Composite fermions, trios, and quartets in a Fermi-Bose mixture

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We consider a model of a Fermi-Bose mixture with strong hard-core repulsion between particles of the same sort and attraction between particles of different sorts. In this case, besides the standard anomalous averages of the type $\langle b\rangle$, $\langle bb\rangle$, and $\langle cc\rangle$, a pairing between fermions and bosons of the type bc is possible. This pairing corresponds to the creation of composite fermions in the system. At low temperatures and equal densities of fermions and bosons composite fermions are further paired in quartets. At higher temperatures trios, which consist of composite fermions and elementary bosons, are also present in the system. Our investigations are important in connection with the recent observation of weakly bound dimers in magnetic and optical dipole traps at ultralow temperatures and with the observation of the collapse of a Fermi gas in an attractive Fermi-Bose mixture of neutral particles.

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I. INTRODUCTION

The model of a Fermi-Bose mixture is very popular nowadays in connection with different problems in condensed matter physics such as high-T_c superconductivity (sc), superfluidity in ^{3}He-^{4}He mixtures [1], fermionic superfluidity in magnetic traps, and so on.

In high-T_c superconductivity this model was first proposed by Ranninger and co-workers [2,3] to describe simultaneously the high transition temperature and short coherence length of SC pairs on the one hand and the presence of a well-defined Fermi surface on the other. Later on Anderson [4] reformulated this model, introducing bosonic degrees of freedom (holons) and fermionic degrees of freedom (spinons), which, according to his ideas, experience in strongly correlated models the phenomenon of spin-charge separation.

Since then, a lot of prominent scientists have tried to prove the ideas of Anderson in the framework of two-dimensional (2D) Hubbard and *t-J* models. In this context it is necessary to mention first of all the ideas of Laughlin and co-workers [5,6] and Lee *et al.* [7,8]. These ideas are based on an anionic picture or slave boson method. However, even these important papers do not contain a rigorous proof of spin-charge separation in the whole parameter region of the phase diagram of high-T_c superconductors. Moreover, the photoemission experiments [9] and numerical calculations of Ohta *et al.* [10] show that at least at low temperatures the superconductive pairs in high-T_c materials are very much the same as in ordinary superconductors.

In this paper we show that a Fermi-Bose mixture with attractive interaction between fermions and bosons is unstable towards the creation of composite fermions $f=bc$. Moreover, for low temperatures and equal densities of fermions and bosons the composite fermions are further paired in the quartets $\langle ff\rangle$. Note that a matrix element $\langle f\rangle=\langle bc\rangle$ is nonzero only for the transitions between the states with $|N_B;N_F\rangle$ and $\langle N_B-1;N_F-1|$, where N_B and N_F are numbers of particles of elementary bosons and fermions, respectively. For superconductive state a matrix element $\langle ff\rangle\neq 0$ only for the transitions between the states with $|N_B;N_F\rangle$ and $\langle N_B-2;N_F-2|$. Our results are interesting not only for the physics of high-T_c superconductors, but also for the Fermi-Bose mixtures in magnetic and optical dipole traps as well as in optical lattices, where we can easily tune the parameters of the system such as the particle density and the sign and the strength of the interparticle interaction [11,12].

II. THEORETICAL MODEL

The model of a Fermi-Bose mixture has the following form on a lattice:

$$H = H_F + H_B + H_{BF,}$$

$$H_F = -t_F\sum_{\langle ij\rangle,\sigma} c^+_{i\sigma}c_{j\sigma} + U_{FF}\sum_i n^F_{i\uparrow}n^F_{i\downarrow} - \mu_F\sum_{i\sigma} n^F_{i\sigma},$$

$$H_B = -t_B\sum_{\langle ij\rangle} b^+_i b_j + \frac{1}{2}U_{BB}\sum_i n^B_i n^B_i - \mu_B\sum_i n^B_i,$$

$$H_{BF} = -U_{BF}\sum_{i\sigma} n^B_i n^F_{i\sigma}. \qquad (1)$$

This is a lattice analog of the standard Hamiltonian considered for example in Ref. [13] by Efremov and Viverit. Here t_F and t_B are fermionic and bosonic hopping ampli-

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tudes, $c_{i\sigma}^{+}$, $c_{i\sigma}$ and b_i^{+}, b_i are fermionic and bosonic creation and annihilation operators. The Hubbard interactions U_{FF} and U_{BB} correspond to hard-core repulsions between particles of the same sort. The interaction U_{BF} corresponds to the attraction between fermions and bosons. $W_F=8t_F$ and $W_B=8t_B$ are the bandwidths in 2D. Finally, μ_F and μ_B are fermionic and bosonic chemical potentials. For the square lattice the spectra of fermions and bosons after Fourier transformation read $\xi_{p\sigma}=-2t_F(\cos p_x d+\cos p_y d)-\mu_F$ for fermions and $\eta_p=-2t_B(\cos p_x d+\cos p_y d)-\mu_B$ for bosons, where d is a lattice constant. In the intermediate coupling case $W_{BF}/\ln(W_{BF}/T_{0BF})<U_{BF}<W_{BF}$ the energy of the bound state reads

$$|E_b| = \frac{1}{2m_{BF}d^2}\frac{1}{\exp\left[\dfrac{2\pi}{m_{BF}U_{BF}}\right]-1}, \tag{2}$$

where $m_{BF}=m_B m_F/(m_B+m_F)$ is an effective mass, $W_{BF}=4/m_{BF}d^2$, and $T_{0BF}=2\pi n/m_{BF}$. For simplicity we consider the case of equal densities $n_B=n_F=n$ which is more relevant for the physics of holons and spinons.

Note that in the intermediate coupling case the binding energy between fermions and bosons $|E_b|$ is larger than bosonic and fermionic degeneracy temperatures $T_{0B}=2\pi n_B/m_B$ and $T_{0F}=2\pi n_F/m_F\equiv\varepsilon_F$, but smaller than the bandwidths W_B and W_F. In this case the pairing of fermions and bosons $\langle bc\rangle\neq 0$ takes place earlier (at higher temperatures) than both Bose-Einstein condensation of bosons (or bibosons) ($\langle b\rangle\neq 0$ or $\langle bb\rangle\neq 0$) and superconductive pairing of fermions ($\langle cc\rangle\neq 0$). Note that in the case of a very strong attraction $U_{BF}>W_{BF}$ we have a natural result $|E_b|=U_{BF}$, and the effective mass $m_{BF}^{*}=m_{BF}U_{BF}/W_{BF}\gg m_{BF}$ is additionally enhanced on the lattice [14]. Note also that the Hubbard interactions U_{FF} and U_{BB} satisfy the inequalities $U_{FF}>W_F/\ln(W_F/|E_b|)$ and $U_{BB}>W_B/\ln(W_B/|E_b|)$.

Now let us consider the temperature evolution of the system. It is governed by the corresponding Bethe-Salpeter (BS) equation. After analytical continuation $i\omega_n\to\omega+i0$ (see Ref. [15]) the solution of this equation acquires a form

$$\Gamma(\mathbf{q},\omega) = \frac{-U_{BF}}{1-U_{BF}\displaystyle\int\frac{d^2p}{(2\pi)^2}\frac{1-n_F(\xi(\mathbf{p}))+n_B(\eta(\mathbf{q}-\mathbf{p}))}{\xi(\mathbf{p})+\eta(\mathbf{q}-\mathbf{p})-\omega-i0}}, \tag{3}$$

where $\xi(\mathbf{p})=p^2/2m_F-\mu_F$ and $\eta(\mathbf{p})=p^2/2m_B-\mu_B$ are spectra of fermions and bosons at low densities $n_F d^2\ll 1$ and $n_B d^2\ll 1$. Note that in the pole of BS equation enters the temperature factor $1-n_F(\xi(\mathbf{p}))+n_B(\eta(\mathbf{q}-\mathbf{p}))$ in contrast with the factor $1-n_F(\xi(\mathbf{p}))-n_F(\xi(\mathbf{q}-\mathbf{p}))$ for two-fermion superconductive pairing and $1+n_B(\eta(\mathbf{p}))+n_B(\eta(\mathbf{q}-\mathbf{p}))$ for two-boson pairing. The pole of the Bethe-Salpeter equation corresponds to the spectrum of the composite fermions:

$$\omega\equiv\xi_{\mathbf{p}}^{*}=\frac{p^2}{2(m_B+m_F)}-\mu_{comp}. \tag{4}$$

Note that in Eq. (4)

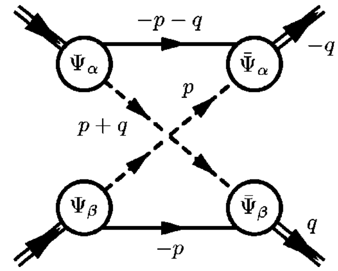

FIG. 1. The skeleton diagram for the coefficient b near Ψ^4 in the effective action. The dashed lines correspond to bosons, the solid lines correspond to fermions.

$$\mu_{comp}=\mu_B+\mu_F+|E_b| \tag{5}$$

is a chemical potential of composite fermions. Note also that composite fermions are well-defined quasiparticles, since the damping of quasiparticles equals to zero in the case of the bound state ($E_b<0$), but it becomes nonzero and is proportional to E_b in the case of the virtual state ($E_b>0$). The process of a dynamical equilibrium (boson + fermion $\rightleftarrows$ composite fermion) is governed by the standard Saha formula [16]. In the 2D case it reads

$$\frac{n_B n_F}{n_{comp}}=\frac{m_{BF}T}{2\pi}\exp\left\{-\frac{|E_b|}{T}\right\}. \tag{6}$$

The crossover temperature T_* is defined, as usual, from the condition that the number of composite fermions equals the number of unbound fermions and bosons: $n_{comp}=n_B=n_F=n$. This conditions yields

$$T_*\simeq\frac{|E_b|}{\ln(|E_b|/2T_{0BF})}\gg\{T_{0B};T_{0F}\}. \tag{7}$$

Note that in the Boltzmann regime $|E_b|>\{T_{0B};T_{0F}\}$, in fact we deal with the pairing of two Boltzmann particles. That is why this pairing does not differ drastically from the pairing of two particles of the same type of statistics. Indeed, if we substitute $\mu_B+\mu_F$ in Eq. (5) on $2\mu_B$ or $2\mu_F$ we will get the familiar expressions for chemical potentials of composite bosons consisting either of two elementary bosons [17,18] or of two elementary fermions [19,20]. The crossover temperature T_* plays the role of a pseudogap temperature, so the Green functions of elementary fermions and bosons acquire a two-pole structure below T_* in similarity with Ref. [20].

For lower temperatures $T_0<T<T_*$ [where $T_0=2\pi n/(m_F+m_B)$ is the degeneracy temperature of composite fermions] the numbers of elementary fermions and bosons are exponentially small. The chemical potential of composite fermions reads $\mu_{comp}=-T\ln(T/T_0)$. Hence $|\mu_{comp}|\ll|E_b|$ for $T\ll T_*$.

By performing the Hubbard-Stratonovich transformation, the original partition function $Z=\int\mathcal{D}\bar{b}\mathcal{D}b\,\mathcal{D}\bar{c}\mathcal{D}c\exp\{-\beta F\}$ can be written in terms of the composite fermions, only $Z=\int\mathcal{D}\bar{\Psi}_\alpha\mathcal{D}\Psi_\alpha\exp\{-\beta F_{\mathrm{eff}}\}$. This procedure gives the magnitude of the interaction between the composite fermions. The lowest order of the series expansion is given in Fig. 1. Analytically this diagram is given by

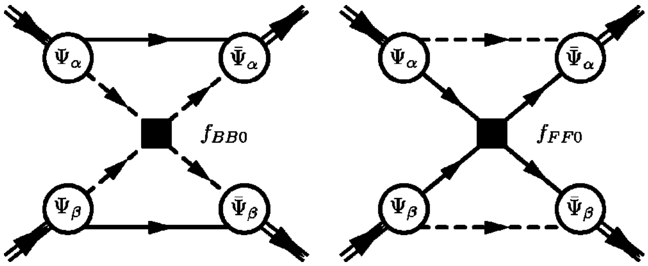

FIG. 2. The corrections to the coefficients b containing boson-boson and fermion-fermion interactions.

$$-\frac{1}{2}\sum_n \int \frac{d^2p}{(2\pi)^2}\{G_F^2(\mathbf{p};i\omega_{nF})G_B^2(-\mathbf{p};-i\omega_{nB}) + G_F^2(-\mathbf{p};-i\omega_{nF})G_B^2(\mathbf{p};i\omega_{nB})\}, \qquad (8)$$

where $G_F=1/[i\omega_{nF}-\xi(\mathbf{p})]$ and $G_B=1/[i\omega_{nB}-\eta(\mathbf{p})]$ are fermion and boson Matsubara Green functions, and $\omega_{nF}=(2n+1)\pi T$ and $\omega_{nB}=2n\pi T$ are fermion and boson Matsubara frequencies. In fact this integral determines the coefficient b near Ψ^4 in the effective action. Evaluation of integral (8) yields

$$b \simeq -N(0)/|E_b|^2, \qquad (9)$$

where $N(0)=m_{BF}/2\pi$. The corrections to the coefficient b are presented in Fig. 2. They contain explicitly the T matrices for boson-boson and fermion-fermion interactions. In the intermediate-coupling case these diagrams are small and governed by the small parameters $f_{BB0}\sim 1/\ln(W_B/|E_b|)$ and $f_{FF0}\sim 1/\ln(W_F/|E_b|)$. So the exchange diagram really provides the main contribution to the coefficient b.

The coefficient near quadratic term Ψ^2 in an effective action in agreement with general rules of diagrammatic technique (see Ref. [15]) is given by

$$a + cq^2/2(m_B+m_F) = -1/\Gamma(q;0), \qquad (10)$$

where $\Gamma(q;0)$ is given by Eq. (3). The solution of Eq. (10) for $q\neq 0$ yields $c=N(0)/|E_b|$.

If we want to rewrite the effective action with gradient terms

$$\Delta F = a\overline{\Psi}_\alpha\Psi_\alpha + \frac{c}{2(m_F+m_B)}(\nabla\overline{\Psi}_\alpha)(\nabla\Psi_\alpha) + \frac{1}{2}b\overline{\Psi}_\alpha\overline{\Psi}_\beta\Psi_\beta\Psi_\alpha \qquad (11)$$

in the form of the energy functional of a nonlinear Schrödinger equation for the composite particle with the mass m_B+m_F, we have to introduce the effective order parameter $\Delta_\alpha=\sqrt{c}\,\Psi_\alpha$. Accordingly in terms of Δ_α the new coefficients $\tilde{a}$ and $\tilde{b}$ near quadratic and quartic terms read $\tilde{a}=a/c$ and $\tilde{b}=b/c^2$. Note that the Grassman field Δ_α corresponds to the composite fermions and is normalized according to the condition $\Delta_\alpha^+\Delta_\alpha=n_{comp}$. Hence the coefficient $\tilde{b}$ plays the role of the effective interaction between composite particles. From Eqs. (9) and (10), $\tilde{b}=-1/N(0)$.

This result coincides in absolute value, but is different in sign with the results of Drechsler and Zwerger [21], who calculated in the 2D case the residual interaction between two composite bosons, each one consisting of two elementary fermions. The sign difference between these two results is due to the different statistics of elementary particles in both cases. It is also important to calculate $b(q)$, where the momenta of the incoming composite fermions equal, respectively, $(\mathbf{q},-\mathbf{q})$. It is easy to find that

$$b(q) = -\frac{1}{2}\sum_n \int \frac{d^2p}{(2\pi)^2}\{G_B(\mathbf{p};i\omega_{nB})G_F(\mathbf{p};-i\omega_{nF}) \times G_B(\mathbf{p}+\mathbf{q};i\omega_{nB})G_F(\mathbf{p}-\mathbf{q};-i\omega_{nF}) + G_B(\mathbf{p};-i\omega_{nB})G_F(\mathbf{p};i\omega_{nF})G_B(\mathbf{p}-\mathbf{q};-i\omega_{nB}) \times G_F(\mathbf{p}+\mathbf{q};i\omega_{nF}). \qquad (12)$$

A straightforward calculation for small q yields in the case of equal masses $m_B=m_F=m$

$$b(q) = -\frac{m}{4\pi(|E_b|+q^2/4m)^2}. \qquad (13)$$

Accordingly,

$$\tilde{b} = \frac{b}{c^2} \approx -\frac{4\pi}{m(1+q^2/4m|E_b|)^2}, \qquad (14)$$

where $|E_b|=1/ma_0^2$ and a_0 is an s-wave scattering length. An analogous result in the 3D case was obtained by Pieri and Strinati [22]. Hence, the four-particle interaction has a Yukawa form in the momentum space. Therefore $U_4(r)\approx -1/ma_0^2\sqrt{2r/a_0}\exp(-2r/a_0)$ corresponds to an attractive potential with the radius of interaction equal to $a_0/2$. We can calculate now the binding energy of quartets $|E_4|$. A straightforward calculation absolutely similar to the calculation of $|E_b|$ yields

$$1 = \frac{|\tilde{b}|(m_B+m_F)}{2\pi}\int_0^{2/a_0}\frac{q\,dq}{q^2+(m_B+m_F)|E_4|}. \qquad (15)$$

Hence,

$$|E_4| = \frac{4}{a_0^2(m_B+m_F)\left[\exp\left(\frac{4\pi}{|\tilde{b}|(m_B+m_F)}\right)-1\right]}. \qquad (16)$$

For equal masses $m_B=m_F$ a coupling constant $|\tilde{b}|(m_B+m_F)/4\pi=1/2$ and thus

$$|E_4| = \frac{2|E_b|}{(e^{1/2}-1)} \approx 3|E_b|. \qquad (17)$$

The process of dynamical equilibrium (composite fermion + composite fermion ⇄ quartet) is again governed by the Saha formula of the type

$$\frac{n_{comp}^2}{n_4} = \frac{m_4T}{2\pi}\exp\left\{-\frac{|E_4|}{T}\right\}, \qquad (18)$$

where $m_4=(m_B+m_F)/2$. The number of composite fermions equals half the number of quartets $n_4=n_2/2$ for the crossover temperature:

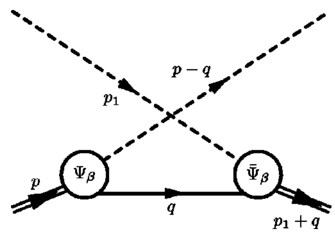

FIG. 3. The exchange diagram for the three-particle interaction.

$$T_{**}^{(4)} = \frac{|E_4|}{\ln(|E_4|/2T_0)}. \tag{19}$$

Below this temperature quartets of the type $\langle f_{i\uparrow}b_i; f_{j\downarrow}b_j\rangle$ play the dominant role in the system. Note that $T_{**}^{(4)} > T_*$, so quartets are dominant over pairs (composite fermions) in the entire temperature interval. Note also that the quartets are in a spin-singlet state. The creation of spin-triplet quartets is prohibited or at least strongly reduced by the Pauli principle. The triplet p-wave pairs of composite fermions are possibly created in a strong coupling case $|E_b| > W$, when the corrections to the coefficient b given by the diagrams in Fig. 2 are large and repulsive. However, in this case small parameters are absent and it is very dificult to control the diagrammatic expansion.

III. THREE-PARTICLE PROBLEM

If we consider the scattering process of an elementary fermion on a composite fermion, we get a repulsive sign of the interaction regardless of the relative spin orientation of composite and elementary fermions. The same result in 3D for scattering of elementary fermions on a dimer consisting of two fermions was obtained by Petrov *et al.* [23]. However, for the scattering process of elementary bosons on a composite fermion, we get an attractive sign of the interaction. Moreover, a Fourier component of the three-particle interaction for $m_B = m_F = m$ reads in the 2D case (see Fig. 3)

$$U_3(q) = \frac{1}{c}G_F(0,q) = -\frac{8\pi}{m(1+q^2a_0^2)}, \tag{20}$$

where $G_F(0,q)$ is the Green function of an elementary fermions and $c = N(0)/|E_b|$. Hence,

$$U_3(r) \sim -\frac{1}{ma_0^2}K_0(r/a_0) \sim -\frac{1}{ma_0^2}\sqrt{\frac{a_0}{r}}\, e^{-r/a_0} \tag{21}$$

again corresponds to an attractive potential of the Yukawa type, but now with a range of the interaction equal to a. Calculation of the three-particle bound-state energy yields

$$1 = \frac{|U_3(0)|}{2\pi}\int_0^{1/a_0} \frac{qdq}{q^2/2m_B + q^2/2(m_B+m_F) + |E_3|}. \tag{22}$$

Hence for $m_B = m_F = m$

$$|E_3| = \frac{3}{4\, ma_0^2}\frac{1}{\left[\exp\left(\frac{3\pi}{m|U_3|}\right) - 1\right]} = \frac{3|E_b|}{4(e^{3/8}-1)} \approx 1.7|E_b|. \tag{23}$$

Note that we are studying trios and quartets in the zeroth-order exchange approximation. A more rigorous solution of the three- and four-particle problems requires a full analysis of the Skorniakov-Ter-Martirosian type of equations [24]. This investigation will be the subject of a separate publication. The dynamical equilibrium of the type composite fermion + boson $\rightleftarrows$ trio is again governed by the following Saha formula:

$$\frac{n_B\, n_{comp}}{n_3} = \frac{m_3 T}{2\pi}\exp\left\{-\frac{|E_3|}{T}\right\}, \tag{24}$$

where $m_3 = m_B(m_B+m_F)/(2m_B+m_F)$. Accordingly, trios dominate over unbound bosons for temperatures $T < T_{**}^{(3)}$, where

$$T_{**}^{(3)} = \frac{|E_3|}{\ln(|E_3|/2T_0)}. \tag{25}$$

Note that $T_{**}^{(3)} < T_{**}^{(4)}$, so trios are not so important as quartets.

As a result for $T < T_{**}^{(4)}$ there are mostly quartets in the system. The quartets are Bose condensed at the critical temperature $T_c = T_0/[8 \ln \ln(4/na^2)]$ in the case of equal masses. It is important to note that in the Feshbach resonance scheme [11,12,25] we are usually in the regime $T \sim T_0$, where quartets prevail over trios and pairs. In this scheme the particles are at first cooled to very low temperatures $T < T_0$ and only then is the sign of the scattering length changed by a magnetic field to support the formation of bound pairs. Let us emphasize that in the restricted geometry of magnetic or optical dipole traps our theory is valid under the condition $T_c > \omega$, where ω is the level spacing in the trap. For a large number of particles, $N \gg 1$, in the 2D trap $\omega \sim T_0/N^{1/2}$ ($\omega \sim T_0/N^{1/3}$ in 3D traps), so this condition is easily satisfied. Note also that octets are not formed in the system because two quartets repel each other due to the Pauli principle similar to the results of Refs. [21,26].

IV. THREE-DIMENSIONAL CASE

Let us consider now the 3D case, which is more actual for the physics of magnetic and dipole traps. Let us also concentrate on the situation in a free space; that is, let us neglect the lattice and consider bosonic particles with the spectrum $\eta = p^2/2m_B - \mu_B$ and fermionic particles with the spectrum $\xi = p^2/2m_F - \mu_F$. In this case the role of the gas parameter is played by the product $(3\pi^2 n)^{1/3} r_0 \ll 1$ [27], where r_0 is the range of the potential and $n = n_B = n_F$ is the particle density. The analytical theory for composite fermions and quartets in this case can be developed for a shallow level or for a resonant interaction. For these cases $|E_b| = 1/2m_{BF}\, a_0^2 \ll 1/2m_{BF}\, r_0^2$, and hence $a_0 \gg r_0$. Note that effectively a quantity $1/2m_{BF}\, r_0^2$ plays the role of a bandwidth W_{BF} in free space.

Let us calculate now the coefficients c and b in the effective action in the 3D case. The calculation of the total vertex $\Gamma(\mathbf{q},0)$ in the 3D case yields

$$-\frac{1}{\Gamma(\mathbf{q},0)}=a+\frac{cq^2}{2(m_B+m_F)}, \tag{26}$$

where

$$c=\frac{(2m_{BF})^{3/2}}{8\pi\sqrt{|E_b|}}. \tag{27}$$

Note that for fermion-boson interaction described by the rectangular spherically symmetrical potential well of the width r_0 and of the depth U the energy of the shallow bound state reads

$$|E_b|=\frac{\pi^2}{16}U_c\left(\frac{U-U_c}{U_c}\right)^2 \ll \frac{1}{m_{BF}r_0^2}, \tag{28}$$

where

$$U_c=\frac{\pi^2}{8}\frac{1}{m_{BF}r_0^2}\sim\frac{1}{m_{BF}r_0^2} \tag{29}$$

is the threshold value of the interaction for the formation of the bound state [28]. It is necessary to point out that the result $|E_b|\sim(U-U_c)^2/U_c$, where $U_c\sim 1/m_{BF}r_0^2$ is a universal one for a shallow level in 3D and does not depend upon an exact form of the fermion-boson potential. The exact form of the potential defines only the numerical coefficient in front of U_c in Eq. (29) and in front of $1/m_{BF}r_0^2$ in Eq. (30).

The calculation of the exchange diagram on Fig. 1 yields in the 3D case

$$b=-\frac{m_{BF}^3}{2\pi(2m_{BF}|E_b|)^{3/2}}. \tag{30}$$

Note that the corrections to the coefficient b given by the diagrams in Fig. 2, which explicitly contain boson-boson and fermion-fermion interactions, are again small in the 3D case for the shallow level or in a resonance situation. Their smallness is governed by the parameter $r_0/a_0 \ll 1$ in the 3D case. The effective interaction between two composite fermions for $m_B=m_F=m$ is again given by

$$\tilde{b}=\frac{b}{c^2}=-\frac{2\pi}{m}(2a_0). \tag{31}$$

Note that by an absolute value the magnitude of the interaction coincides with the mean-field result of Haussmann [26]; however, there is again, as in the 2D case, an important difference in sign between our result and the result of Haussmann [26].

To answer the question about the possibility of quartet formation, we have to calculate $\tilde{b}(q)$ again. In the 3D case for small q it acquires a form

$$\tilde{b}(q)\approx-\frac{2\pi}{m}\frac{2a_0}{(1+q^2/4m|E_b|)^{3/2}}. \tag{32}$$

The solution of the Bethe-Salpeter equation for quartet formation yields

$$\sqrt{\frac{ma_0^2|E_4|}{2}}\arctan\left(\sqrt{\frac{2}{ma_0^2|E_4|}}\right)=1-\frac{\pi^2 a_0}{2m|\tilde{b}(0)|}>0. \tag{33}$$

Hence in the mean-field approximation $|E_4|\approx 0.7|E_b|$.

Note that in the more rigorous calculation of Pieri and Strinati [22] for the case of the interaction between two composite bosons, each one consisting of two elementary fermions,

$$\tilde{b}=\frac{2\pi}{m}(0.75a_0), \tag{34}$$

so the effective scattering length is $a_{eff}=0.75a_0$ instead of $a_{eff}=2a_0$ which it is in the calculation of Haussmann. In even the more rigorous calculation of Petrov *et al.* [23] it is even smaller $a_{eff}=0.6a_0$.

Hence a shallow bound state of quartets in a more rigorous approach exists in the 3D case only if

$$a_{eff}\gtrsim\frac{\pi a_0}{4}. \tag{35}$$

Note that a rigorous calculation of a_{eff} in the 3D case as well as a rigorous analysis of trio formation requires again the exact solution of Skorniakov–Ter-Martirosian integral equations for a Fermi-Bose mixture and, as we already mentioned in Sec. III, will be the subject of a separate publication.

V. CONCLUSIONS

In conclusion we considered the appearance and pairing of composite fermions in a Fermi-Bose mixture with an attractive interaction between fermions and bosons.

At equal densities of elementary fermions and bosons, the system is described at low temperatures by a one-component attractive Fermi gas for composite fermions and is unstable towards quartet formation.

The problem which we considered is important for a theoretical understanding of high-temperature SC materials and for the investigation of Fermi-Bose mixtures of neutral particles at low and ultralow temperatures. In high-T_c superconductors the role of bosons is played by holons and the role of fermions is played by spinons. At high temperatures spinons and holons are unbound. At lower temperatures they are bound in composite fermions and, moreover, the composite fermions are further paired in quartets (singlet superconductive pairs). The radius of the quartets (the coherence length of the superconductive pair) is governed by the binding energy of the quartets $|E_4|$. If $|E_4|$ is larger than T_0, then the quartets are local: $p_F a_0<1$. Finally for $T_c=T_0/[8\ln\ln(4/na_0^2)]$ the local quartets are Bose condensed and the system becomes superconductive. Note thatat higher temperatures $T>T_0$ besides quartets some amount of trios is also present in the system. The role of trios is usually neglected in the standard theories of high-T_c superconductivity.

Note also that we consider a low-density limit $|E_b|\gg T_0$. In the opposite case of higher densities $T_0\gg|E_b|$, Bose-Einstein condensation of holons or biholons (see Refs.

[7,18,8]) takes place earlier than the creation of composite fermions and quartets. Such a state can be distinguished from the ordinary BCS superconductor by measuring the temperature dependence of the specific heat and normal density.

In Fermi-Bose mixtures our investigations enrich the superfluid phase diagram in magnetic and optical dipole traps and are important in connection with recent experiments, where weakly bound dimers $^6Li_2$ and $^{40}K_2$, consisting of two elementary fermions, were observed [29,30]. Note that in an optical dipole trap it is possible to get an attractive scattering length of the fermion-boson interaction with the help of Feshbach resonance [25]. Note also that even in the absence of Feshbach resonance it is experimentally possible now to create a Fermi-Bose mixture with an attractive interaction between fermions and bosons. For example in Refs. [31,32] such a mixture of ^{87}Rb (bosons) and ^{40}K (fermions) was experimentally studied. Moreover, the authors of Refs. [31,32] experimentally observed the collapse of a Fermi gas with the sudden disappearance of fermionic ^{40}K atoms when the system enters into the degenerate regime. We cannot exclude in principle that it is just a manifestation of the creation of quartets $\langle bc;bc\rangle$ in the system. Note that in the regime of a strong attraction between fermions and bosons phase separation with the creation of larger clusters or droplets is also possible. Note also that a much slower collapse in the Bose subsystem of ^{87}Rb atoms can be possibly explained by the fact that the number of Rb atoms in the trap is much larger than the number of K atoms, so after the formation of composite fermions a lot of residual bosons are still present in the system. A more thorough comparison of our results with an experimental situation will be the subject of a separate publication.

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