhancement factor in the case of semiconductor lasers are not easily applicable in the case of solid-state lasers due to their very different spectral and dynamical characteristics. Hence, the width of the spontaneous-emission spectrum is only three or four free spectral ranges, which makes it difficult to apply the method of Henning and Collins [12], based on net gain measurement and wavelength shift. Moreover, the relaxation frequency lies in the MHz range, which implies the use of an ultrahigh-resolution spectrometer in order to apply the method of rf modulation [13]. Our method avoids also the use of a tunable injection laser [14].

The plan of this paper is as follows. We first introduce the model of a laser with external feedback, and derive the expressions for the intensity and the relaxation frequency as a function of the α factor and the feedback parameters. Then, the experimental results obtained for a different level of feedback are compared with the theoretical predictions. The value of the amplitude-phase coupling coefficient α is then determined.

In the case of weak optical feedback, the dynamical behavior of a reinjected laser can be described by the following set of equations [9,15]:

$$\frac{dN(t)}{dt} = \gamma_1(N_0 - N) - BNE_c^2(t), \tag{1a}$$

$$\frac{dE_c(t)}{dt} = \frac{1}{2}(BN - \gamma_c)E_c(t) + \gamma_{ext}E_c(t-\tau)\cos[\phi_c(t-\tau) - \phi_c(t) - \omega\tau], \tag{1b}$$

$$\frac{d\phi_c(t)}{dt} = \omega_c - \omega + \frac{\alpha}{2}BN + \gamma_{\mathrm{ext}}\frac{E_c(t-\tau)}{E_c(t)}\sin[\phi_c(t-\tau) - \phi_c(t) - \omega\tau]. \tag{1c}$$

$N(t)$ is the population inversion with decay rate γ_1. $E_c(t)$ and $\phi_c(t)$ are the amplitude (in photon units) and the phase, respectively, of the electric field in the laser cavity; γ_c is the photon cavity decay rate. ω_c is the laser cavity frequency at the atomic transition and ω is the optical running frequency. B is the Einstein coefficient and $\gamma_1 N_0$ is the pumping rate. We have added to the model of Ref. [9] the phase-amplitude coupling parameter α [7].

The optical feedback is characterized by two parameters: (i) the photon round-trip time between the laser and an external feedback mirror, $\tau = 2d/c$ (with d the distance between laser and mirror), and (ii) the reinjection rate of the electric field of the feedback $\gamma_{\mathrm{ext}} = \gamma_c\sqrt{R_{\mathrm{eff}}}$, where R_{eff} represents the effective reflectivity of the mirror coupled to an attenuator.

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The stationary lasing conditions can be obtained from Eqs. (1) by setting the population inversion N, the amplitude E_c, and the phase ϕ_c of the electric field to be constant. The steady-state solutions in the presence of feedback are given by

$$N_s = N_{s_0}\left(1 - 2\frac{\gamma_{\text{ext}}}{\gamma_c}\cos(\omega_s\tau)\right), \tag{2a}$$

$$I_s = I_{s_0}\frac{1 + \dfrac{2}{\eta-1}\dfrac{\gamma_{\text{ext}}}{\gamma_c}\cos(\omega_s\tau)}{1 - 2\dfrac{\gamma_{\text{ext}}}{\gamma_c}\cos(\omega_s\tau)}, \tag{2b}$$

$$\omega_s = \omega_c + \frac{\alpha}{2}\gamma_c - \gamma_{\text{ext}}[\alpha\cos(\omega_s\tau) + \sin(\omega_s\tau)], \tag{2c}$$

where $N_{s_0}=\gamma_c/B$, $I_{s_0}=|E_{s_0}|^2=(\gamma_1/B)(\eta-1)$, and $\eta=N_0/N_{s_0}$ are, respectively, the steady-state values of the population inversion, intensity, and pumping rate of the laser without optical feedback.

To determine the relaxation frequency of the laser with external feedback, we proceed to a linear stability analysis of the steady-state solutions of Eqs. (2) of the system equations (1). This approach leads to the following characteristic polynomials [2,9,16]:

$$\begin{aligned} P(\lambda) = {} & \left[\lambda + \frac{2\Gamma_{R_0}}{1 - 2\dfrac{\gamma_{\text{ext}}}{\gamma_c}\cos(\omega\tau)}\right][\lambda^2 + 2\gamma_{\text{ext}}\cos(\omega\tau) \\ & \times(1 - e^{-\lambda\tau})\lambda + \gamma_{\text{ext}}^2(1 - e^{-\lambda\tau})^2] \\ & + \Omega_{R_0}^2\left[1 + \frac{2}{\eta-1}\frac{\gamma_{\text{ext}}}{\gamma_c}\cos(\omega\tau)\right] \\ & \times[\lambda + \gamma_{\text{ext}}(\cos(\omega\tau) - \alpha\sin(\omega\tau))(1 - e^{-\lambda\tau})], \end{aligned} \tag{3}$$

where $\Gamma_{R_0}=(\gamma_1\eta)/2$ and $\Omega_{R_0}=\sqrt{\gamma_1\gamma_c(\eta-1)}$ are, respectively, the damping rate and the relaxation frequency of the laser without feedback. In the case of weak optical feedback, $\gamma_{ext}\tau \ll 1$, the solution of Eq. (3) will satisfy $\lambda\tau \ll 1$, and the term $[1-\exp(-\lambda\tau)]$ can be approximated as $\lambda\tau$ [2]. Moreover, for a class B laser, $\gamma_1 \ll \gamma_c$ and the roots of Eq. (3) are

$$\lambda_1 = 0 \quad \text{and} \quad \lambda_{2,3} \approx -\Gamma \pm i\Omega_R, \tag{4}$$

$$\Gamma_R = \frac{\Gamma_0}{1 - 2\dfrac{\gamma_{\text{ext}}}{\gamma_c}\cos(\omega\tau)}, \tag{5}$$

$$\begin{aligned} \Omega_R = {} & \Omega_{R_0}\left[1 + \frac{2}{\eta-1}\frac{\gamma_{\text{ext}}}{\gamma_c}\cos(\omega\tau)\right]^{1/2} \\ & \times\left[\frac{1 + \gamma_{\text{ext}}\tau(\cos(\omega\tau) - \alpha\sin(\omega\tau))}{1 + 2\gamma_{\text{ext}}\tau\cos(\omega\tau) + \gamma_{\text{ext}}^2\tau^2}\right]^{1/2}. \end{aligned} \tag{6}$$

The trivial root λ_1 expresses the arbitrary choice of the laser phase [17]. The real part of the complex-conjugate roots $\lambda_{1,2}$ is positive for weak feedback, which means that the stationary solution is stable. Note that a Hopf bifurcation can appear under some conditions [18]. The imaginary part, Ω_R, gives the relaxation frequency and we see that it depends on the α parameter.

In the case of very weak optical feedback ($\gamma_{\text{ext}}\tau \ll 1$), the relaxation frequency and the steady-state solutions can be approximated as

$$\begin{aligned} \Omega_R \approx \Omega_{R_0}\Bigg\{1 - \frac{\gamma_{\text{ext}}\tau_0}{2}\Bigg[& \alpha\sin(\omega_s\tau) \\ & + \left(1 - \frac{2}{(\eta-1)\gamma_c\tau_0}\right)\cos(\omega_s\tau)\Bigg]\Bigg\}, \end{aligned} \tag{7a}$$

$$I_s \approx I_{s_0}\left(1 + \frac{2\eta\dfrac{\gamma_{\text{ext}}}{\gamma_c}}{\eta-1}\cos(\omega_s\tau)\right), \tag{7b}$$

$$\omega_s \approx \omega_c, \tag{7c}$$

where $\tau=\tau_0+\delta\tau$, with τ_0 the mean value of the feedback delay and $\delta\tau \ll \tau_0$.

The relative variations induced by a weak feedback of the relaxation frequency and of the intensity are

$$\begin{aligned} \frac{\Delta\Omega_R}{\Omega_{R_0}} = \left|\frac{\Omega_R - \Omega_{R_0}}{\Omega_{R_0}}\right| & = \frac{\gamma_{\text{ext}}\tau_0}{2}[\alpha\sin(\omega_c\tau) + \beta\cos(\omega_c\tau)] \\ & = \frac{\gamma_{\text{ext}}\tau_0}{2}\sqrt{\alpha^2+\beta^2}\cos(\omega_c\tau - \psi) \end{aligned} \tag{8}$$

$$\frac{\Delta I_s}{I_{s_0}} = \left|\frac{I_s - I_{s_0}}{I_{s_0}}\right| = \frac{2\eta}{\eta-1}\frac{\gamma_{\text{ext}}}{\gamma_c}\cos(\omega_c\tau), \tag{9}$$

where $\beta=1-2/(\eta-1)\gamma_c\tau_0$ and $\psi=\arctan(\alpha/\beta)$.

The quantities ΔI_s and $\Delta\Omega_R$ are periodic functions of the feedback delay τ and are phase-shifted. The phase shift is directly related to the linewidth enhancement factor α; the coefficient β depends only on the known parameters η, τ_0, and γ_c. Therefore, the value of α can be determined by varying the feedback delay $\delta\tau$ and measuring the phase shift induced between $\Delta\Omega_R$ and ΔI_s. The accuracy of the method is limited by the precicion of the measurement of $\Delta\Omega_R$ and ΔI_s.

The experimental setup is shown in Fig. 1. The laser is a Nd^{3+}:YAG microchip laser with a cavity length of 1.43 mm and lasing at 1.064 μm. The pumping laser is a 810 nm diode laser. The diode and the microchip are in the same TO3 package (Northrop-Grumman ML-00038). In typical operating conditions the output power is of the order of tens of mW. The optical feedback is provided by a mirror located at a distance d from the laser output. The mirror is mounted on a piezoelectric transducer. Variable optical density allows us to adjust the level of feedback. The output laser signal is detected by a silicon photodiode. The position of the feedback mirror d, and consequently the feedback delay τ, is slowly modulated over several wavelengths around its mean position d_0 by applying a triangular signal at 20 Hz on the piezoelectric transducer to induce a periodic variation of the

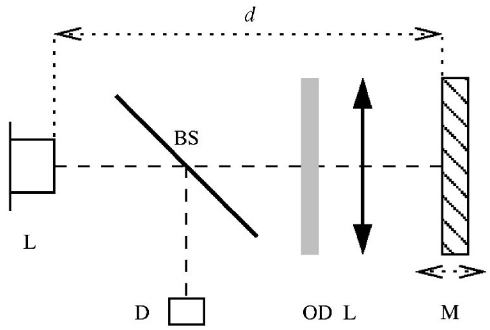

FIG. 1. Experimental setup: L, microchip laser; BS, beam splitter; OD, variable optical density; L, lens; M, moving mirror; D, detector.

relaxation frequency and of the intensity as explained in the preceding section.

A typical laser output signal obtained for very weak modulated optical feedback is shown on Fig. 2. This signal exhibits the well known quantum noise-driven relaxation oscillations that are already present without optical feedback [7]. The inset of Fig. 2 shows that the period of the relaxation oscillation is around 0.6 μm. The signal envelope is characterized by a slow periodic modulation that is induced by the modulation of the feedback mirror position. The slow varying part I_s of the signal intensity is obtained by a low pass filtering of the signal. To obtain the temporal evolution of the relaxation frequency, we proceed to the following analysis: first, we compute the output intensity variation $\Delta I(t)=I(t)-I_s$ which is a real signal of the form $\Delta I(t)=A(t)\cos[\Phi(t)]$. Second, from this real signal we construct the analytic signal $S(t)=A(t)\exp[i\Phi(t)]$, using the relation $S(t)=\Delta I+i\mathcal{H}(\Delta I)$, where $\mathcal{H}(f)$ denotes the Hilbert transform of the function f. The relaxation frequency $\Omega_R(t)$ is obtained from the derivative of the phase

$$\Omega_R(t)=\frac{d\Phi(t)}{dt}=\mathrm{Im}\left(\frac{1}{S(t)}\frac{dS(t)}{dt}\right), \tag{10}$$

where $\mathrm{Im}(z)$ denotes the imaginary part of z.

The temporal evolutions of the intensity I_s and of the relaxation frequency Ω_R are represented in Fig. 3. They correspond to sine waves as expected from Eq. (8). The phase shift between the two sine waves of Fig. 3 is better evidenced by plotting the relaxation frequency $\Delta\Omega_R/\Omega_{R_0}$ versus the intensity $\Delta I_s/I_{s_0}$ that gives the central ellipse of Fig. 4. The value of α is determined from the signals ΔI_s, $\Delta\Omega_R$ and Eqs. (8) and (9). We find $\alpha=1\pm0.2$, the parameters being $\eta=1.2$, $\tau_0=2.5\times10^{-9}$ s, and $\gamma_c=6\times10^{9}$ s^{-1}. We determine also $\gamma_{\mathrm{ext}}=2.75\times10^{7}$ s^{-1}.

When the amount of feedback is increased, the representation $\Delta\Omega_R$ versus ΔI_s becomes a deformed ellipse (see Fig. 4, external curve). In this condition, the linear approximations of Eq. (7) become less valid and the full expression of Ω_R, given by Eq. (6), as well as the complete steady-state expressions, Eq. (2), have to be taken into account. In both cases (weak and increased feedback), the theoretical curves are in good agreement with the experimental ones, Fig. 4.

At higher feedback level, we emphasize that the effect of α can be directly evidenced on the waveform of the laser average intensity I_s. Indeed, when the level of feedback is high, the approximation $\omega_s\approx\omega_c$ is no longer valid, and ω_s has to be obtained by solving the transcendental equation (6). As a result, $\omega_s\tau$ does not evolve linearly with τ and the intensity I_s is a deformed sine depending on the level of feedback and of the value of α [19]. The evolution of I_s for different values of α is displayed in Fig. 5(a), underlining the deformation of the sine with the increasing value of α. The deformation of the sine is observable only for the large value of feedback as shown theoretically in Fig. 5(b) and experi-

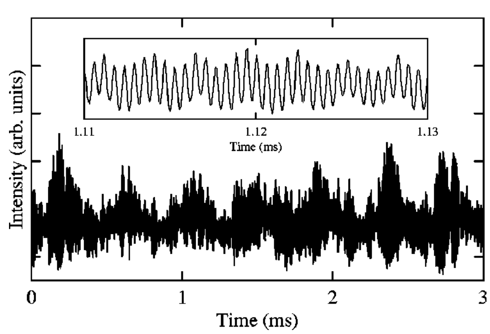

FIG. 2. Temporal output intensity obtained for very weak feedback. The insert displays a temporal zoom of the signal.

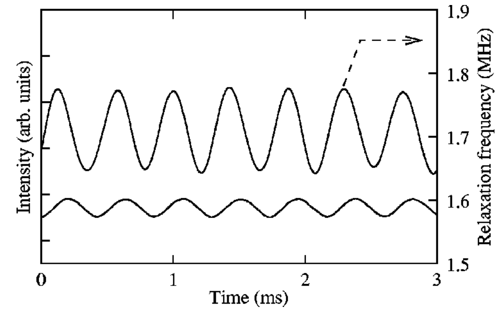

FIG. 3. Lower trace: Low pass filtered intensity I_s of the signal of Fig. 2. Upper trace: Relaxation frequency of the same signal.

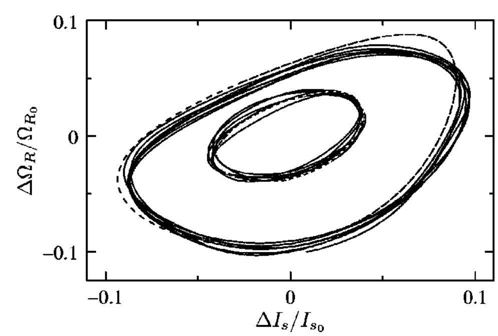

FIG. 4. Plots of the relaxation frequency variation vs the intensity variation. Solid lines: experiments, the central curve is obtained with the signal of Fig. 3; the external curve correspond to the higher feedback level. Dashed line: plot of the analytical expression (6) vs Eq. (2b) for $\gamma_{\mathrm{ext}}=2.75\times10^{7}$ s^{-1} (central curve), $\gamma_{\mathrm{ext}}=6.3\times10^{7}$ s^{-1} (external curve). The other parameters are $\eta=1.2$, $\gamma_c=6\times10^{9}$ s, $\tau_0=2.5\times10^{-9}$, $\alpha=1$.

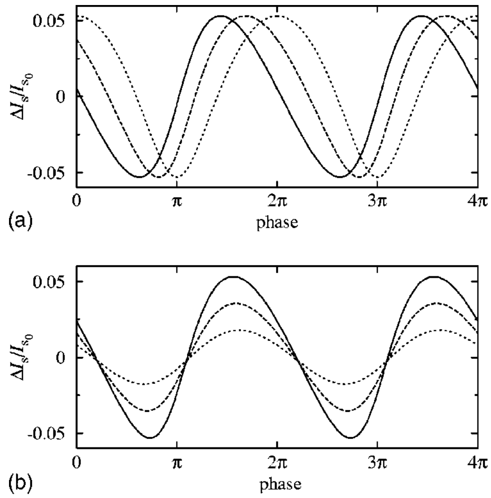

FIG. 5. Plot of the laser average intensity $\Delta I_s/I_{s_0}$ obtained from Eq. (2b) with ω_s solution of Eq. (2c) and $\eta=1.2$. (a) Evolution as a function of α: 10 (solid line), 1 (dashed line), 0 (dotted line), $\sqrt{R_{\text{eff}}}=0.3$ in the three plots. (b) Evolution as a function of $\sqrt{R_{\text{eff}}}$: 0.3 (solid line), 0.2 (dashed line), 0.1 (dotted line), $\alpha=2$ in the three plots.

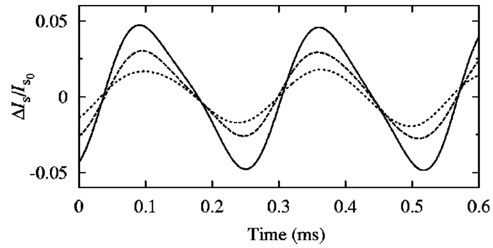

FIG. 6. Experimental evolution of the laser average intensity for three different level of feedback.

mentally in Fig. 6. In this latter case, we found a value of $\alpha \approx 2$. Note that the value of α found in the experiments can be different depending on the operating condition of the laser (temperature, pump current) but also on the condition of feedback (tilt of the mirror, level of feedback, etc.), which can change the emission wavelength and therefore α [20]. To clarify the situation, a more careful treatment of the injection (taking into account spatial dimension, for example) would be necessary, however this is beyond the scope of this paper.

In this paper, we evidence experimentally that a nonsemiconductor laser, here a Nd^{3+}:YAG microchip laser, can have a non-negligible linewidth enhancement factor (α in the range of unity). Since standard techniques used to determine the linewidth enhancement factor in the case of semiconductor lasers are not easily applicable in the case of solid-state lasers, we used an original method to measure this factor. This method consists in measuring the phase shift between the variation of the relaxation frequency and the variation of the output intensity when the laser is subjected to a modulated round-trip time feedback. In the case of large feedback, the effect of α is directly observable on the laser average intensity. Since microchip lasers are widely used in practical applications of imaging and sensing based on feedback, this work motivates further studies on the effect of α on the imaging and sensing efficiency. In addition, due to their lower relaxation frequency (in the MHz range, instead of GHz for semiconductor lasers), microchip lasers are convenient devices to study the effects of injection and optical feedback on lasers. Most of the phenomena predicted or observed for semiconductor lasers should be observable in microchip lasers.

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[1] C. H. Henry, IEEE J. Quantum Electron. **18**, 259 (1982).
[2] R. Lang and K. Kobayashi, IEEE J. Quantum Electron. **16**, 347 (1980).
[3] G. H. M. van Tartwijk and D. Lenstra, Quantum Semiclassic. Opt. **7**, 87 (1995).
[4] T. Erneux, A. Gavrielides, and M. Sciamanna, Phys. Rev. A **66**, 033809 (2002).
[5] R. Lang, IEEE J. Quantum Electron. **18**, 976 (1982).
[6] A. Brignon, L. Loiseau, C. Larat, J.-P. Huignard, and J.-P. Pocholle, Appl. Phys. B: Lasers Opt. **69**, 159 (1999).
[7] T.-S. Lim, T.-H. Yang, J.-L. Chern, and K. Otsuka, IEEE J. Quantum Electron. **37**, 1215 (2001).
[8] E. Lacot, R. Day, and F. Stoeckel, Opt. Lett. **24**, 744 (1999).
[9] E. Lacot, R. Day, J. Pinel, and F. Stoeckel, Opt. Lett. **26**, 1483 (2001).
[10] K. Otsuka, R. Kawai, Y. Asakawa, and T. Fukazawa, Opt. Lett. **24**, 1862 (1999).
[11] K. Otsuka, K. Abe, J.-Y. Ko, and T.-S. Lim, Opt. Lett. **27**, 1339 (2002).
[12] I. D. Henning and J. V. Collins, Electron. Lett. **19**, 927 (1983).
[13] C. Harder, K. Vahala, and A. Yariv, Appl. Phys. Lett. **42**, 328 (1983).
[14] K. Iiyama, K.-I. Hayashi, and Y. Ida, Appl. Phys. Lett. **17**, 1128 (1992).
[15] Y. I. Khanin, *Principles of Laser Dynamics* (Elsevier, Amsterdam, 1995), Eq. 2.73.
[16] A. Ritter and H. Haug, J. Opt. Soc. Am. B **10**, 130 (1993).
[17] E. Lacot, R. Day, and F. Stoeckel, Phys. Rev. A **64**, 043815 (2001).
[18] E. Lacot, O. Hugon, and F. Stoeckel, Phys. Rev. A **67**, 053806 (2003).
[19] G. A. Acket, D. Lenstra, A. J. Den Boef, and B. H. Verbeek, IEEE J. Quantum Electron. **20**, 1163 (1984).
[20] T. Heil, I. Fischer, and W. Elsäßer, Phys. Rev. A **60**, 634 (1999).