Controlled hole burning in the Fock space via conditional measurements on beam splitters

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We present a simple method to generate controlled holes in the statistical distribution of quantized electromagnetic fields. This scheme extends previous approaches for the cavity field to a traveling one. It relies on combining a coherent state with a one-photon state, both impinging a single beam splitter plus a single detector.

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Various proposals and experiments preparing interesting states of the quantized electromagnetic field are found nowadays [1–3]. These procedures, named "quantum-state engineering" (QSE), were motivated by the technological advances and striking results appearing in the literature [4] as well as the potential applications of nonclassical states to relevant topics, such as teleportation [5], quantum computation [6], quantum communication [7], quantum cryptography [8], quantum lithography [9], decoherence of states [10], measurements of field properties [11,12], etc.

Among the various interesting states studied in quantum optics, one class of them deserves special attention in the present context—namely, that of states having controlled holes in their photon-number distribution (PND), previously studied in [13]. States having holes in their PND exemplify a theoretical result by Mandel and Wolf [14], showing that an arbitrary field state $\hat{\rho}=\int P(\alpha)|\alpha\rangle\langle\alpha|d^2\alpha$ has its PND given by $P_n=\int P(\alpha)|\langle n|\alpha\rangle|^2 d^2\alpha$; since $|\langle n|\alpha\rangle|^2>0$, then all $P_n\neq 0$ when $P(\alpha)$ is a true probability density. Hence $P_n=0$ for certain values of n imply the state having no classical analog, also corroborating a theorem by Hillery [15].

As argued in [13] these states are candidates having potential applications in optical data storage and optics communication, with each hole of the state being associated with some signal (yes, 1, or +) and the absence of a hole being associated with an opposite signal (no, 0, or −). The generation of such states has been discussed in the context of cavity QSE [16], but the idea has been also extended to traveling waves [17], more appropriate for quantum communication.

In the present Brief Report we will consider an alternative and simpler scheme generating the mentioned states in traveling waves. Figure 1 shows a sketch of the experimental setup producing holes in the PND. In this figure a single-photon field (mode a), obtained from a parametric down conversion process [18], and the arbitrary field (mode b) enter the symmetric beam splitter (BS), both constituting the input state $|\Psi\rangle_{in}=|1\rangle_a|\phi\rangle_b$.

The action of BS is described by the unitary operator [19]

$$\hat{R}_{ab}=\exp[i\theta(\hat{a}^\dagger\hat{b}+\hat{a}\hat{b}^\dagger)]. \tag{1}$$

Thus, just after the BS the output state of the system, $|\Psi\rangle_{out}=\hat{R}_{ab}|\Psi\rangle_{in}$, results

$$|\Psi\rangle_{out}=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\binom{n}{k}^{1/2}C_nT^kR^{n-k}\times(R\sqrt{k+1}|k+1\rangle_c|n+k\rangle_d + T\sqrt{n-k+1}|k\rangle_c|n-k+1\rangle_d), \tag{2}$$

where $T=\cos\theta$ and $R=i\sin\theta$ stand for the BS reflection and transmission coefficients, respectively.

By measuring the field mode b in the state $|1\rangle_b$, we synthesize the projection of the field mode a in the state

$$|\Psi\rangle_a={}_b\langle 1|\Psi\rangle_{out}=\sum_{n=0}^{\infty}C_n\left[R^{n-1}T^2\left(\frac{R^2}{T^2}+n\right)\right]|n\rangle_a. \tag{3}$$

Next, the substitution in Eq. (3) of the coefficients R and T in the form (with $|T|^2+|R|^2=1$)

$$R=R_1=i\sqrt{\frac{n_1}{1+n_1}},\quad T=T_1=\frac{1}{\sqrt{1+n_1}} \tag{4}$$

allows us to write it as

$$|\psi\rangle_a^1=\sum_{n=0}^{\infty}C_n[R_1^{n-1}T_1^2(n-n_1)]|n\rangle_a, \tag{5}$$

which shows the integer n_1 determining the position of a hole in the PND.

According to Eqs. (4) and (5) the hole position is controlled by the transmission coefficient of the BS. In particular, when the initial component $|\phi\rangle_b$, related to the mode b, is mixed in the BS with a single-photon state and a one-photon measurement is obtained in one of the output channels of the BS (mode b), the state in the other output mode collapses into the state $|\psi\rangle_a^{(1)}=\hat{B}_1|\phi\rangle_b$, with $\hat{B}_1=R^{\hat{n}}(\hat{n}-n_1)\sum_{n'=0}^{\infty}|n'\rangle_{ab}\langle n'|$ being an effective operator.

The successive application [20] of the operator $\hat{B}_i$, $i=1,2,3,\ldots,N$, leads the input state $|\phi\rangle_b$ to the output one:

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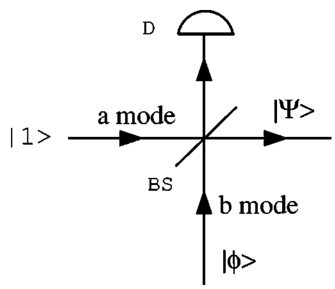

FIG. 1. Schematic illustration of the experimental setup.

$$|\psi\rangle_a^{(N)} = \hat{B}_N \cdots \hat{B}_2 \hat{B}_1 |\phi\rangle_b = \mathcal{N}_N \sum_{n=0}^{\infty} C_n \prod_{i=1}^{N} [R_i^n (n - n_i)] |n\rangle_a, \quad (6)$$

where R_i is the reflection coefficient of the ith BS, n_i determines the position of the ith hole in PND, and $\mathcal{N}_N$ is the normalization factor, given by

$$\mathcal{N}_N = \frac{\exp\{[|\alpha|^2(1 - |\prod_{i=1}^{N} R_i|^2)]/2\}}{\sqrt{\sum_{i=0}^{2N} a_i^{(N)} F_i(|\alpha \prod_{i=1}^{N} R_i|^2)}}, \quad (7)$$

with $a_i^{(N)}$ determined by the expression

$$\prod_{i=1}^{N} (x - n_i)^2 = \sum_{i=0}^{2N} a_i^{(N)} x^i. \quad (8)$$

In Eq. (7) the function $F_i(y)$ is obtained from the recurrence relation

$$F_i(y) = y[F'_{i-1}(y) + F_{i-1}(y)], \quad (9)$$

with $F_0(y)=1$. So, setting the initial state $|\phi\rangle$ equal to a coherent state [21], the PND of the field state given by $P_n^{(N)} = |\langle n | \psi\rangle_a^{(N)}|^2$ results:

$$P_n^{(N)} = \frac{\mathcal{N}_N^2}{n!} e^{-|\alpha|^2} |\alpha|^{2n} \prod_{i=1}^{N} |R_i|^{2n} (n - n_i)^2. \quad (10)$$

In the present case the success probability $\mathcal{P}^{(N)} = \prod_{i=1}^{N} \mathcal{P}^{(i)}$, where $\mathcal{P}^{(i)} = |_b\langle 1 | \Psi\rangle_{out}^{(i)}|^2$ stands for the ith BS. In this way, a little algebra furnishes

$$\mathcal{P}^{(N)} = \frac{1}{\mathcal{N}_N^2} \prod_{i=1}^{N} \frac{T_i^4}{(1 - T_i^2)}. \quad (11)$$

Note in Eq. (6) that the creation of each hole neither affects nor is affected by the presence of others. In practice, this generalization for many holes is implemented through the use of the successive BS setups, as described above. Now, the control of each hole position n_i is obtained via the manipulation of the transmission coefficients T_i of the ith BS.

To illustrate results we have plotted Figs. 2(a)–2(c) showing the controlled production of holes in the PND, with success probability found in Eq. (11): $\mathcal{P}^{(1)} = 6.2\%$, $\mathcal{P}^{(2)} = 0.5\%$, and $\mathcal{P}^{(3)} = 0.03\%$, respectively.

In summary, we have discussed a new scheme to produce a sequence of N controlled holes in the PND of a field state. The scheme is obtained using a BS acted upon by a combination of coherent and single-photon states, with subsequent one-photon detections being performed through highly effi-

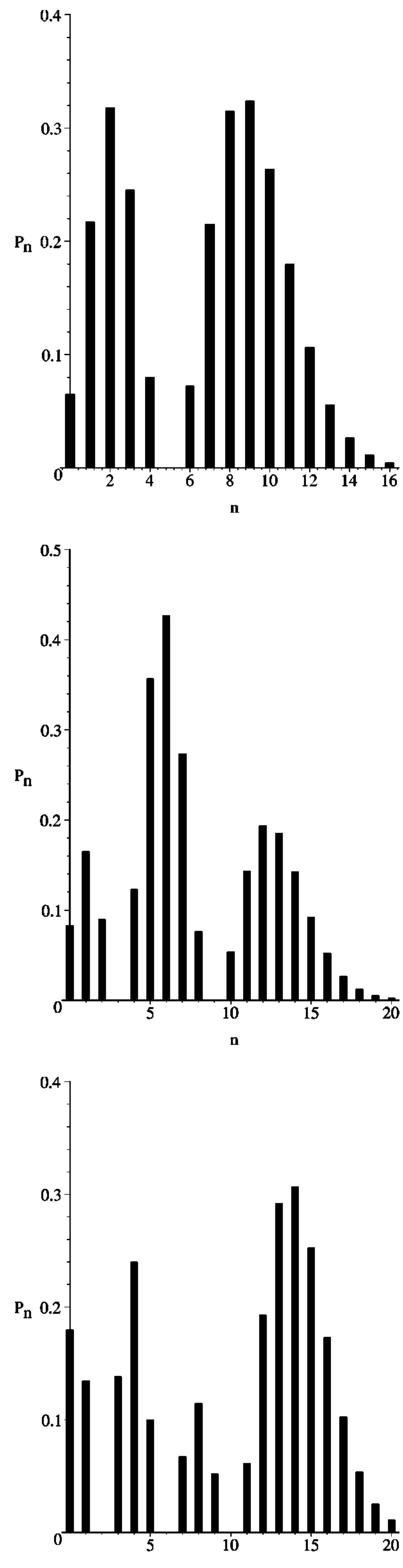

FIG. 2. (a) Holes in the PND at $|n\rangle = |5\rangle$, for $\alpha = 2.5$. (b) Same as in (a), with holes at $|3\rangle$ and $|9\rangle$, for $\alpha = 2.9$. (c) Same as in (a), with holes at $|2\rangle$, $|6\rangle$, and $|10\rangle$, for $\alpha = 3.2$.

cient avalanche photodiodes. This scheme, valid for traveling fields, is an extension of a previous work [16] for fields trapped inside a high-Q cavity. While in Ref. [16] the control of holes is obtained by manipulating Ramsey zones and transient times of atoms crossing a cavity, here this control is gotten by manipulating the transmission coefficients of the BS's. When this control is achieved, both schemes offer potential applications in quantum communication. The present procedure may be compared with another in the recent literature [17]: it is simpler since it dispenses the use of Mach-Zehnder interferometer and the Kerr medium employed in [17], thus going one step further. In addition, it is more general, since it also works for many holes.

As final remark, it is worth mentioning that we have assumed the detectors as being ideal. Using detectors having low efficiencies destroys the fidelity of the state being generated. However, that assumption is not drastic nowadays since recent technological advances have achieved photodetectors with efficiency near 100% [22].

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