Nonlinear magneto-optical rotation produced by atoms near a $J=1\rightarrow J=0$ transition

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The nonlinear magneto-optical rotation in a medium consisting of $J=1\rightarrow J=0$ atoms placed in a static magnetic field is studied. The density matrix approach and irreducible atomic basis are used to describe the state of the atomic system. The stationary propagation equations for two collinear laser beams with perpendicular circular polarizations are derived and analyzed in the case of the magnetic field perpendicular to the light propagation direction. The effect of the linear polarization rotation toward the direction parallel or perpendicular to the magnetic field vector and lossless propagation of the resulting light are predicted. The conversion of the circularly polarized beam into linearly polarized one is shown. The propagation of the leading edges of switched on cw-laser beams and their stationary propagation are analyzed numerically. The dependence of the considered effects on the light detuning and on the additional magnetic field component parallel to the light propagation direction is discussed. The destructive role of the collisional relaxation is demonstrated.

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I. INTRODUCTION

The nonlinear modification of the polarization of light due to its traveling through an atomic medium placed in a magnetic field has been the subject of extensive studies, both experimental and theoretical [1–3]. Usually the nonlinear Faraday effect when the linearly polarized light propagates along the magnetic field direction is considered. The different experimental geometry in which the magnetic field is perpendicular to the direction of propagation—e.g., the Voigt effect [4]—is rarely considered.

In typical experiments the magneto-optical rotation is studied as a function of light detuning in the presence of the constant magnetic field or the change of the rotation angle as a function of magnetic field intensity for the laser light tuned to the atomic resonance is measured. Spectra obtained in such a way are valuable sources of atomic data. In general, the modification of radiation polarization due to propagation in a medium is studied.

Recently the nonlinear magneto-optical rotation is a subject of growing interest [1]. It is linked to coherent population trapping (CPT) and electromagnetically induced transparency (EIT). For example, in the atom with the states $J=1\rightarrow J=0$, CPT occurs in the absence of the magnetic field. A nonzero magnetic field parallel to the light beam direction disturbs the CPT and nonlinear polarization rotation appears [5]. In the same system the subluminal and superluminal propagation was predicted [6] under the assumption that linearly polarized pump and probe fields and the magnetic field are mutually orthogonal. It was shown that in this system one can generate a large atomic coherence to enhance the resonant nonlinear magneto-optical effect by several orders of magnitude [7].

The atomic systems $J=1/2\rightarrow J=1/2$, $J=0\rightarrow J=1$, and $J=1\rightarrow J=0$ are frequently used as realistic models for an explanation of various physical phenomena. They are the simplest four-level systems but they are usually called degenerate two-level systems. In many cases their degeneracy is not important; e.g., when the $J=1/2\rightarrow J=1/2$ atom is driven by a linearly polarized light it behaves like a two-level atom. However, in many interesting (from the physical point of view) effects the degeneracy of these atoms is very important or plays a crucial role. The magneto-optical rotation is one of such phenomena [1,2,5].

In the present paper we study the nonlinear optical rotation in the medium composed of $J=1\rightarrow J=0$ atoms. We solve the density matrix equations for the atomic state. We use the conventional expansion in a basis of multipole moments, which are represented by contractions of tensors constructed from light polarizations vectors and atomic irreducible tensor operators. We solve the Maxwell equations in the framework of the slowly varying amplitude and phase approximation to account for light propagation effects. We restrict practically our considerations to steady-state solutions of the Liouville equation for the density matrix and to stationary solutions of the propagation equations. Even in such a case the exact solutions are cumbersome or impossible to obtain. Therefore some of our results are calculated numerically.

If the collisional relaxation processes are absent, there is no Faraday effect in the $J=1\rightarrow J=0$ system. This is due to the optical pumping which transfers all atoms to a dark state. However, even very small collisional relaxation redistributes atoms among Zeeman sublevels of the ground state and the Faraday rotation appears [4,7].

When the magnetic field is perpendicular to the propagation direction—e.g., for the Voigt geometry—there is no dark state and even in the absence of the collisional relaxation the magneto-optical rotation occurs. However, as we show, during stationary propagation the linear polarization of light rotates to the final one for which the medium is transparent.

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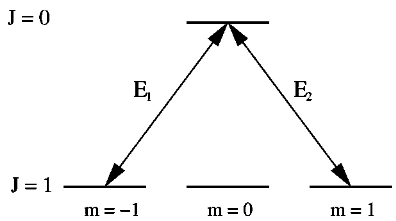

FIG. 1. The $J=1-J=0$ atomic system interacting with two circularly polarized laser beams.

Moreover, we show that circularly polarized light is converted in the course of stationary propagation into the linearly polarized one propagating without losses. We also study the influence of the detuning, nonzero longitudinal magnetic field component and the collisional relaxation on the considered nonlinear magneto-optical rotation effect.

II. THEORETICAL MODEL

We consider atoms illuminated by two collinear laser beams with perpendicular circular polarizations. Both beams have the same frequency and are resonant with the $J=1 \rightarrow J=0$ transition (see Fig. 1). This configuration can be experimentally realized for example in atomic samarium [8]. The atoms are placed in a static magnetic field. The laser electric field, propagating along the z axis, is given by

$$\vec{\mathcal{E}}(z,t) = \frac{1}{2}[E_1(z,t)\vec{\epsilon}_1 + E_2(z,t)\vec{\epsilon}_2]e^{i(\omega t-kz)} + \text{c.c.}, \tag{1}$$

where $\vec{\epsilon}_1=(1/\sqrt{2})(\hat{x}+i\hat{y})$ and $\vec{\epsilon}_2=(1/\sqrt{2})(\hat{x}-i\hat{y})$ are orthogonal circular polarization vectors. We expand the static magnetic field according to

$$\vec{B} = B_-\vec{\epsilon}_1 + B_+\vec{\epsilon}_2 + B_z\hat{z}, \tag{2}$$

where $B_\pm=(1/\sqrt{2})(B_x \pm iB_y)=(1/\sqrt{2})\sqrt{B_x^2+B_y^2}e^{\mp i\phi_B}$ and ϕ_B is the angle between the x axis and the transverse component of the magnetic field.

The interaction of the atom with the electric and magnetic fields is in the dipole approximation given by [1]

$$V = -\vec{d}\cdot\vec{\mathcal{E}} - \vec{\mu}\cdot\vec{B}, \tag{3}$$

where $\vec{d}$ and $\vec{\mu}$ are the electric and magnetic dipole moments, respectively.

The state of the atom is described by the density matrix $\rho(z,t)$ which we expand in a Liouville space basis $\{e_i\}$ of rotationally irreducible tensors [9–11] according to

$$\rho(z,t) = \sum_i \rho_i(z,t)e_i. \tag{4}$$

This basis consisting of 16 vectors, which are contractions of spherical vectors constructed from the polarization vectors and atomic operators, is given by

$$e_1 = e_{11}(00),$$
$$e_2 = e_{22}(00),$$
$$e_3 = \sum_m (-1)^m(\hat{z})_{-m}e_{11}(1m),$$
$$e_4 = \sqrt{6}\sum_m (-1)^m(\vec{\epsilon}_1,\vec{\epsilon}_2)^2_{-m}e_{11}(2m),$$
$$e_5 = e_6^\dagger = \sum_m (-1)^m(\epsilon_1,\epsilon_1)^2_{-m}e_{11}(2m),$$
$$e_7 = e_8^\dagger = \sum_m (-1)^m(\vec{\epsilon}_1)_{-m}e_{12}(1m),$$
$$e_9 = e_{10}^\dagger = \sum_m (-1)^m(\vec{\epsilon}_2)_{-m}e_{12}(1m),$$
$$e_{11} = e_{12}^\dagger = \sum_m (-1)^m(\hat{z})_{-m}e_{12}(1m),$$
$$e_{13} = e_{14}^\dagger = \sum_m (-1)^m(\vec{\epsilon}_1)_{-m}e_{11}(1m),$$
$$e_{15} = e_{16}^\dagger = \sqrt{2}\sum_m (-1)^m(\hat{z},\vec{\epsilon}_1)^2_{-m}e_{11}(2m),$$

where $(\vec{a})_m$ is the spherical component of a vector $\vec{a}$:

$$e_{ki}(Jm) = \sum_{m_i,m_k} (-1)^{J_k-m_k}C(J_iJ_kJ;m_i,-m_k,m)|J_im_i\rangle\langle J_km_k| \tag{5}$$

and

$$(\vec{a},\vec{b})^2_m = -\sum_{m_1,m_2} C(112,m_1,m_2,m)(\vec{a})_{m_1}(\vec{b})_{m_2}. \tag{6}$$

The symbol $C(J_1J_2J;m_1,m_2,m)$ denotes the Clebsch-Gordan coefficient and $|Jm\rangle$ is the atomic state from the Hilbert space.

The components of the density matrix defined by Eq. (4) have defined physical meaning. The populations of the ground and excited levels are given by $\sqrt{3}\rho_1$ and ρ_2, respectively. The atomic coherences between these levels are described by ρ_7, ρ_9, and ρ_{11} and their complex conjugates. The orientation in the ground level is given by ρ_3, ρ_{13}, and ρ_{14}. The components ρ_4, ρ_5, ρ_6, ρ_{15}, and ρ_{16} represent the alignment in the ground state.

The evolution of the density operator ρ is governed by the Liouville equation [1,10]

$$i\frac{d}{dt}\rho = \hat{L}\rho = [H,\rho] + i\hat{\Phi}\rho = (-\hat{H} + i\hat{\Phi})\rho, \tag{7}$$

where $\hat{L}$ and $H=H_A+V$ denote the Liouvillian and the Hamiltonian of the system, respectively (we put $\hbar=1$). The operator H_A stands for the atomic Hamiltonian and V represents the interaction of the atom with the light beams and magnetic field. In our approach the relaxation operator $\hat{\Phi}$ is determined phenomenologically by spontaneous damping rates and experimental collisional cross sections. We neglect the effects of atomic motion.

We construct the matrix $\mathcal{L}=-i(e_i,\hat{L}e_j)$, where $(A,B) = \text{Tr}(A^\dagger B)$ denotes the scalar product in Liouville space [10]. This matrix governs the evolution of the density matrix in the framework of the rotating-wave approximation (RWA):

$$\mathcal{L} = \begin{pmatrix} A & D \\ -D^\dagger & C \end{pmatrix}, \tag{8}$$

where

$$A=\begin{pmatrix}
0 & \frac{\gamma}{\sqrt{3}} & 0 & 0 & 0 & 0 & \frac{v_1^*}{\sqrt{3}} & \frac{v_1}{\sqrt{3}} & \frac{v_2^*}{\sqrt{3}} & \frac{v_2}{\sqrt{3}} \\
0 & -\gamma & 0 & 0 & 0 & 0 & -v_1^* & -v_1 & -v_2^* & -v_2 \\
0 & 0 & -\Gamma_1 & 0 & 0 & 0 & -\frac{v_1^*}{\sqrt{2}} & -\frac{v_1}{\sqrt{2}} & \frac{v_2^*}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} \\
0 & 0 & 0 & -\Gamma_2 & 0 & 0 & \frac{v_1^*}{\sqrt{6}} & \frac{v_1}{\sqrt{6}} & \frac{v_2^*}{\sqrt{6}} & \frac{v_2}{\sqrt{6}} \\
0 & 0 & 0 & 0 & -\Gamma_2-2ib_z & 0 & v_2^* & 0 & 0 & v_1 \\
0 & 0 & 0 & 0 & 0 & -\Gamma_2+2ib_z & 0 & v_2 & v_1^* & 0 \\
-\frac{v_1}{\sqrt{3}} & v_1 & \frac{v_1}{\sqrt{2}} & -\frac{v_1}{\sqrt{6}} & -v_2 & 0 & -\Gamma+i\Delta_- & 0 & 0 & 0 \\
-\frac{v_1^*}{\sqrt{3}} & v_1^* & \frac{v_1^*}{\sqrt{2}} & -\frac{v_1^*}{\sqrt{6}} & 0 & -v_2^* & 0 & -\Gamma-i\Delta_- & 0 & 0 \\
-\frac{v_2}{\sqrt{3}} & v_2 & -\frac{v_2}{\sqrt{2}} & -\frac{v_2}{\sqrt{6}} & 0 & -v_1 & 0 & 0 & -\Gamma+i\Delta_+ & 0 \\
-\frac{v_2^*}{\sqrt{3}} & v_2^* & -\frac{v_2^*}{\sqrt{2}} & -\frac{v_2^*}{\sqrt{6}} & -v_1^* & 0 & 0 & 0 & 0 & -\Gamma-i\Delta_+
\end{pmatrix},$$

$$D=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & ib_+ & -ib_- & 0 & 0 \\
0 & 0 & 0 & 0 & -i\sqrt{3}b_+ & i\sqrt{3}b_- \\
0 & 0 & 0 & 0 & i\sqrt{2}b_- & 0 \\
0 & 0 & 0 & 0 & 0 & -i\sqrt{2}b_+ \\
ib_- & 0 & 0 & 0 & 0 & 0 \\
0 & -ib_+ & 0 & 0 & 0 & 0 \\
-ib_+ & 0 & 0 & 0 & 0 & 0 \\
0 & ib_- & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$C=\begin{pmatrix}
-\Gamma+i\Delta & 0 & \frac{v_2}{\sqrt{2}} & -\frac{v_1}{\sqrt{2}} & -\frac{v_2}{\sqrt{2}} & -\frac{v_1}{\sqrt{2}} \\
0 & -\Gamma-i\Delta & -\frac{v_1^*}{\sqrt{2}} & \frac{v_2^*}{\sqrt{2}} & -\frac{v_1^*}{\sqrt{2}} & -\frac{v_2^*}{\sqrt{2}} \\
-\frac{v_2^*}{\sqrt{2}} & \frac{v_1}{\sqrt{2}} & -ib_z-\Gamma_1 & 0 & 0 & 0 \\
\frac{v_1^*}{\sqrt{2}} & -\frac{v_2}{\sqrt{2}} & 0 & ib_z-\Gamma_1 & 0 & 0 \\
\frac{v_2^*}{\sqrt{2}} & \frac{v_1}{\sqrt{2}} & 0 & 0 & -ib_z-\Gamma_2 & 0 \\
\frac{v_1^*}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & 0 & 0 & 0 & ib_z-\Gamma_2
\end{pmatrix}.$$

Here $\Gamma=\gamma/2+\gamma_{coll}$ denotes the perpendicular relaxation rate. The spontaneous relaxation rate is denoted by γ and γ_{coll} is the collisional dephasing rate. The detuning of the driving fields from the atomic transition is given by $\Delta=\omega-\omega_{21}$ and $\Delta_{\pm}=\Delta\pm b_z$. The collisional damping of the orientation and alignment in the ground state are described by the rates Γ_1 and Γ_2, respectively. The couplings of the light beams with the atom are given by $v_i=\vec{d}\cdot\vec{E}_i/\sqrt{3}$, $i=1,2$, and the coupling with the magnetic field is given by $b_{\pm,z}=\mu_0 g B_{\pm,z}$ (μ_0 is the Bohr magneton and g denotes the Lande factor).

In general the amplitudes E_1 are E_2 are complex-valued functions and they can be expressed in the exponential form

$$E_i = E_{0i}e^{i\phi_i}, \quad i=1,2. \tag{9}$$

The total electric field can be rewritten as

$$E_1\vec{\epsilon}_1 + E_2\vec{\epsilon}_2 = 2E_{01}e^{i\phi_+}\vec{\epsilon}_{12} + (E_{02}-E_{01})\vec{\epsilon}_2 e^{i\phi_2}, \tag{10}$$

where $\phi_+=(\phi_1+\phi_2)/2$, $\vec{\epsilon}_{12}=\vec{x}\cos[(\phi_1-\phi_2)/2]-\vec{y}\sin[(\phi_1-\phi_2)/2]$. In other words, the total electric field being generally elliptically polarized can be treated as a sum of linearly and circularly polarized components.

The electric field of the light induces the medium polarization $\vec{P}$ which is described by the components of the density matrix ρ representing the atomic coherences:

$$\begin{aligned}\vec{P} &= (P_1\vec{\epsilon}_1 + P_2\vec{\epsilon}_2)e^{i(\omega t-kz)} + \text{c.c.}\\ &= \frac{Nd^2}{3}(\rho_{v_1}\vec{\epsilon}_1 + \rho_{v_2}\vec{\epsilon}_2)e^{i(\omega t-kz)} + \text{c.c.},\end{aligned} \tag{11}$$

where N is the density of atoms, $\rho_{v_1}=\rho_7$ and $\rho_{v_2}=\rho_9$.

The propagation of the fields $\vec{E}_1$ and $\vec{E}_2$ can be treated independently and the respective propagation equations in the framework of the slowly varying amplitude and phase approximation are as follows:

$$\frac{dv_i(z,\tau)}{dz} = \frac{2\pi\omega}{c}P_i(z,\tau) = \alpha\rho_{v_i}(z,\tau), \tag{12}$$

where for simplicity we use the couplings instead of the field amplitudes, $i=1,2$, and $\alpha=(2\pi/3c)N\omega d^2$. The retarded time is denoted by $\tau=t-z/c$.

III. RESONANT STATIONARY PROPAGATION

Let us assume that both propagating cw-laser beams are in resonance with the atomic transition ($\Delta=0$). Moreover, we assume that the light interacts with the medium being in the steady state [12–14]. It means that the intensities of both circular components are functions of the distance only—i.e., $E_i=E_i(z)$, $i=1,2$. One can imagine that two collinear cw-laser beams enter the medium. The leading edge of the field drives the atoms to the steady steady in the time much greater than the atomic lifetime $1/\gamma$ and then the light propagates in the medium being in the steady state.

The steady-state density matrix is the solution of the steady-state Liouville equation $\hat{L}\rho=0$ [see Eq. (7)] with the normalization $\text{Tr}\rho=\sqrt{3}\rho_1+\rho_2=1$. We assume that the static magnetic field is perpendicular to the light propagation direction ($B_z=0$) and that collisional effects can be neglected ($\Gamma=\gamma/2$, $\Gamma_1=\Gamma_2=0$). Since we are interested in the light propagation, we present only the density matrix components ρ_{v_1} and ρ_{v_2}:

$$\rho_{v_1} = -\frac{4\mathcal{A}}{b^2\mathcal{B}}, \tag{13}$$

$$\mathcal{A} = \gamma b_-(3b_-v_1^* + b_+v_2^*)(b_+^2v_1^2 - b_-^2v_2^2),$$

$$\begin{aligned}\mathcal{B} = {} & 8(v_{01}^2 + v_{02}^2 + 2v_{01}v_{02}\cos 2\phi_-)[2(v_{01}^2+v_{02}^2-b^2)^2\\ & + 3b^2(v_{01}^2+v_{02}^2-2v_{01}v_{02}\cos 2\phi_-)] + b^2\gamma^2[5(v_{01}^2+v_{02}^2)\\ & + 6v_{01}v_{02}\cos 2\phi_-],\end{aligned}$$

$$\rho_{v_2} = \rho_{v_1}(v_1\to v_2, v_2\to v_1, b_{\pm}\to -b_{\mp}), \tag{14}$$

where $v_{0i}=|v_i|$, $i=1,2$, $b=\sqrt{2}|b_{\pm}|$, and $\phi_-=(\phi_1-\phi_2)/2-\phi_B$. Note that $\mathcal{B}$ is always positive. Obviously, when the magnetic field is absent, the atoms are in the dark state due to the optical pumping and the medium is transparent for the laser beams.

We derive from Eq. (12) equations describing propagation of the moduli and phases of the respective laser fields,

$$\frac{dv_{0i}}{dz} = \alpha\,\text{Re}[\rho_{v_i}e^{-i\phi_i}], \tag{15}$$

$$v_{0i}\frac{d\phi_i}{dz} = \alpha\,\text{Im}[\rho_{v_i}e^{-i\phi_i}], \tag{16}$$

and we rewrite them using new variables

$$v_{\pm} = v_{01} \pm v_{02} \tag{17}$$

and the phases ϕ_+ and ϕ_-. Finally we obtain

$$\begin{aligned}\frac{dv_-}{dz} = {} & -\alpha\frac{\gamma b^2}{4\mathcal{B}}[(9+4\cos 2\phi_- + 3\cos 4\phi_-)v_+^2\\ & + 6\sin^2(2\phi_-)v_-^2]v_-,\end{aligned} \tag{18}$$

$$\begin{aligned}\frac{dv_+}{dz} = {} & -\alpha\frac{\gamma b^2}{4\mathcal{B}}[(9-4\cos 2\phi_- + 3\cos 4\phi_-)v_-^2\\ & + 6\sin^2(2\phi_-)v_+^2]v_+,\end{aligned} \tag{19}$$

$$\frac{d\phi_-}{dz} = \alpha\frac{\gamma b^2}{2\mathcal{B}}\frac{(v_+^2+v_-^2)^2}{v_+^2-v_-^2}\mathcal{D}, \tag{20}$$

$$\frac{d\phi_+}{dz} = -\alpha\frac{\gamma b^2}{\mathcal{B}}\frac{v_+v_-(v_+^2+v_-^2)}{v_+^2-v_-^2}\mathcal{D}, \tag{21}$$

where

$$\mathcal{D} = -\left(1 + 3\frac{v_+^2-v_-^2}{v_+^2+v_-^2}\cos 2\phi_-\right)\sin 2\phi_-. \tag{22}$$

Since the right-hand sides of Eqs. (18)–(21) do not depend on the mean phase ϕ_+, its stationary propagation is

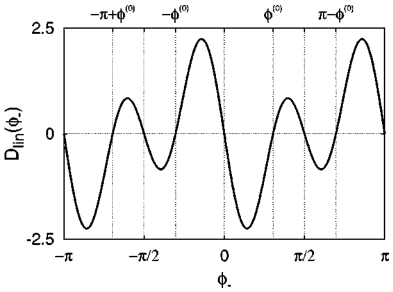

FIG. 2. The function $\mathcal{D}_{lin}$ Eq. (23), in the interval $[-\pi,\pi]$.

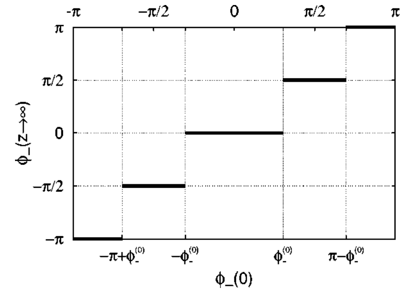

FIG. 3. The asymptotic relative phase difference $\phi_-(z\to\infty)$ versus the initial one $\phi_-(0)$ for beams giving linearly polarized light.

described by the amplitudes v_+ and v_- and by the relative phase ϕ_-. Therefore we restrict our considerations to Eqs. (18)–(20).

The analysis of the propagation equation for v_- [see Eq. (18)] shows that this difference of the amplitudes tends to zero. It means that the intensities of both fields equalize and the light polarization becomes linear along the direction described by the phase difference [see Eq. (10)].

The sign of the derivative $d\phi_-/dz$ depends on the function $\mathcal{D}$ which for the laser beams giving linearly polarized light ($v_-=0$) is a function of ϕ_- only:

$$\mathcal{D}_{lin}=\mathcal{D}(v_-=0)=-(1+3\cos 2\phi_-)\sin 2\phi_-. \tag{23}$$

This function has eight roots in the interval $[-\pi,\pi)$ (see Fig. 2):

$$\mathcal{D}_{lin}(\phi_-)=0 \ \text{ for } \begin{cases}\phi_-=\dfrac{k\pi}{2}, & k=\pm 1,\pm 2\\ \phi_-=\pm\phi_-^{(0)}, & \pm(\pi-\phi_-^{(0)}),\end{cases} \tag{24}$$

where $\phi_-^{(0)}\approx 0.304\pi$.

The character of the stationary propagation of the linearly polarized light is determined by the function $\mathcal{D}_{lin}$. It is easy to notice that the polarization vector rotates until it becomes parallel or perpendicular to the direction of the static transverse magnetic field. If we assume that the linearly polarized light enters the medium, the final polarization direction is related to the initial relative phase difference $\phi_-(z=0)$ in the following way (see Fig. 3):

$$\lim_{z\to\infty}\phi_-=\begin{cases}-\pi & \text{for } -\pi<\phi_-(0)<-\pi+\phi_-^{(0)},\\ -\dfrac{\pi}{2} & \text{for } -\pi+\phi_-^{(0)}<\phi_-(0)<-\phi_-^{(0)},\\ 0 & \text{for } -\phi_-^{(0)}<\phi_-(0)<\phi_-^{(0)},\\ \dfrac{\pi}{2} & \text{for } \phi_-^{(0)}<\phi_-(0)<\pi-\phi_-^{(0)},\\ \pi & \text{for } \pi-\phi_-^{(0)}<\phi_-(0)<\pi.\end{cases} \tag{25}$$

The sum of the amplitudes v_+ also decreases in the course of the propagation [see Eq. (19)]. However, if the relative phase ϕ_- achieves its asymptotic value and both propagating fields have equal amplitudes ($v_-=0$), one can expect the lossless propagation—i.e., $v_+=\text{const}$.

Let us assume that only one circularly polarized beam enters the medium $[v_2(z=0)=0]$. In such a case an atomic coherence

$$\rho_{v_2}=\frac{\gamma b^2 v_{01}e^{i(\phi_1-2\phi_B)}}{8[2(v_{01}^2-b^2)^2+3b^2v_{01}^2]+5\gamma^2b^2} \tag{26}$$

appears for $z=0$ [see Eq. (14)]. According to Eq. (12) this coherence is the source of the circularly polarized light described by the coupling v_2 and having an initial phase $\phi_2(0)=\phi_1(0)-2\phi_B$—i.e., $\phi_-(0)=0$. It should be stressed that this coherence is created only when the transverse magnetic field is present $(b\neq 0)$.

Since the amplitudes of both circularly polarized fields equalize during stationary propagation [compare Eq. (18)], circularly polarized light entering the medium is converted into light polarized linearly in the direction parallel to the transverse magnetic field.

IV. NUMERICAL SIMULATION

A. Nonstationary propagation

In order to verify obtained results we have solved the set of equations (7) and (12) numerically. We have implemented the method described in [15]. We assume that initially all atoms of the medium are in the ground state with equally populated Zeeman sublevels; i.e., only $\rho_1=1/\sqrt{3}$ is nonzero. Two circularly polarized laser beams propagate through this medium. Their polarizations are orthogonal. The medium is placed in the static magnetic field $\vec{B}$ perpendicular to the propagation direction. The time and distance are measured in γ^{-1} and $z_0=\alpha^{-1}$ units, respectively. The rest of necessary parameters are related to the spontaneous decay rate γ.

The beams with the envelopes, which are represented by the function

$$f(\tau)=\begin{cases}\dfrac{v_0}{2}\left[1-\cos\left(\dfrac{\tau}{\Delta\tau}\pi\right)\right] & \text{for } 0\leqslant\tau\leqslant\Delta\tau,\\ v_0 & \text{for } \tau\geqslant\Delta\tau,\end{cases} \tag{27}$$

enter the medium at $z=0$. The parameters v_0 and $\Delta\tau$ describe the final amplitude and the switching on time of the laser beam, respectively. Obviously the leading edge of such an envelope is absorbed in the course of the propagation but later the field amplitude stabilizes (see Fig. 4) and its change is described by the stationary propagation equations

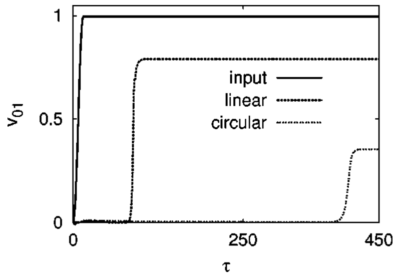

FIG. 4. The envelope of the field v_{01} for $z=0$ and $z=200z_0$. The solid line: the envelope of the incident light. The dotted line: the final envelope in the absence of the incident field $v_2(z=0)=0$. The dashed line: the final envelope in the presence of the incident field with $v_{01}(z=0)=v_{02}(z=0)$ and $\phi_1(z=0)=\phi_2(z=0)=0$. The static magnetic field is characterized by $b=2\gamma$ and $\phi_B=\pi/6$. The switching on time $\Delta\tau=10/\gamma$.

(18)–(21). Since we have chosen a relative weak input field with $v_{01}=\gamma$ for our simulation, the leading edge of the beam behaves relatively smoothly during propagation. For stronger incident light damped oscillations, due to the Rabi oscillations, appear on this edge. However, the stationary regime is achieved approximately at the same time. If we use much weaker beam, we have to wait much longer for the steady state of the medium atoms.

Let us consider two cases. In the first one the field v_1 enters the medium in the presence of the field $v_2(z=0)=v_1^*(z=0)$; i.e., both components form the linearly polarized beam. In the second case $v_2^*(z=0)=0$; i.e., the incident light is circularly polarized. As is shown in Fig. 4 the component v_1 of the linearly polarized incident beam has at the same distance much greater amplitude than this field in the case when the incident beam is circularly polarized. In the second case the orthogonal circular field appears and increases due to the energy transfer from the field v_1 and the stationary amplitudes of these two fields are practically equal for the distance $z=200z_0$. As it is expected the resulting light is linearly polarized.

B. Stationary propagation

The process of conversion of the circularly polarized light into linearly polarized one in the course of the stationary propagation is presented in Fig. 5 where we have plotted the stationary amplitudes of both circularly polarized fields as

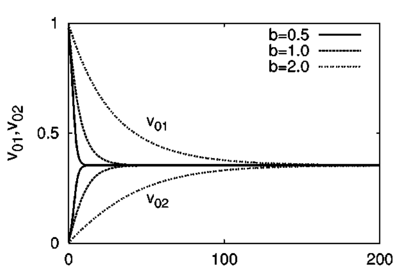

FIG. 5. The stationary propagation of the amplitudes v_{01} and v_{02} when only the field v_1 with the phase $\phi_1=\pi/18$ enters the medium. The direction of the static magnetic is defined by $\phi_B=\pi/6$.

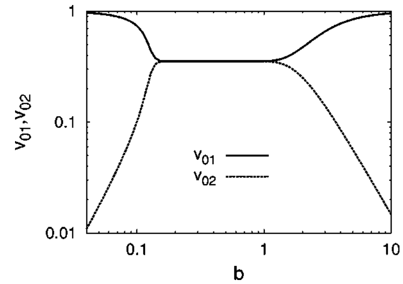

FIG. 6. The amplitudes v_{01} and v_{02} vs magnetic field for $z=50z_0$. Only the beam v_1 enters the medium.

functions of the propagation distance for three values of the magnetic field. Only the field v_1 enters the medium but later the field v_2 grows up and finally these two fields have equal amplitudes. The phase ϕ_- is always equal to zero.

In such a case we can derive from Eqs. (18) and (19) the following constant of motion:

$$2v_+^2(z) - v_-^2(z) = \text{const}. \tag{28}$$

Since the function $v_-(z)$ monotonically tends to zero and as we assume only the field v_1 enters the medium, the asymptotic values of the amplitudes of both circularly polarized beams are equal (see Figs. 4 and 5):

$$v_{01}(z \to \infty) = v_{02}(z \to \infty) = \frac{1}{2\sqrt{2}} v_{01}(z=0). \tag{29}$$

The role of the magnetic field is more complicated than it is suggested by the results presented in Fig. 5, where it is shown that the propagation distance necessary for the equalization of both circularly polarized fields increases with the magnetic field. In Fig. 6 we present the beam amplitudes as functions of the static magnetic field intensity for the fixed propagation distance $z=50z_0$ when only one circularly polarized beam enters the medium. It is clear that for this distance the amplitude equalization is practically finished in the intermediate range of the magnetic fields (plateau in Fig. 6). Outside this interval this process is much slower.

We can look at these last results from the other point of view. Let us assume that the length of medium is finite. Changing the intensity of the magnetic field we can control the intensities of both beams. In this way we can influence the polarization of the resulting field.

In the course of the propagation the amplitudes of both circularly polarized components of the linearly polarized light behave in the same way—i.e., $v_-(z)=0$ (see Fig. 7). Therefore only the rotation of the polarization direction occurs; i.e., $\phi_-(z)$ tends to zero. For sufficiently large distances the propagation is practically lossless. The intensity of such a beam depends on the initial light intensity for $z=0$ and on the initial relative phase $\phi_-(z=0)$.

Using Eqs. (19) and (20) we derive the following constant of motion in the case of linearly polarized light:

$$v_+(z)\sqrt{1 + 3\cos 2\phi_-(z)} = \text{const}, \tag{30}$$

from which we obtain

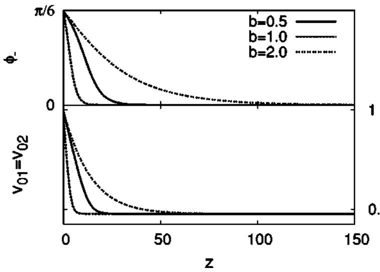

FIG. 7. The stationary propagation of the linearly polarized laser beam ($v_1 = v_2^*$) with initial phase $\phi_- = \pi/6$. The direction of the static magnetic field is characterized by $\phi_B = \pi/6$.

$$v_+(z) = v_+(0)\sqrt{\frac{1+3\cos[2\phi_-(0)]}{1+3\cos[2\phi_-(z)]}}. \tag{31}$$

Asymptotically the phase $\phi_-(z)$ (for $z \to \infty$) tends to the one of four values given by Eq. (25) which depends on $\phi_-(0)$ (see also Fig. 3). Therefore the amplitude of the losslessly propagating linearly polarized light is determined by these values according to Eq. (31) (see Fig. 8). The incident light energy losses are biggest for $\phi_-(0)$ close to the zeros of the function D_{lin}. Obviously, this light is not influenced by the stationary propagation if the phase $\phi_-(0)$ is equal to one of its four possible asymptotic values.

Also the final direction of the linear polarization depends on the initial value of the relative phase $\phi_-(0)$ (see Fig. 9). When the input phase $|\phi_-(0)|$ is less than $\phi_-^{(0)}$ the linear polarization vector has the same direction as the magnetic field. But when $\phi_-(0)$ passes $\phi_-^{(0)}$ the final polarization direction suddenly changes orientation to the perpendicular one. This effect repeats for the bigger values of the initial relative phase $\phi_-(0)$. It should be noted that to orient its polarization in direction parallel to the magnetic field, the light has to travel much longer distance than to orient itself perpendicularly.

In next subsections we analyze the influence of additional factors: the light detuning, the longitudinal magnetic field, and the collisional relaxation on the magneto-optical rotation in the $J=1 \to J=0$ system in the presence of the transverse magnetic field.

C. $\Delta \neq 0$

If linearly polarized light is slightly detuned from the resonance and the magnetic field is parallel to the light

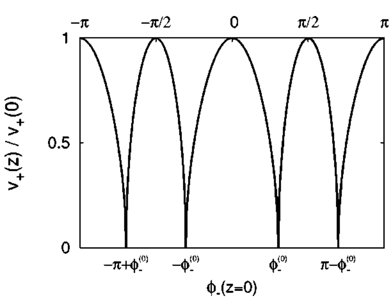

FIG. 8. The relative sum of the amplitudes of the stationary and losslessly propagating linearly polarized light vs the relative phase $\phi_-(z=0)$ [see Eq. (31)].

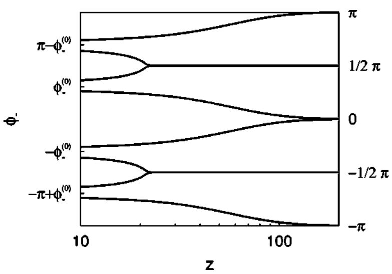

FIG. 9. The stationary propagation of the relative phase ϕ_- for the initial relative phases shifted from the zeros of the function D_{lin} by $\pm\pi/18$.

propagation direction, the Macaluso-Corbino effect can be observed [2]. As a result the incident linearly polarized light evolves into elliptically polarized one.

When the magnetic field is absent self-rotation of resonant elliptically polarized light can take place [16]. However, the systems $J=1 \to J=0$ and $J=0 \to J=1$ do not exhibit self-rotation.

In order to find whether the effect of equalization of the intensities of circular components shown in Fig. 5 is still present when the incident circular beam is detuned from the resonance we performed numerical simulation of the stationary propagation of the light in such conditions (Fig. 10). As in the previous case circularly polarized light entering the medium is finally converted into a linearly polarized one with direction parallel to the magnetic field—i.e., $\phi_- = 0$. But now this process has an oscillatory character. The intensities of both circular components oscillate with decreasing amplitudes which tend to the asymptotic value. The relative phase $\phi_- = 0$ is not now equal to zero from the beginning but tends oscillating to zero. It means that during stationary propagation the elliptical polarization of the light changes periodically. Asymptotically the resulting linearly polarized light propagates without losses.

D. $B_z \neq 0$

It is well known that the efficiency of the optical pumping is decreased by the presence of a transverse magnetic field. In order to increase this efficiency one uses a longitudinal magnetic field. However, the effects described in the previous sections are caused by the transverse magnetic field and are modified by the magnetic field component B_z. Let us

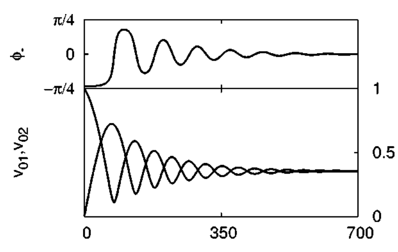

FIG. 10. The stationary propagation of the amplitudes v_{01} and v_{02} when only the field v_1 with the phase $\phi_1 = \pi/18$ enters the medium. The direction of the static magnetic field is defined by $\phi_B = \pi/6$ and its intensity by $b=2$. The detuning $\Delta = 5$.

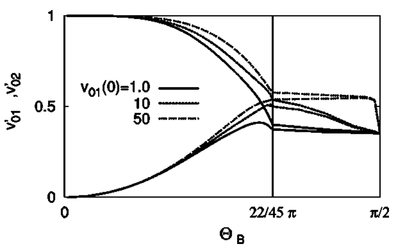

FIG. 11. The asymptotic relative amplitudes $v_{0i}'=v_{0i}(z)/v_{0i}(0)$, $i=1,2$, as functions of the magnetic field direction for different input field amplitudes. The relative phase difference $\phi_-=0$. Only beam v_1 enters the medium. The transverse component of the magnetic field is described by the relation $\sqrt{b^2+b_z^2}=2$. The scale in the right part of the figure is magnified.

define an angle $\Theta_B=\arctan(b/b_z)$ which is the angle between the light propagation direction and the direction of the magnetic field. The stronger is the longitudinal component of the magnetic field in relation to the transverse one, the smaller is the angle Θ_B.

In principle, one can find the steady-state density matrix and derive the stationary propagation equations in such a case. However, they are so complex that practically only a numerical treatment is possible.

Let us assume that only one circularly polarized beam enters the medium. As is expected a beam with orthogonal circular polarization is produced during stationary propagation (see Fig. 11). The initial relative phase $\phi_-(0)$ is not zero, but as the numerical simulation shows, it tends to zero. The amplitudes do not equalize asymptotically and the resulting beam propagating without losses is elliptically polarized. One can say using Eq. (10) that in general the contribution of the circular component to this elliptically polarized light decreases when the longitudinal magnetic field component decreases but this behavior is strongly influenced by the incident light intensity when the magnetic field becomes the transverse one.

Assuming that $\phi_-(0)=0$ we derive from the propagation equations the following constant of motion:

$$v_-(z)^{2k}[2(k+1)v_+^2(z) - v_-^2(z)] = \text{const}, \qquad (32)$$

where $k=\frac{3}{2}\cos^2\Theta_B$. Our numerical calculations showed that Eq. (32) is fulfilled asymptotically for every considered values of the phases $\phi_1(0)$ and ϕ_B.

The lossless propagation takes place when $\phi_-(0)=0$ and

$$v_- = \pm v_+ \cos\Theta_B, \qquad (33)$$

which explains the relation between the respective curves presented in Fig. 11.

Let us consider a linearly polarized beam entering the medium. In such a case the light finally also propagates without losses. When the longitudinal magnetic field is relatively large in comparison with the transverse one the final polarization direction tends to be perpendicular to the transverse magnetic field component—i.e., $\phi_-=\pm\pi/2$ (see Fig. 12). Obviously when the longitudinal magnetic field component is very small final linear polarization direction can be also parallel to the transverse magnetic field component.

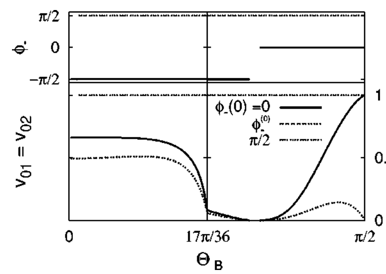

FIG. 12. The final amplitudes v_{10}, v_{20} and final relative phase ϕ_- vs the angle Θ_B for different values of the input phase $\phi_-(0)$. The transverse component is given by the relation $\sqrt{b^2+b_z^2}=2$ and $v_{01}(z=0)=v_{02}(z=0)$. The scale in the right part of the figure is magnified.

The asymptotic amplitude of this linearly polarized light depends on the initial relative phase $\phi_-(0)$ but in general it decreases when B_z component decreases. In the region where B_z is very small this amplitude increases since for vanishing B_z the medium should be transparent (compare Figs. 12 and 8).

E. Collisional relaxation

If the collisional relaxation is present the lossless stationary propagation is not possible. The collisions redistribute population among the ground-state sublevels. In other words the orientation and alignment in the ground state are destroyed. Therefore the propagating light is always absorbed.

Let us assume that the magnetic field is perpendicular to the light beam propagation direction and that collisional dumping rates are of the order of the spontaneous relaxation rate γ. We solve the stationary propagation equations numerically. Our calculations show that the rotation of the polarization direction still occurs (Fig. 13) but now the polarization tends to be perpendicular to the direction of the transverse magnetic field. When the initial linear polarization of the light is parallel to the magnetic field the polarization does not rotate. However, even a small deviation of the phase $\phi_-(0)$ from zero causes the light polarization rotation. It means that the polarization perpendicular to the transverse magnetic field is privileged.

V. DISCUSSION AND CONCLUSION

We have investigated the effect of the static magnetic field on the nonlinear stationary and nonstationary propagation of

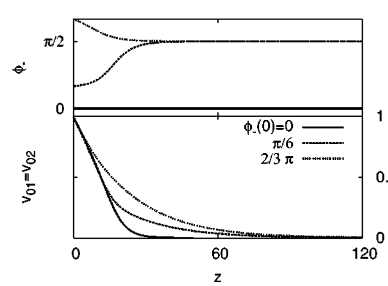

FIG. 13. The steady-state propagation of the linearly polarized laser beam ($v_1=v_2^*$) for different initial phases $\phi_-(0)$. The transverse magnetic field is characterized by $b=2$. The collisional relaxation is described by the rates $\Gamma=\Gamma_1=1$ and $\Gamma_2=1.2$.

orthogonally polarized circular light beams in the medium composed of the $J=1 \rightarrow J=0$ atoms. We have assumed that the magnetic field is perpendicular to the propagation direction. Using the density matrix technique we have derived the stationary propagation equations for the amplitudes and phases of the light beams for the case in which we can neglect collisional relaxation and the motion of the atoms.

The analysis of these equations has shown that the circularly polarized beam, which traveled sufficiently long distance in the medium, is transformed into linearly polarized one with the precisely defined amplitude and with the polarization parallel to the direction of the magnetic field. The lossless propagation of such light has been predicted.

When two circularly polarized beams contributing into the linearly polarized light enter the medium the effect of the polarization rotation occurs. The final result of this rotation depends on the relation between incident beam phases and the magnetic field direction. The asymptotic polarization direction can be parallel or perpendicular to the magnetic field vector. If one choses the direction of the magnetic field as the quantization axis, the linear polarization parallel to the magnetic field means that the light couples the sublevels with $m=0$ and due the optical pumping the atoms occupy the sublevels of the ground-state sublevel with $m=\pm 1$, whereas at the linear polarization perpendicular to the magnetic field the sublevels with $m=\pm 1$ are coupled to the upper level and the atoms are repumped to the ground-state sublevel with $m=0$. In both cases the atoms are in the dark state. One can say that during stationary propagation a consistency between the light parameters and the steady state of the atoms of the medium appears. As a result the medium becomes transparent.

A similar effect was predicted for the homogeneously broadened $1/2 \rightarrow 1/2$ transition [17,18]. The stationary interaction of a resonant elliptically polarized light with atoms in the ground state placed in a longitudinal magnetic field was considered. It was shown that for the propagation distances tending to infinity the light polarization due to the self-rotation always tends to be circular; i.e., a resonance medium of $1/2 \rightarrow 1/2$ atoms is always permeable for an elliptically polarized light. This result is also valid in the presence of the longitudinal magnetic field and when the atomic motion is taken into account [18]. Analogous transparency was predicted also for the $1 \rightarrow 1$ transition [19]. In these two systems optical multipole moments in the ground state couple the circular components of the light.

In the considered here system $J=1 \rightarrow J=0$ the orientation and alignment in the ground state couple the circular components of the light but only when the transverse magnetic field component is present. This coupling disturbs the optical pumping process but results in modification of the intensity and polarization of light and the medium properties. One can say that induced by the transverse magnetic field transparency appears.

It seems that the medium consisting of samarium atoms is a reasonable system where these effects could be observed. This atom was used in the experiment in which inversionless amplification of picosecond pulses due Zeeman coherence was demonstrated [8]. Unfortunately in this experiment the collisional damping of the orientation and alignment in the ground state due to the collisions with the argon buffer gas atoms could not be avoided. However, as we have shown (see Fig. 13) the nonlinear polarization rotation occurs also in such a case but the medium is not permeable.

Our numerical calculations show that the thermal motion of optically active atoms does not change qualitatively the features of the effect. The polarization rotation and asymptotic transparency take place but the Doppler broadening changes the distance necessary for transformation of the incident beam.

The stationary propagation of light in the considered medium is influenced by several factors: the incident light intensity, polarization, phase and its detuning from the resonance transition, the intensity and direction of the magnetic field, the density and the temperature of the atomic vapor, and the pressure of the buffer gas. It seems that these parameters can be adjusted in an optimal way to enable the experimental observation of effects predicted in the present papers.

There are several mechanisms of inducing transparency of the medium, e.g., the self-induced transparency—the envelope of propagating in a two level resonant inhomogeneously broadened medium pulse is transformed into the $2n\pi$ one which propagates without losses, the electromagnetically induced transparency—pumping light changes, due to the quantum interference, the absorption coefficient for probe light, and the optical pumping—pumping light transfers atoms to a dark state. We have shown that when the optical pumping process in the system $J=1 \rightarrow J=0$ is disturbed by the transverse magnetic field the polarized light changes its intensity and polarization to reestablish the conditions for the optimal atomic population transfer to a dark state. However, the atomic orientation and alignment generated in this way are different from the ones which would be generated in the absence of this magnetic field.

In our opinion such induced by the magnetic field transparency occurs also in the system with different level configurations. It seems that the effect described by us can be important in the interpretation of the experimental results and can be used in the engineering of the light and medium states.

ACKNOWLEDGMENT

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