First-order quantum phase transition driven by rotons in a gaseous Bose-Einstein condensate irradiated by a laser

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O'Dell *et al.* in recent work [Phys. Rev. Lett. **90**, 110402 (2003)] have proposed that laser-induced dipole-dipole interactions may drive a gaseous Bose-Einstein condensate to an instability against a periodic density modulation. We discuss the possibility that the instability is preceded by a first-order quantum phase transition to a modulated state.

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In a recent stimulating contribution O'Dell *et al.* [1] have shown the emergence of a roton minimum in a dilute boson assembly under intense laser irradiation. In their model, the application of a suitably polarized laser beam induces an effective dipole-dipole interaction among the atoms, giving rise to an intensity-dependent attractive part of the interparticle potential $v(k)$ for a narrow range of wave vectors k. Using the Bijl-Feynman model relation between the excitation dispersion relation $\omega(k)$ and structure factor $S(k)$ to display $S(k)$ for various values of laser intensity I, O'Dell *et al.* note that an instability against a periodic density modulation occurs as I is increased to 0.654 W/cm^2, where $\omega(k)$ touches the k axis and the main peak of the structure factor diverges.

Here, we appeal to the density wave theory of freezing [2], which yields a Verlet-like criterion [3] that a first-order phase transition occurs from a fluid to a periodically modulated state when the height of the main peak in the structure factor attains a certain value. For classical fluids this value is known to be around 2.8 [4]. For quantum fluids the available body of information is much less, but appeal to data on the freezing of superfluid ^{4}He [5] suggests a value of about one-half of the classical result. Such a phase transition should actually be characterized in a quantum fluid by the height of the main peak in the density-density response function $\chi(k)$ [6], but in fact both $S(k)$ and $\chi(k)$ are closely related to the dispersion relation $\omega(k)$ within the Bijl-Feynman model.

Therefore, to estimate the location of the phase transition in the model of O'Dell *et al.* [1] we show in Fig. 1 a plot of the maxima of the structure factor *versus* laser intensity I. The points in this plot are obtained from the relation $S(k) = \hbar k^2/[2m\omega(k)]$ using the dispersion relation calculated from the model for various laser intensities as shown in Fig. 2. The corresponding values of the structure factor and of the static density-response function $\chi(k)=n\hbar k^2/[2m\omega^2(k)]$ are shown in Figs. 3 and 4. The points nearest to $I_{\max} = 0.654$ W/cm^2 in Fig. 1 can be fitted by the relation

$$S_{\max}(I) \propto (I_{\max} - I)^{-\alpha}, \qquad (1)$$

where α is found to be 0.51, which suggests a "mean-field-like" exponent for the instability in the model of O'Dell *et al.* [1]. An explicit analytical proof is given below [7].

For a crude estimate of the location of the first-order phase transition we appeal to the measured value $S_{\max} \simeq 1.4$ for superfluid ^{4}He at freezing [5]. From the plot in Fig. 1 this locates the transition in a laser-irradiated Bose-Einstein condensate at a laser intensity $I = 0.635$ W/cm^2. This value corresponds to the lowest curve for the dispersion relation in Fig. 2 and to the uppermost curves for the structure factor and the density-response function in Figs. 3 and 4. It is worth remarking that at this laser intensity a free energy cost is still to be met in modulating the fluid density at the wave number corresponding to the peak in $S(k)$ and $\chi(k)$. However this free energy expense, according to the density wave theory of freezing [2], is balanced by a free energy gain coming from a discontinuous change in the average particle density, which should accompany the phase transition and make it a first-order one.

A related result was found by Pomeau and Rica for a

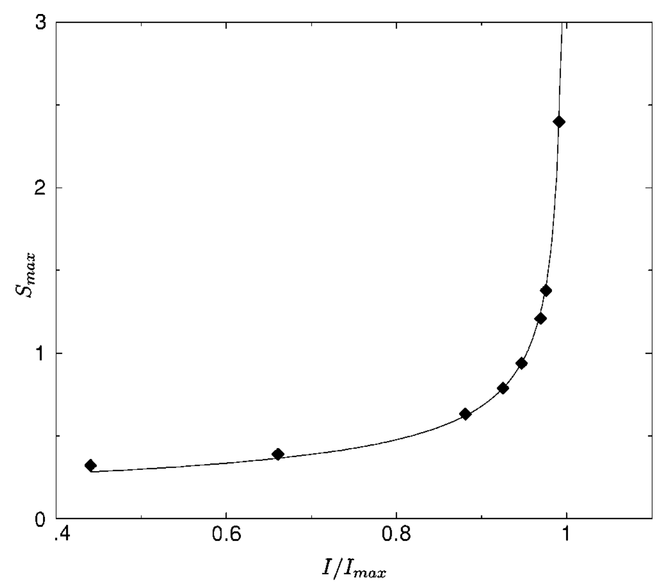

FIG. 1. Peak value $S_{\max}$ of the structure factor $S(k)$ (in arbitrary units) as a function of laser intensity I (in units of the intensity $I_{\max}$ for which the roton gap vanishes). Diamonds: numerical calculation using the model and the system parameters of O'Dell *et al.* [1]. Solid line: fit with the function $S_{\max}(I) = S_0/(1 - I/I_{\max})^{\alpha}$.

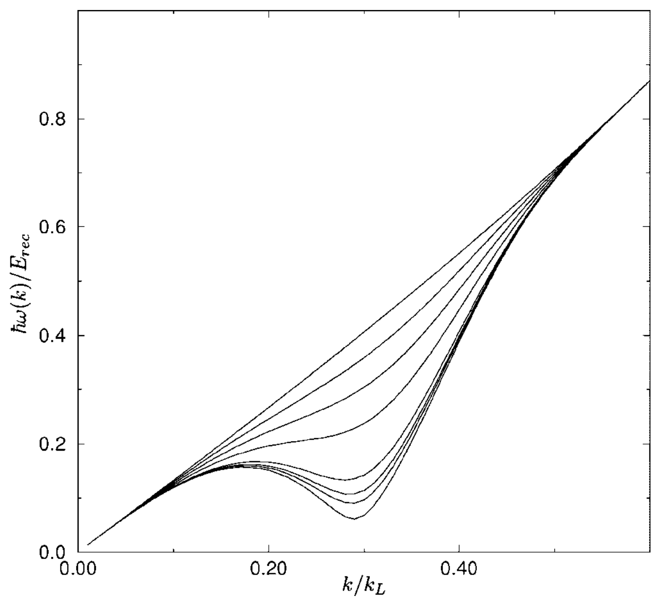

FIG. 2. Bogoliubov dispersion relation (in units of the recoil energy) as a function of wave number k (in units of laser wave number k_L) for a dipolar gas using the model and the system parameters of O'Dell *et al.* [1]. The curves from top to bottom are for various values of the laser intensity: $I/I_{\max}=0$, 0.22, 0.44, 0.66, 0.88, 0.92, 0.947, and 0.975.

dilute Bose gas interacting via a soft-sphere potential [8]. In their model they found that at increasing density a roton minimum develops in the dispersion relation and the fluid undergoes a first-order transition to a spatially modulated phase as the roton energy gap decreases below a critical value.

A further remarkable feature of the model of O'Dell *et al.*

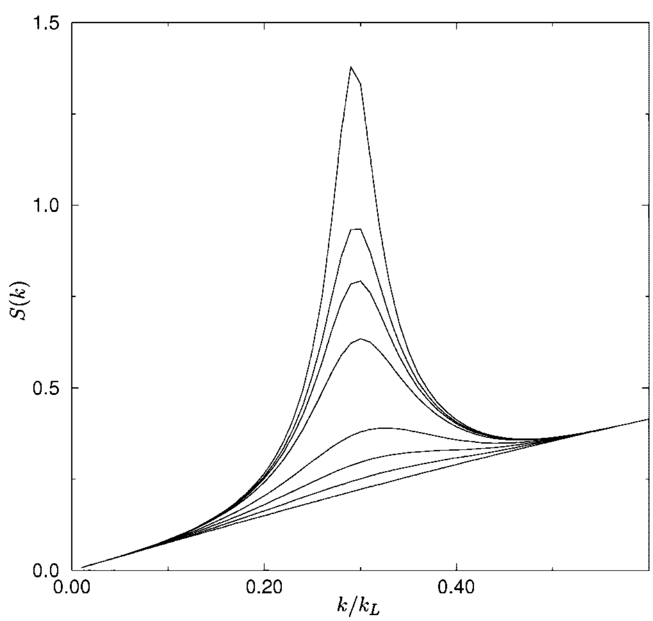

FIG. 3. Structure factor $S(k)$ (in arbitrary units) as a function of wave number k (in units of k_L) for a dipolar gas using the model and the system parameters of O'Dell *et al.* [1]. The curves from bottom to top are for the same values of the laser intensity as in Fig. 2.

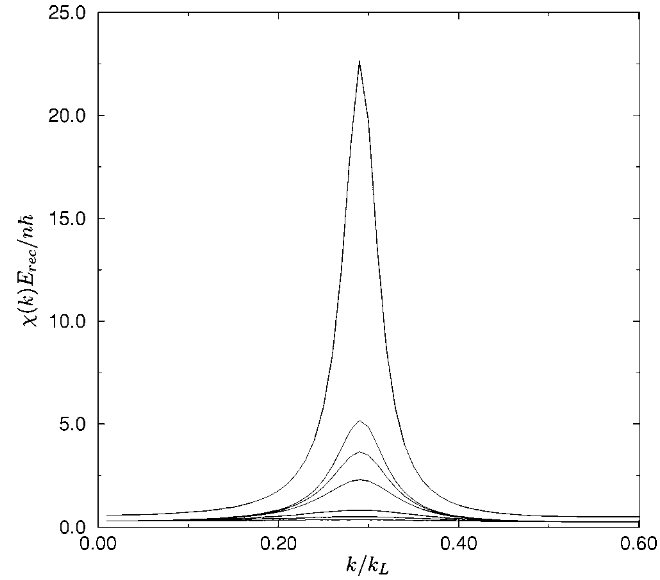

FIG. 4. Static density response function $\chi(k)$ (in units of $n\hbar/E_{rec}$) as a function of wave number k (in units of k_L) for a dipolar gas using the model and the system parameters of O'Dell *et al.* [1]. The curves from bottom to top are for the same values of the laser intensity as in Fig. 2.

is that, when we consider the structure factor $S(k)$ in Fig. 3 in relation to the curves in Fig. 3 of Ref. [1], we can clearly write as a useful representation

$$S = S_0 + S_r(I), \tag{2}$$

where S_0 is obtained from the Bogoliubov dispersion relation $\omega_0(k)$ for $I=0$, and $S_r(I)$ is the "remainder." Why the separation made in Eq. (2) is interesting physically is because, as O'Dell *et al.* stress, the laser irradiation at intensity I alters the dipole-dipole interactions in the boson assembly. It is seen from Fig. 3 that $S_r(I)$ has appreciable magnitude only over a finite range of wave number k. An interesting question is, therefore, whether analytical predictions from simple models such as that proposed by O'Dell *et al.* [1] for an elongated condensate or its spherical analog utilized recently by Granik and Chapline [9,10] may have some import on the more difficult structural problem posed by the strongly interacting, dense superfluid ^{4}He in its ground state.

The natural starting point for the ensuing, relatively brief discussion is the pioneering neutron diffraction experiment of Henshaw [11]. There, it became plain that as liquid ^{4}He was cooled through the λ transition, changes in the measured $S(k)$ were truly small. As Henshaw emphasized, the small changes observed were in fact such that there is less spatial ordering below the λ point than is found for $T > T_\lambda$. Further neutron diffraction measurements that were subsequently made by Svensson *et al.* [12] confirmed Henshaw's findings and their results were compared with computer simulation data obtained by Whitlock *et al.* [13] using a pair potential of the simple Lennard-Jones form. Both the measured and model structure factors depend on the force law through the sound velocity in the long-wavelength region and, in the large-k region for the dense bulk liquid, through the size and hardness of the atomic core. What is remarkable in the light of the discussion given above on the basis of Eq. (2) is that, from Fig. 15 in Ref. [12], the further details of the force law are apparently playing a discernible role only in the neighborhood of the main peak of $S(k)$ in liquid ^{4}He. Although the computer data refer to the ground state—i.e., $T=0$ while the experiments are at 1 K—the possibility of temperature effects was discounted by Svensson *et al.* [12]. Further computer studies of the role of the details of the force law in determining the detailed shape of the liquid structure factor of ^{4}He may thus be worthwhile.

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[1] D. H. J. O'Dell, S. Giovanazzi, and G. Kurizki, Phys. Rev. Lett. **90**, 110402 (2003).

[2] T. V. Ramakrishnan and M. Yussouff, Solid State Commun. **21**, 389 (1977).

[3] J. P. Hansen and L. Verlet, Phys. Rev. **184**, 151 (1969).

[4] See, for example, N. H. March and M. P. Tosi, *Coulomb Liquids* (Academic Press, London, 1984).

[5] A. D. B. Woods and R. A. Cowley, Rep. Prog. Phys. **36**, 1135 (1973).

[6] G. Senatore and G. Pastore, Phys. Rev. Lett. **64**, 303 (1990).

[7] By using the dispersion relation of the model of O'Dell *et al.* [1] $\omega(k)=\{[c_0^2+IF(k)]k^2+k^4\}^{1/2}$ (ω in units of the recoil energy E_{rec}, k in units of laser wave number k_L), where c_0^2 is the sound velocity for $I=0$ and $F(k)$ is a negative function describing the light-atom interactions and showing a minimum at around $k/k_L \simeq 0.3$ for our choice of parameters, Eq. (1) with exponent $\alpha=1/2$ is easily obtained under the approximation that the minimum of $\omega(k)$ is independent of the laser light intensity I.

[8] Y. Pomeau and S. Rica, Phys. Rev. Lett. **72**, 2426 (1994).

[9] A. Granik and G. Chapline, Phys. Lett. A **310**, 252 (2003).

[10] See also G. Chapline, E. Hohlfeld, R. B. Laughlin, and D. I. Santiago, Philos. Mag. B **81**, 235 (2001).

[11] D. G. Henshaw, Phys. Rev. **119**, 9 (1960).

[12] E. C. Svensson, V. F. Sears, A. D. B. Woods, and P. Martel, Phys. Rev. B **21**, 3638 (1980).

[13] P. A. Whitlock, D. M. Ceperley, G. V. Chester, and M. H. Kalos, Phys. Rev. B **19**, 5598 (1979).