Quantum mechanics in dissipative systems with a strong magnetic field

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Quantum mechanics in dissipative systems with a magnetic field is discussed. For strong magnetic fields the system exhibits an oscillatory behavior around the classical trajectory of the electron which should generate emissions in the millimeter range of the electromagnetic spectrum.

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I. INTRODUCTION

The principles of quantum mechanics were developed nearly a century ago, with a large impact on our understanding of nature. Even today, quantum mechanics can be good for surprises and some problems have been resolved only recently. For example, quantum mechanical systems *with dissipation* were difficult to formulate. One might oppose that microscopic systems in the physical world are not dissipative, but most systems are submerged in a dissipative environment. There exist different approaches for the treatment of dissipative systems, classically as well as quantum mechanically. The more conventional approaches take the system-plus-reservoir point of view, i.e., they couple the system of interest to an environment given by a very large number of external degrees of freedom that can be represented, e.g., by harmonic oscillators, and consider the whole as a closed conservative Hamiltonian system. In the quantum mechanical context, much progress in this direction was achieved in the Heisenberg picture (see, e.g., [1,2]) as well as in the Schrödinger picture (see, e.g., [3]). Usually, one is only interested in the dissipative subsystem and the explicit microscopic reservoir variables are eliminated from the description, e.g., by means of projection operator techniques or tracing procedures (see, e.g., [4–6]). As a result, the reservoir enters only through a few parameters, such as the friction coefficient. On the one hand, due to the large number of environmental degrees of freedom, the *ab initio* calculations are often cumbersome and the approximations involved at different stages are often afflicted with problems of mathematical or physical origin that are not yet solved quite satisfactorily (see, e.g., [7]). On the other hand, why should one carry a large number of variables along part of the calculations if the information contained in them will be eliminated later on anyway and enters only through some parameters?

Therefore several attempts have been made in the direction of an effective description of dissipative systems. These approaches try to find a Hamiltonian description of the dissipative subsystems alone, without taking the environment explicitly into account. The most well-known, and often quoted, approach is that of Caldirola [8] and Kanai [9] which uses an explicitly time-dependent Hamiltonian. This model and its quantum analog, obtained by canonical quantization, have been discussed extensively in the literature (see, e.g., [7,11–15]). Although it was applied frequently for the description of dissipative systems, it was also criticized by various authors [16–18]. The most serious point of criticism raised against this model is that after quantization, this Hamiltonian leads to an apparent violation of Heisenberg's uncertainty principle. A puzzling situation arose when Yu and Sun [19,20] showed that the same Hamiltonian operator could be obtained by standard procedures, just starting from the above-mentioned conventional system-plus-reservoir approach and should, therefore, be physically equivalent to this. This puzzle was solved by showing that it is actually not the Hamiltonian operator that leads to the violation of the uncertainty relation, but, the inappropriate treatment of the corresponding wave functions. Since the transition to the Caldirola-Kanai Hamiltonian involves classically a noncanonical and quantum mechanically nonunitary transformation, the wave functions must also be transformed accordingly and have a different physical meaning in the Caldirola-Kanai model from their meaning in the physical system described by the Schrödinger equation (SE) (in the conservative case); for a detailed discussion see [21,22].

However, the problems of violation of the uncertainty principle and transformation of the wave function do not occur in our model, which uses a nonlinear modification of the SE that describes the system alone, i.e., similar to the Caldirola-Kanai model but, in our case, the interpretation of the wave function and the quantities calculated with them (such as position and momentum uncertainties) are the same as for the usual SE. Moreover, it has been shown [21] that our nonlinear Schrödinger equation (NLSE) can be transformed into the Caldirola-Kanai equation with the help of a well-defined transformation that establishes the physical equivalence of the two models and, thus, also the physical equivalence between our description using a NLSE and the corresponding system-plus-reservoir approach. An advantage of our approach, compared to that of the system-plus-reservoir, is that it immediately yields exact analytical solutions in the cases where the corresponding conservative problem does, and it also does not suffer the shortcomings of several other approaches [10,23,24] also using NLSEs for the description of dissipative systems (see also [11]). A short outline of how to obtain our nonlinear model is given in Sec. II.

The objective of this contribution is to investigate the motion of a charged particle, e.g., an electron, represented by a wave packet, through a dissipative medium in the presence of a magnetic field. This problem is far from trivial and has also been dealt with in Refs. [25–27], but with a different

method and emphasis. We will show that, on average, the particle follows the classical path augmented by a quantum mechanical contribution. The discussion will focus on the situation where a strong magnetic field is present, such that the cyclotron frequency $\omega_c = qB/mc$ is larger than the friction coefficient γ. The motion of an electron in this strong magnetic field exhibits, in addition to the classical part, an oscillation of the wave packet width that was not obtained or discussed in Refs. [25–27]. This oscillation might be measurable through the emission of waves in the millimeter range of the electromagnetic spectrum.

This contribution is organized as follows: after the short resumé of the derivation of our NLSE (for details we refer to the literature [28–30]) given in Sec. II, the distinct cases (i) $\omega_c < \gamma$, (ii) $\omega_c = \gamma$, and (iii) $\omega_c > \gamma$ will be discussed in Sec. III. The last case shows the oscillatory behavior and is investigated in detail and physical consequences are outlined. Finally, in Sec. IV conclusions are drawn.

II. DERIVATION AND SOLUTION OF THE LOGARITHMIC NLSE IN THE PRESENCE OF A MAGNETIC FIELD

The derivation of our logarithmic NLSE is based on the work of Madelung and Mrowka [31,32] where the Schrödinger equation is deduced from three basic axioms, taken from observation: (i) the validity of the Heisenberg uncertainty relation, (ii) the existence of interference phenomena for material systems, and (iii) the Ehrenfest theorem (or the principle of correspondence).

The first condition leads to a statistical description, i.e., there exists a density function ρ, normalized to unity in the whole space, which allows for the determination of the average values of observables $\mathbf{O}$,

$$\langle \mathbf{O} \rangle = \int dV \rho(\mathbf{r},t)\mathbf{O}, \tag{1}$$

where dV is the volume element. Conservation of probability can be achieved if the continuity equation

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{j} = 0 \tag{2}$$

is fulfilled. The second condition (the observation of interference phenomena) implies, in analogy to optics (where intensity is a quadratic function of the amplitudes), that the density ρ and the convection current $\mathbf{j}$ can be expressed as bilinear forms of some complex field amplitudes, e.g., $\rho = \alpha\beta$ (for $\mathbf{j}$ see below), where α and β are complex functions of position and time. The last condition (Ehrenfest's theorem) implies that the mean value of the force, $\langle \mathbf{F} \rangle$, obeys the classical equations of motion. It can then be shown by a separation ansatz that α fulfills the time-dependent SE with $\alpha \equiv \Psi(\mathbf{r},t)$ and β the conjugate complex equation.

For dissipative systems, a similar procedure can be followed. Instead of the continuity equation one uses a Fokker-Planck-type equation which has the same appearance as the continuity equation but the current has an additional diffusion term $\mathbf{j}_D = -D\Delta\rho$, where D is the diffusion coefficient. Under the assumption of separability, i.e., $-D\Delta\rho/\rho = F_1(\alpha) + F_2(\beta)$, which can be fulfilled by the ansatz $-D\Delta\rho/\rho = \gamma(\ln\rho - \langle \ln\rho \rangle)$, one arrives at the NLSE [28,30,33]

$$i\hbar\frac{\partial \Psi}{\partial t} = \frac{1}{2m}\left(\frac{\hbar}{i}\boldsymbol{\nabla} - \frac{q}{c}\mathbf{A}\right)^2\Psi + q\Phi\Psi + \gamma\frac{\hbar}{i}[\ln\Psi - \langle \ln\Psi \rangle]\Psi - \gamma\frac{q}{c}(\mathbf{A}(\langle \mathbf{r} \rangle)\cdot\mathbf{r})\Psi, \tag{3}$$

where the case of the NLSE within an electromagnetic field is chosen and $\langle \cdots \rangle = \int \Psi^* \cdots \Psi d\mathbf{r}$. The charge of the particle is given by q. $\mathbf{A}(\mathbf{r},t)$ is the vector potential and Φ the scalar potential of electrodynamics. The term $\langle \ln\Psi \rangle$ assures that, on average, the friction term, depending on Ψ, vanishes and that despite the imaginary contribution from the nonlinear term, Ψ still can be normalized.

The vector potential for a constant magnetic field along the z axis is given by $\mathbf{A} = \frac{1}{2}(\mathbf{B}\times\mathbf{r})$. The diffusion coefficient is assumed to be isotropic. The NLSE can be separated into a longitudinal and a transversal differential equation via the ansatz

$$\Psi_{\mathrm{WP}}(\mathbf{r},t) = \Psi_{\mathrm{WP}_\parallel}(z,t)\Psi_{\mathrm{WP}_\perp}(\mathbf{r}_\perp,t), \tag{4}$$

where the index "WP" stands for "wave packet," since Gaussian wave packets can be found as analytical solutions of the NLSE. The symbols $\parallel$ and $\perp$ refer to the contributions parallel and perpendicular to the z axis. The $\mathbf{r}_\perp$ is the vector in the plane perpendicular to the z axis. The wave packet in the transversal direction is

$$\Psi_{\mathrm{WP}_\perp}(\mathbf{r}_\perp,t) = N(t)\exp\left[iY_x(t)\tilde{x}^2 + \frac{i}{\hbar}L_x(t)\tilde{x} + iY_y(t)\tilde{y}^2 + \frac{i}{\hbar}L_y(t)\tilde{y} + iK(t)\right], \tag{5}$$

where $\tilde{x} = x - \eta_x$, with x the position in the x direction and η_x the classical position, $\eta_x = \langle x \rangle$. A similar definition holds for the y direction. The imaginary parts of the complex quantities $Y_k(t)$, for $k = x, y$ are, by definition, inversely proportional to the square of the width of the wave packet.

Substituting the ansatz (5) into the transversal NLSE

$$i\hbar\frac{\partial \Psi_{\mathrm{WP}_\perp}}{\partial t} = \frac{1}{2m}\left(\frac{\hbar}{i}\boldsymbol{\nabla}_\perp - \frac{q}{c}\mathbf{A}_\perp(\mathbf{r}_\perp)\right)^2\Psi_{\mathrm{WP}_\perp} + \gamma\frac{\hbar}{i}[\ln\Psi_{\mathrm{WP}_\perp} - \langle \ln\Psi_{\mathrm{WP}_\perp} \rangle]\Psi_{\mathrm{WP}_\perp} - \gamma\frac{q}{c}[\mathbf{A}_\perp(\boldsymbol{\eta}_\perp)\cdot\mathbf{r}_\perp]\Psi_{\mathrm{WP}_\perp}, \tag{6}$$

with $\Psi_{\mathrm{WP}_\perp} = \Psi_{\mathrm{WP}_\perp}(\mathbf{r}_\perp,t)$ and where $\boldsymbol{\eta}_\perp$ is the classical position vector in the (x,y) plane, we obtain a polynomial in $\tilde{x}^2$, $\tilde{x}$, $\tilde{y}^2$, $\tilde{y}$, and $\tilde{x}\tilde{y}$, assuming an isotropic system leads to $Y_x = Y_y$ and the mixing term vanishes. Equating the factor of the $\tilde{k}$ term to zero leads to the classical equations of motion

$$\ddot{\eta}_x - \omega_c\dot{\eta}_y + \gamma\dot{\eta}_x = 0, \tag{7}$$

$$\ddot{\eta}_y + \omega_c \dot{\eta}_x + \gamma \dot{\eta}_y = 0. \tag{8}$$

Equating the terms proportional to $\tilde{k}^2$ to zero leads to the Riccati equation

$$\frac{2\hbar}{m}\dot{Y}_k + \gamma\frac{2\hbar}{m}Y_k + \left(\frac{2\hbar}{m}Y_k\right)^2 + \left(\frac{\omega_c}{2}\right)^2 = 0. \tag{9}$$

Using the ansatz of a constant solution $2\hbar/m\tilde{Y}_k = \left(-\gamma/2 \pm \frac{1}{2}\sqrt{\gamma^2-\omega_c^2}\right)$ plus a time-dependent $w_k(t)$,

$$\frac{2\hbar}{m}Y_k = \frac{2\hbar}{m}[\tilde{Y}_k + w_k(t)], \tag{10}$$

we arrive at the Bernoulli equation

$$\frac{2\hbar}{m}\dot{w}_k + \left(\gamma + \frac{4\hbar}{m}\tilde{Y}_k\right)\frac{2\hbar}{m}w_k + \left(\frac{2\hbar}{m}w_k\right)^2 = 0. \tag{11}$$

The solution of Eq. (11) is

$$\frac{2\hbar}{m}w_k(t) = \left[\left(\frac{1}{A} + \frac{1}{\frac{2\hbar}{m}w_{k0}}\right)e^{At} - \frac{1}{A}\right]^{-1}, \tag{12}$$

where the w_{k0} is the initial condition of $w_k(t)$ at $t=0$ and $A = \pm\sqrt{\gamma^2-\omega_c^2}$.

With this, the total solution $Y_k(t)$ can be obtained. It is, in general, complex and Y_k can be written as real and imaginary parts, i.e., $Y_k = Y_{kR} + iY_{kI}$. The imaginary part must be positive in order to obtain a normalizable wave packet solution. Both parts are related to the width of the wave packet via

$$\frac{2\hbar}{m}Y_{kI} = \frac{1}{\alpha_k^2} = \frac{\hbar}{2m}\frac{1}{\langle\tilde{k}^2\rangle},$$

$$\frac{2\hbar}{m}Y_{kR} = \frac{\dot{\alpha}_k}{\alpha_k} - \frac{1}{2}\gamma, \tag{13}$$

with the mean square deviation $\langle\tilde{k}^2\rangle = \langle k^2\rangle - \langle k\rangle^2$.

Inserting Eq. (13) into Eq. (9), we obtain for α_k the so-called Ermakov differential equation

$$\ddot{\alpha}_k + \left(\frac{\omega_c^2-\gamma^2}{4}\right)\alpha_k = \frac{1}{\alpha_k^3}. \tag{14}$$

Using Eqs. (10) and (13) we can relate $\alpha_k(t)$ with $w_k(t)$ whose solution is given by Eq. (12).

As will be seen in a moment, the $\dot{\alpha}_k/\alpha_k$ is related to the quantum mechanical contribution of the current $\mathbf{j}=\rho\mathbf{V}$, with $\mathbf{V}=\mathbf{v}+\mathbf{v}_D$ and $\mathbf{v}_D$ being the diffusion velocity.

The current density in the (x,y) plane is given by $\mathbf{j}_\perp = \rho\mathbf{V}_\perp$, where the velocity has two components: one, $\mathbf{v}_\perp$, contains the drift or convection contributions and has the form $\mathbf{v}_\perp = (\hbar/2im)\nabla_\perp \ln(\Psi/\Psi^*) - (q/mc)\mathbf{A}_\perp(\mathbf{r}_\perp)$ and another takes into account the diffusion via $\mathbf{v}_{D\perp} = -D(\nabla_\perp\rho/\rho)$. With this, the total velocity is given by

$$\mathbf{V}_\perp = \begin{bmatrix} \dot{\eta}_x + \left(\frac{\dot{\alpha}_x}{\alpha_x}\right)\tilde{x} + \frac{\omega_c}{2}\tilde{y} \\ \dot{\eta}_y + \left(\frac{\dot{\alpha}_y}{\alpha_y}\right)\tilde{y} - \frac{\omega_c}{2}\tilde{x} \end{bmatrix}. \tag{15}$$

Note that the quantum mechanical contribution is expressed by the dependence on α_k, while the classical part is given by the dependence on $\dot{\eta}_k$ in Eq. (15).

With the help of α_k the quantum mechanical contribution to the energy can be expressed as

$$\begin{aligned}\langle E_q\rangle &= \frac{m}{2}(\langle\mathbf{v}^2\rangle - \langle\mathbf{v}\rangle^2) = \frac{\langle\tilde{p}_x^2\rangle}{2m} + \frac{m}{2}\left(\frac{\omega_c}{2}\right)^2\langle\tilde{y}^2\rangle \\ &\quad + \frac{\langle\tilde{p}_y^2\rangle}{2m} + \frac{m}{2}\left(\frac{\omega_c}{2}\right)^2\langle\tilde{x}^2\rangle \\ &= \frac{\hbar}{4}\left[\alpha_x^2\left(\frac{\dot{\alpha}_x}{\alpha_x} - \frac{\gamma}{2}\right)^2 + \frac{1}{\alpha_x^2} + \left(\frac{\omega_c}{2}\right)^2\alpha_x^2\right] \\ &\quad + \frac{\hbar}{4}\left[\alpha_y^2\left(\frac{\dot{\alpha}_y}{\alpha_y} - \frac{\gamma}{2}\right)^2 + \frac{1}{\alpha_y^2} + \left(\frac{\omega_c}{2}\right)^2\alpha_y^2\right].\end{aligned} \tag{16}$$

From this form, it is obvious that $\langle E_q\rangle$ is always positive but it still can be time dependent due to the time dependence of α_k. This time dependence can be expressed with the help of the mean values of the position and momentum fluctuations as

$$\frac{d}{dt}\langle E_q\rangle = -\frac{\gamma}{4}\frac{d}{dt}(\langle[\tilde{p}_x,\tilde{x}]_+\rangle + \langle[\tilde{p}_y,\tilde{y}]_+\rangle), \tag{17}$$

where $\tilde{p}_k = (p_k - \langle p_k\rangle)$ and the symbol $[\ldots,\ldots]_+$ denotes the anticommutator.

III. DISCUSSION OF THE RESULTS AND POSSIBLE APPLICATIONS

We can distinguish three different regimes:
(1) $\gamma > \omega_c$, which implies $A = \pm\sqrt{\gamma^2-\omega_c^2}$;
(2) $\gamma = \omega_c$, i.e., $A=0$; and
(3) $\gamma < \omega_c$, i.e., $A = \pm i\sqrt{\omega_c^2-\gamma^2} = \pm i\tilde{\omega}$.

The latter defines a reduced frequency $\tilde{\omega}$ and it contains as a special case, $\gamma=0(\tilde{\omega}_c=\omega_c)$. Since in this paper we are mainly interested in case (3), we will only briefly mention the solution for case (1) and refer to [34] for case (2).

For $\gamma > \omega_c$ we obtain the following solution for $\dot{\alpha}_k/\alpha_k$:

$$\frac{\dot{\alpha}_k}{\alpha_k} = \frac{A}{2}\frac{e^{At} + \left(\frac{2\beta_{k0}}{A}\right)^2\sinh\left(\frac{A}{2}t\right)\cosh\left(\frac{A}{2}t\right)}{\left[e^{At} + \left(\frac{2\beta_{k0}}{A}\right)^2\sinh^2\left(\frac{A}{2}t\right)\right]}, \tag{18}$$

with $\beta_{k0} = 1/\alpha_{k0}^2$ and the index 0 refers to the initial condition, i.e., the width of the wave packet at $t=0$. As there are two solutions for A, namely $\pm\sqrt{\gamma^2-\omega_c^2}$, there are also two corresponding solutions for $\dot{\alpha}_k/\alpha_k$ possible and, hence, two different contributions to the quantum mechanical current and to the energy.

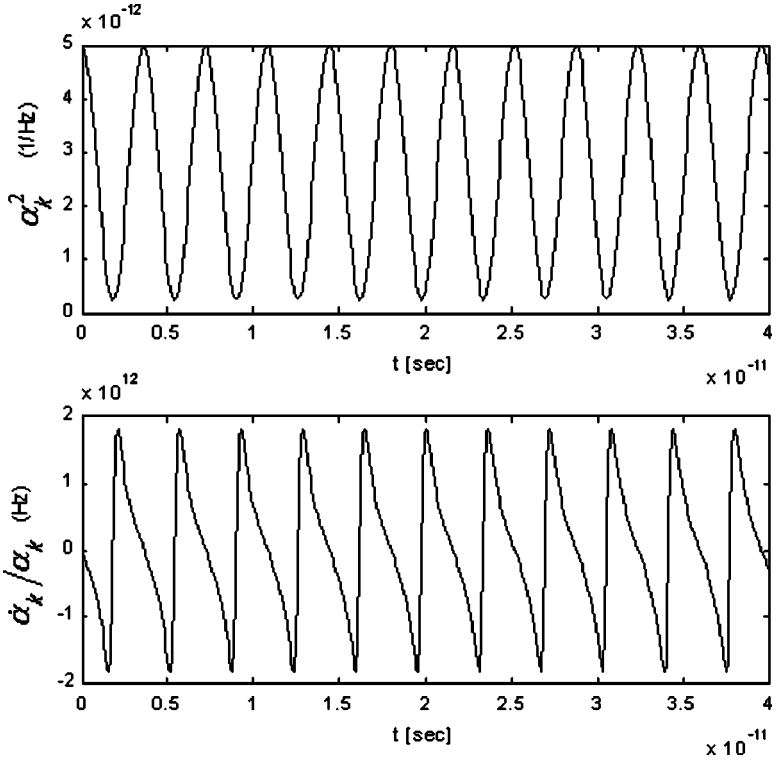

FIG. 1. The α_k^2 (top) and $\dot{\alpha}_k/\alpha_k$ (bottom) as a function of time.

We now focus on the case $\gamma<\omega_c$. There, the $A=\pm i\sqrt{\omega_c^2-\gamma^2}=\pm i\tilde{\omega}$ is imaginary. Since $(2\hbar/m)Y_k=A/2-(2\hbar/m)w_k(t)$, without the time-dependent contribution $w_k(t)$ the negative imaginary A produces a divergent wave packet, which is unphysical. Including $w_k(t)$, the constant negative imaginary part can be overcompensated and for both signs of A physically reasonable solutions can be obtained that show the same qualitative oscillatory behavior. Therefore in the following we will discuss in detail only one case, namely the positive sign of A. In this case and for the vanishing initial value of $\dot{\alpha}_k/\alpha_k$ and with $\beta_{k0}\neq\tilde{\omega}/2$, the wave packet width and the solution for the part of the current depending on α_k are oscillating:

$$\alpha_k^2=\alpha_{k0}^2\left\{\beta_{k0}^2\left[\frac{\sin\left(\frac{\tilde{\omega}}{2}t\right)}{\frac{\tilde{\omega}}{2}}\right]^2+\cos^2\left(\frac{\tilde{\omega}}{2}t\right)\right\}, \tag{19}$$

$$\frac{\dot{\alpha}_k}{\alpha_k}=\frac{\left(\frac{\beta_{k0}^2}{\frac{\tilde{\omega}}{2}}-\frac{\tilde{\omega}}{2}\right)\cos\left(\frac{\tilde{\omega}}{2}t\right)\sin\left(\frac{\tilde{\omega}}{2}t\right)}{\left\{\beta_{k0}^2\left[\frac{\sin\left(\frac{\tilde{\omega}}{2}t\right)}{\frac{\tilde{\omega}}{2}}\right]^2+\cos^2\left(\frac{\tilde{\omega}}{2}t\right)\right\}}. \tag{20}$$

In Fig. 1 we show the quantum mechanical contribution α_k^2 and $\dot{\alpha}_k/\alpha_k$ for $k=x,y$. The following values for the occurring parameters have been chosen: $\gamma=2\times10^{11}$ Hz, $\omega_c=1.7585\times10^{12}$ Hz, $B_z=10$ T, $E_x=100$ N/C, and $\alpha_{k0}=2.236\times10^{-6}$ s$^{1/2}$. Both graphs show a clear oscillatory behavior. The width of the packet, proportional to α_k^2, changes from a maximum value to a very small one. The $\dot{\alpha}_k/\alpha_k$ contribution to the current provokes an oscillatory motion, discussed further below.

Since the mean value of the anticommutator of position and momentum fluctuations can be written as

$$\langle[\tilde{p}_k,\tilde{k}]_+\rangle=\hbar\left(\dot{\alpha}_k\alpha_k-\frac{\gamma}{2}\alpha_k^2\right), \tag{21}$$

it follows from Eqs. (17), (19), and (20) that also the quantum mechanical contribution to the energy is oscillating without exponential decrease.

Even if this small contribution to the energy might not be measurable, we are confident that the oscillatory contribution to the current has a chance of being observed. When the electrons in a solid oscillate, they form with the background of the ions small oscillating dipoles, which emit radiation. Calculating the energy radiated and its frequency permits the proposal of an experiment which can measure the effect. Of course, care must be taken that the energy emitted is significantly smaller than the energy of the motion because of the assumption made that the radiation does not affect the motion of the particle. The condition is satisfied for the system under discussion.

The power radiated by a dipole is given by $P=\mu_0q^2\tilde{\omega}^4d^2/12\pi c$, where d is a characteristic length of the dipole. The $\tilde{\omega}$ is the reduced frequency introduced above and it can be read off Fig. 1 to be approximately $2\pi\times0.2793\times10^{12}$ Hz, which gives a wave length in the millimeter range. We obtain for the power $P\approx27\times10^{-6}d^2$ W/m^2 radiated by one electron. Taking into account that the number of

electrons in a small sample is about 10^{23} and that the characteristic scale of a dipole is of the order of 1 Å, the total power radiated is

$$P = 0.027 \times 10^{20} d^2 \frac{\mathrm{W}}{\mathrm{m}^2} = 0.027\ \mathrm{W}, \tag{22}$$

a number which can be measured with today's methods [35].

IV. CONCLUSIONS

We have investigated the quantum mechanical motion of a charged particle with dissipation within a constant magnetic field. Due to the dissipation, we used an approach with a logarithmic NLSE for the quantum mechanical description. Solutions in the form of Gaussian wave packets were obtained which are very similar to those in the corresponding conservative case. In particular, solutions with time-dependent wave packet width and, hence, quantum mechanical contributions to the velocity field (15) can be found. In the isolated case without dissipation, this time dependence shows oscillatory behavior where the frequency of oscillation is identical with the cyclotron frequency ω_c. It is not obvious—*a priori*—that this effect will also endure if the system is coupled to a dissipative environment and should thus be experimentally observable. Indeed, if the cyclotron frequency ω_c is smaller than the friction coefficient γ, corresponding to small magnetic fields, the oscillatory effect is destroyed and the time dependence asymptotically approaches a constant value. However, if $\omega_c > \gamma$, i.e., for strong magnetic fields, the width of the wave packet and the velocity field start oscillating with the reduced frequency $\tilde{\omega} = \sqrt{\omega_c^2 - \gamma^2}$. If the cyclotron frequency ω_c is about one order of magnitude larger than the friction coefficient γ, as in the example we chose, the reduced frequency $\tilde{\omega}$ is very close to the cyclotron frequency and, thus, the effect of the isolated system is almost recovered. However, for a situation with a reasonable friction coefficient of about 10^{12} Hz, this already requires magnetic fields as high as $B_z = 10$ T. Therefore only for such high magnetic fields should the oscillatory effect show up and lead to the predicted radio emission in the millimeter range. The power emitted for a typical device is of the order of a hundredth watt. Such small powers can be measured nowadays [35].

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