Mott scattering in an elliptically polarized laser field

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We study Mott scattering in the presence of a strong elliptically polarized field. Using the first Born approximation and the Dirac–Volkov states for the electron, we obtain an analytic formula for the unpolarized differential cross section. This generalizes the results found for the linearly polarized field by Li *et al.* [**67**, 063409 (2003)] and for the circularly polarized field by Attaourti and Manaut [**68**, 067401 (2003)].

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I. INTRODUCTION

In this paper we consider Mott scattering in the presence of a strong elliptically polarized field. Using the first Born approximation, we give complete analytical results; these reduce to those found in earlier work for linearly polarized field by Li *et al.* [1] and for circularly polarized field by Attaourti *et al.* [2,3]. An analytical expression for the spin-unpolarized differential cross section is derived using trace calculations. The electric field strength as well as the frequency of the laser field and the kinetic energy of the incoming electron are key parameters, the study of the process of Mott scattering in the presence of an elliptically polarized laser field introduces another key parameter, namely the degree of ellipticity η. The cross section dependence on this key parameter is reported. The general features of the Mott scattering process are qualitatively modified when a laser field is present and this is particularly true when one studies the spin-dependent relativistic Mott scattering. Not only it is important to take care of the fact that the electron is a Dirac particle but also to describe this particle by the appropriate wave function in a nonperturbative way. This is done by using the Dirac-Volkov wave functions [4] which contain the interaction of the electron with the laser field to all orders. The organization of this paper is as follows. In Sec. II, we present the theory in the first Born approximation. In Sec. III, we discuss the results for the spin-unpolarized differential cross section modified by the laser field and analyze their dependencies on the degree of ellipticity η. We end by a brief summary and conclusion in Sec. IV. Throughout this work, we use atomic units $\hbar=m=e=1$ and work with the metric tensor $g^{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$.

II. THEORY

We treat the laser field classically since we are considering intensities that do not allow pair creation [5]. The four potential corresponding to the laser field satisfies the Lorentz condition $\partial^\mu A_\mu=0$ and is given by

$$A = a_1 \cos(\phi) + a_2 \sin(\phi)\tan(\eta/2), \tag{1}$$

with $\phi=k\cdot x=k^\mu x_\mu=wt\text{-}\mathbf{k}\cdot\mathbf{x}$ and where η is the degree of ellipticity of the laser field. The four vectors a_1 and a_2 satisfy the following relations $a_1^2=a_2^2=a^2$ and the Lorentz condition $k\cdot A$ implies $a_1\cdot k=a_2\cdot k=0$. The linear polarization is obtained for $\eta=0$ and the circular polarization is obtained for $\eta=\pi/2$. The electric field associated with the potential of the laser field is

$$\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{A}. \tag{2}$$

Differential cross section: The interaction potential is the Coulomb potential of a target nucleus of charge Z,

$$A^\mu_{\mathrm{Coul}} = \left(-\frac{Z}{|\mathbf{x}|},0,0,0\right), \tag{3}$$

and in the first Born approximation, the transition matrix element for the transition $(i\rightarrow f)$ is

$$S_{fi} = \frac{iZ}{c}\int d^4x\, \overline{\psi}_{q_f}(x)\frac{\gamma^0}{|\mathbf{x}|}\psi_{q_i}(x). \tag{4}$$

The Dirac-Volkov wave functions $\psi_{q_i}(x)$ and $\psi_{q_f}(x)$ describe the incident and scattered electron, respectively. Such wave functions normalized to the volume V are [4]

$$\psi_q(x) = R(q)\frac{u(p,s)}{\sqrt{2QV}}e^{iS(q,x)}, \tag{5}$$

where $u(p,s)$ represents a free Dirac bispinor normalized as $\overline{u(p,s)}u(p,s)=2c^2$ and $q^\mu=(Q/c,\mathbf{q})$ is the quasimomentum acquired by the electron in the presence of the laser field

$$q^\mu = p^\mu - \frac{1}{2c^2(k\cdot q)}\overline{A^2}k^\mu. \tag{6}$$

The quantity $R(q)=R(p)$ is defined by

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$$\begin{aligned} R(q) &= R(p) \\ &= 1 + \frac{1}{2c(k\cdot q)} \not{k}\not{A} \\ &= 1 + \frac{1}{2c(k\cdot q)} \not{k}[\not{a}_1 \cos(\phi) + \not{a}_2 \sin(\phi)\tan(\eta/2)], \end{aligned} \tag{7}$$

where the Feynmann slash notation is used [6]: for a given four vector A, we have $\not{A}=\gamma^\mu A_\mu$. The phase $S(q,x)$ is such that

$$\begin{aligned} S(q,x) &= -p\cdot x - \int_0^{k\cdot x} \frac{1}{c(k\cdot p)}\left[p\cdot A(\xi) - \frac{1}{2c}A^2(\xi)\right]d\xi \\ &= -q\cdot x - \frac{(a_1\cdot p)}{c(k\cdot p)}\sin(\phi) + \frac{(a_2\cdot p)}{c(k\cdot p)}\cos(\phi)\tan(\eta/2) \\ &= -q\cdot x - \frac{(a_1\cdot q)}{c(k\cdot q)}\sin(\phi) + \frac{(a_2\cdot q)}{c(k\cdot q)}\cos(\phi)\tan(\eta/2). \end{aligned} \tag{8}$$

Finally the averaged squared potential $\overline{A^2}$ is given by

$$\overline{A^2} = a^2[1+\tan^2(\eta/2)]/2, \tag{9}$$

from which one deduces $\overline{A^2}=a^2$ for the case of a circular polarization of the laser field and $\overline{A^2}=a^2/2$ for the case of a linear polarization of the laser field. We transform the exponential of the two phases as

$$e^{-i[S(q_f,x)-S(q_i,x)]} = e^{i[(q_f-q_i)\cdot x - z\sin(\phi-\phi_0)]}. \tag{10}$$

In Eq. (10), the argument z is

$$z = \sqrt{\alpha_1^2+\alpha_2^2}, \tag{11}$$

with

$$\alpha_1 = \frac{(a_1\cdot p_i)}{c(k\cdot p_i)} - \frac{(a_1\cdot p_f)}{c(k\cdot p_f)}, \tag{12}$$

and

$$\alpha_2 = \left[\frac{(a_2\cdot p_i)}{c(k\cdot p_i)} - \frac{(a_2\cdot p_f)}{c(k\cdot p_f)}\right]\tan(\eta/2). \tag{13}$$

Therefore, we can recast the transition matrix element in the form

$$\begin{aligned} S_{fi} = \frac{iZ}{c}\int d^4x \frac{1}{\sqrt{2Q_iV}}\frac{1}{\sqrt{2Q_fV}}\overline{u}(p_f,s_f)[C_0 + C_1\cos(\phi) \\ + C_2\sin(\phi) + C_3\cos(2\phi)]u(p_i,s_i)\exp[i(q_f-q_i)x \\ - iz\sin(\phi-\phi_0)], \end{aligned} \tag{14}$$

where the four coefficients C_i, $i=0$, 1,2,3, are given by

$$\begin{aligned} C_0 &= \gamma^0 - k^0a^2\not{k}c(p_i)c(p_f)[1+\tan^2(\eta/2)], \\ C_1 &= c(p_i)\gamma^0\not{k}\not{a}_1 + c(p_f)\not{a}_1\not{k}\gamma^0, \\ C_2 &= [c(p_i)\gamma^0\not{k}\not{a}_2 + c(p_f)\not{a}_2\not{k}\gamma^0]\tan(\eta/2), \\ C_3 &= k^0a^2\not{k}c(p_i)c(p_f)[\tan^2(\eta/2)-1], \end{aligned} \tag{15}$$

and $c(p)=1/[2c(k\cdot p)]$.

We now invoke the well-known identities for the ordinary Bessel functions [6]

$$\left.\begin{matrix} 1 \\ \cos(\phi) \\ \sin(\phi) \\ \cos(2\phi) \end{matrix}\right\} \times e^{[-iz\sin(\phi-\phi_0)]} = \sum_{n=-\infty}^{+\infty} \left\{\begin{matrix} B_n \\ B_{1n} \\ B_{2n} \\ B_{3n} \end{matrix}\right\} \times e^{(-in\phi)}, \tag{16}$$

with $\phi_0=\arctan(\alpha_2/\alpha_1)$ and

$$\left\{\begin{matrix} B_n \\ B_{1n} \\ B_{2n} \\ B_{3n} \end{matrix}\right\} = \left\{\begin{matrix} J_n(z)e^{in\phi_0} \\ [J_{n+1}(z)e^{i(n+1)\phi_0} + J_{n-1}(z)e^{i(n-1)\phi_0}]/2 \\ [J_{n+1}(z)e^{i(n+1)\phi_0} - J_{n-1}(z)e^{i(n-1)\phi_0}]/2i \\ [J_{n+2}(z)e^{i(n+2)\phi_0} + J_{n-2}(z)e^{i(n-2)\phi_0}]/2 \end{matrix}\right\}. \tag{17}$$

Using the standard procedures of QED [6], we obtain for the spin-unpolarized differential cross section evaluated for $Q_f=Q_i+nw$,

$$\frac{d\overline{\sigma}}{d\Omega_f} = \sum_{n=-\infty}^{+\infty}\frac{d\overline{\sigma}^{(n)}}{d\Omega_f}, \tag{18}$$

with

$$\frac{d\overline{\sigma}^{(n)}}{d\Omega_f} = \frac{Z^2}{c^4}\frac{|\mathbf{q}_f|}{|\mathbf{q}_i|}\frac{1}{|\mathbf{q}_f-\mathbf{q}_i-n\mathbf{k}|^4}\left(\frac{1}{2}\sum_{s_i,s_f}|M_{fi}^{(n)}|^2\right). \tag{19}$$

Using REDUCE for the trace calculations [8,9], we obtain

$$\begin{aligned} \frac{1}{2}\sum_{s_i,s_f}|M_{fi}^{(n)}|^2 = 2\{J_n^2(z)A + [J_{n+1}^2(z)+J_{n-1}^2(z)]B + J_{n+1}(z)J_{n-1}(z)C + J_n(z)[J_{n+1}(z)+J_{n-1}(z)]D + J_n(z)[J_{n+2}(z)+J_{n-2}(z)]E + [J_{n+2}^2(z) \\ + J_{n-2}^2(z)]F + [J_{n-1}(z)J_{n+2}(z) + J_{n+1}(z)J_{n-2}(z)]G + [J_{n+1}(z)J_{n+2}(z) + J_{n-1}(z)J_{n-2}(z)]H\}. \end{aligned} \tag{20}$$

The eight coefficients A, B, C, D, E, F, G and H are given, respectively, by

$$A = c^4 - (q_i\cdot q_f)c^2 + 2Q_iQ_f - \frac{a^2}{2}\left(\frac{(k\cdot q_f)}{(k\cdot q_i)} + \frac{(k\cdot q_i)}{(k\cdot q_f)}\right) + \frac{a^2w^2}{c^2(k\cdot q_i)(k\cdot q_f)}[(q_i\cdot q_f) - c^2][1+\tan^2(\eta/2)]/2 + \frac{(a^2)^2w^2}{c^4(k\cdot q_i)(k\cdot q_f)}\left(\frac{1}{8}\tan^4(\eta/2) + \frac{5}{8} + \frac{1}{4}\tan^2(\eta/2)\right) + \frac{a^2w}{c^2}\left[\frac{Q_f}{(k\cdot q_i)} + \frac{Q_i}{(k\cdot q_f)} - \left(\frac{Q_i}{(k\,.\,q_i)} + \frac{Q_f}{(k\cdot q_f)}\right)[1+\tan^2(\eta/2)]/2\right], \tag{21}$$

$$B = \frac{w^2}{2c^2}\left(\frac{(a_1\cdot q_i)(a_1\cdot q_f)}{(k\cdot q_i)(k\cdot q_f)} + \frac{(a_2\cdot q_i)(a_2\cdot q_f)}{(k\cdot q_i)(k\cdot q_f)}\tan^2(\eta/2)\right) - \left\{\frac{a^2}{2} + \frac{(a^2)^2w^2}{2c^4(k\cdot q_i)(k\cdot q_f)} - \frac{a^2}{4}\left(\frac{(k\cdot q_f)}{(k\cdot q_i)} + \frac{(k\cdot q_i)}{(k\cdot q_f)}\right) + \frac{a^2w^2}{2c^2(k\cdot q_i)(k\,.\,q_f)}[(q_i\cdot q_f) - c^2] - \frac{a^2w}{2c^2}(Q_f - Q_i)\left(\frac{1}{(k\cdot q_f)} - \frac{1}{(k\cdot q_i)}\right)\right\}[1+\tan^2(\eta/2)]/2, \tag{22}$$

$$C = \frac{w^2}{c^2(k\cdot q_i)(k\cdot q_f)}(\cos(2\phi_0)\{(a_1\cdot q_i)(a_1\cdot q_f) - (a_2\cdot q_i)(a_2\cdot q_f)\tan^2(\eta/2)\} + \sin(2\phi_0)\{(a_2\cdot q_i)(a_1\cdot q_f) + (a_1\cdot q_i) \times (a_2\cdot q_f)\}\tan(\eta/2)) - \left\{\frac{a^2}{4}\left(\frac{(k\cdot q_f)}{(k\cdot q_i)} + \frac{(k\cdot q_i)}{(k\cdot q_f)}\right) - \frac{a^2w^2}{2c^2(k\cdot q_i)(k\cdot q_f)}[(q_i\cdot q_f) - c^2] - \frac{(a^2)^2w^2}{2c^4(k\cdot q_i)(k\cdot q_f)} + \frac{a^2w}{2c^2}(Q_i - Q_f) \times\left(\frac{1}{(k\cdot q_f)} - \frac{1}{(k\cdot q_i)}\right) - \frac{a^2}{2}\right\}\cos(2\phi_0)[\tan^2(\eta/2) - 1], \tag{23}$$

$$D = -\frac{c}{2}\left(\frac{(k\cdot q_f)}{(k\cdot q_i)}(\text{Å}\cdot q_i) + \frac{(k\cdot q_i)}{(k\cdot q_f)}(\text{Å}\cdot q_f)\right) + \frac{w}{c}\left(\frac{Q_i(\text{Å}\cdot q_f)}{(k\cdot q_f)} + \frac{Q_f(\text{Å}\cdot q_i)}{(k\cdot q_i)}\right) + \frac{c}{2}[(\text{Å}\cdot q_i) + (\text{Å}\cdot q_f)] - \frac{a^2w^2}{4c^3(k\cdot q_i)(k\cdot q_f)}[\tan^2(\eta/2) - 1][(\text{Å}\cdot q_i) + (\text{Å}\cdot q_f)], \tag{24}$$

$$E = \cos(2\phi_0)[\tan^2(\eta/2) - 1)a^2w\left\{-\frac{(q_i\cdot q_f)w}{4c^2(k\cdot q_i)(k\cdot q_f)} + \frac{1}{4c^2}\left(\frac{Q_i}{(k\cdot q_i)} + \frac{Q_f}{(k\cdot q_f)}\right) - \frac{a^2w}{8c^4(k\cdot q_i)(k\cdot q_f)} + \frac{w}{4(k\cdot q_i)(k\cdot q_f)} - \frac{a^2w}{8c^4(k\cdot q_i)(k\cdot q_f)}\tan^2(\eta/2)\right\}, \tag{25}$$

$$F = [\tan^2(\eta/2) - 1]^2\left(\frac{(a^2)^2w^2}{32c^4(k\cdot q_i)(k\cdot q_f)}\right), \tag{26}$$

$$G = [\tan^2(\eta/2) - 1]\frac{a^2w^2}{8c^3(k\cdot q_i)(k\cdot q_f)}\{\cos(3\phi_0)[(a_1\cdot q_i) + (a_1\cdot q_f)] + \sin(3\phi_0)[(a_2\cdot q_i) + (a_2\cdot q_f)]\tan(\eta/2)\}, \tag{27}$$

$$H = [\tan^2(\eta/2) - 1]\frac{a^2w^2}{8c^3(k\cdot q_i)(k\cdot q_f)}\{\cos(\phi_0)[(a_1\,.\,q_i) + (a_1\cdot q_f)] - \sin(\phi_0)[(a_2\cdot q_i) + (a_2\cdot q_f)]\tan(\eta/2)\}, \tag{28}$$

where $\text{Å} = a_1\cos(\phi_0) + a_2\sin(\phi_0)\tan(\eta/2)$. In the absence of the laser field, all the contributions coming from the sum over n of the various ordinary Bessel functions vanish except for $n=0$ where $J_n(0)=\delta_{n0}$ and we recover the well-known formula for Mott scattering in the absence of the laser field [6]

$$\frac{d\overline{\sigma}}{d\Omega_f} = \frac{1}{4}\frac{Z^2\alpha^2}{|\mathbf{p}|^2\beta^2}\frac{[1-\beta^2\sin^2(\theta/2)]}{\sin^4(\theta/2)}, \tag{29}$$

where $\theta = \widehat{(\mathbf{p}_i, \mathbf{p}_f)}$. It can easily be checked that for the case of a linear polarization of the laser field ($\eta=0$), the phase $\phi_0=0$ and the results of Li *et al.* [1] are straightforward to obtain whereas for the circular polarization ($\eta=\pi/2$), we find the results previously found by Attaourti *et al.* [2]. Equation (18) is the relativistic generalization of the Bunkin and Fedorov [7] treatment and it contains the degree of ellipticity of the laser field.

III. RESULTS AND DISCUSSION

In this section, we discuss the numerical simulations for the differential cross sections of the Mott scattering by an elliptically polarized laser field. We assume without loss of generality that the target is a proton having a charge $Z_p=1$. The dependence of the unpolarized DCS on the atomic charge is quadratic. For an arbitrary target with atomic charge Z, the unpolarized DCS is related to that corresponding to a proton target by $d\sigma(Z)/d\Omega_f=Z^2\, d\sigma(Z_p=1)/d\Omega_f$, The z axis is set along the direction of the field wave vector $\mathbf{k}$, $a_1^\mu=(0,\mathbf{a}_1)$ and $a_2^\mu=(0,\mathbf{a}_2)$ with the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ such that $\mathbf{a}_1=|\mathbf{a}|\ (1,0,0)$ and $\mathbf{a}_2=|\mathbf{a}|\ (0,1,0)$ from which one deduces that $\overline{A^2}=a^2[1+\tan^2(\eta/2)]/2=-|\mathbf{a}|^2[1+\tan^2(\eta/2)]/2$. Thus, as found by Li [1] for the linear polarization, we have $\overline{A^2}=-|\mathbf{a}|^2/2$ and for the case of circular polarization [2], $\overline{A^2}=-|\mathbf{a}|^2$. The laser frequency used is $w=0.043$ (a.u.) which corresponds to a photon energy $\hbar w=1.17$ eV. The incident electron kinetic energy is $T_i=2.7$ keV which corresponds to a relativistic parameter $\gamma=(1-\beta^2)^{-1/2}=1.0053$ and the electric field strength has the value $\mathcal{E}=0.05$ (a.u.)$=2.57\times10^8$ V/cm. Let θ_f and ϕ_f be the polar angles of the three-dimensional momentum $\mathbf{p}_f$ and let θ_i and ϕ_i be the polar angles of the three-dimensional momentum $\mathbf{p}_i$. The various differential cross sections are plotted as functions of the angle θ_f. For small scattering angles, typically $(1°\leqslant\theta_i\leqslant15°,1°\leqslant\phi_i\leqslant15°)$, $(-180°\leqslant\theta_f\leqslant180°,\phi_f=\phi_i+90°)$, the summed spin-unpolarized differential cross sections are sharply peaked around $\theta_f=0°$ and are all close to the corresponding unpolarized laser-free differential cross section given in Eq. (29). The three DCSs corresponding to $\eta=0$, $\eta=2\pi/3$, and $\eta=\pi/2$ are given in Fig. 1 together with the laser-free DCS for the geometry $(\theta_i=15°,\phi_i=15°)$ $(-180°\leqslant\theta_f\leqslant180°,\phi_f=\phi_i+90°)$. In the three figures, the expression scaled in 10^{-m} (where m is an positive integer) means that the DCSs read on the ordinate axis must be multiplied by this number in order to obtain the appropriate results in (a.u.). We obtain four almost indistinguishable curves. The reason why these four curves are indistinguishable is the following: in the nonrelativistic regime, the convergence to the unpolarized laser-free DCS is rapid. For the linear polarization of the laser field, we have summed over ±150 photons, for the circular polarization, we have summed over ±350 photons and finally, for the elliptical polarization of the laser field corresponding to a degree of ellipticity $\eta=2\pi/3$, we have summed over ±1250 photons. The situation is more complex in the relativistic regime since due to a lack of high speed computing facilities, we cannot sum over a very large number of photons exchanged. However, with a Cray computer, this point of interest to researchers can easily be answered to. At small angles and in the nonrelativistic regime, the summed unpolarized differential cross sections are almost unmodified by the laser field and its polarization does not play a key role. The physical explanation of this observation is that classically, when the particles are close to the small angle scattering region, this corresponds to large impact parameters and the incident electron does not deviate notably from its trajectory. For other scattering angles $(45°\leqslant\theta_i\leqslant89°,45°\leqslant\phi_i\leqslant89°)$ $(-180°\leqslant\theta_f\leqslant180°,\phi_f=\phi_i+90°)$, the situation changes drastically since for medium and large scattering angles, the momentum transfer during the Mott scattering is large and a significant number of photons can be exchanged with the laser field. In Fig. 2, we compare the three DCSs corresponding to $\eta=0$, $\eta=2\pi/3$, and $\eta=\pi/2$ together with the laser-free DCS for the geometry $(\theta_i=60°,\phi_i=0°)$ $(-180°\leqslant\theta_f\leqslant180°,\phi_f=90°)$ for an exchange of ±150 photons. In this geometry, the effect of the laser field polarization is clearly shown since the three DCSs are now well distinguishable. One has to sum over a very large number of photons to recover the laser-free DCS. Furthermore, all numerical simulations have shown the following. The DCS for linear polarization is always higher than the two others. The DCS for elliptical polarization is lower or higher than the DCS for circular polarization depending on the value of the degree of ellipticity η. To have an idea about the behavior of the DCS as a function of the degree of ellipticity η, we show in Fig. 3 a three-dimensional curve for a degree of ellipticity η varying from 0 to $\pi/2$. As we used the software SURFER, the axis representing the degree of ellipticity η varies only from 0 to 100, thereby constraining us to scale this axis in multiples of $\pi/200$. To illustrate this, let us take an arbitrary point on this axis, e.g., 20. This means that the actual degree of ellipticity is $20\times\pi/200=\pi/10$. The oscillations of the DCS for the elliptical polarization are shown and the DCS for linear polarization is always the highest DCS. These oscillations decrease as the number of photons exchanged is increased. The convergence towards the laser-free DCS is faster for the linear polarization of the laser field. For the circular polarization, this convergence is easily obtained

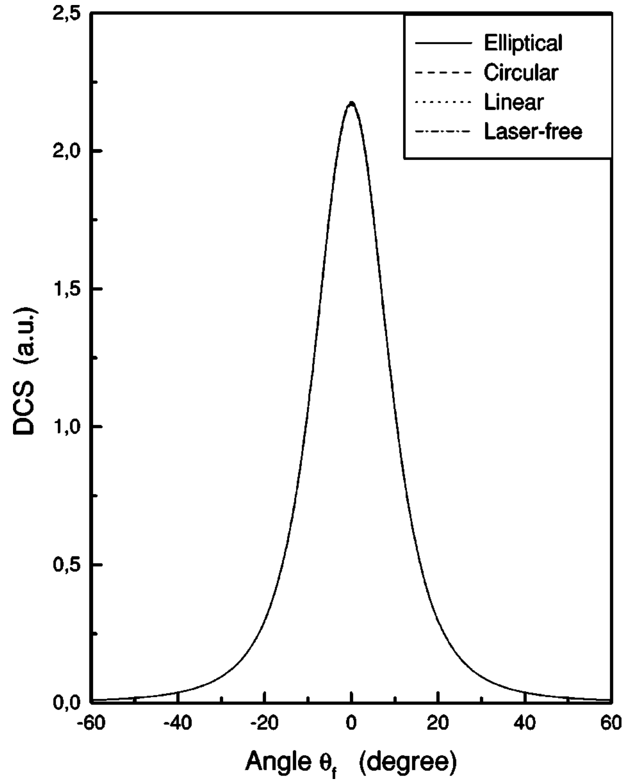

FIG. 1. The summed spin-unpolarized cross sections for an incident electron kinetic energy of 2.7 keV as a function of the angle θ_f scaled in 10^{-2}. The electric field strength is $\mathcal{E}=0.05$ (a.u.) and the laser frequency is $w=0.043$ (a.u.). The parameters of the geometry are $\theta_i=\phi_i=15°$, $\phi_f=105°$.

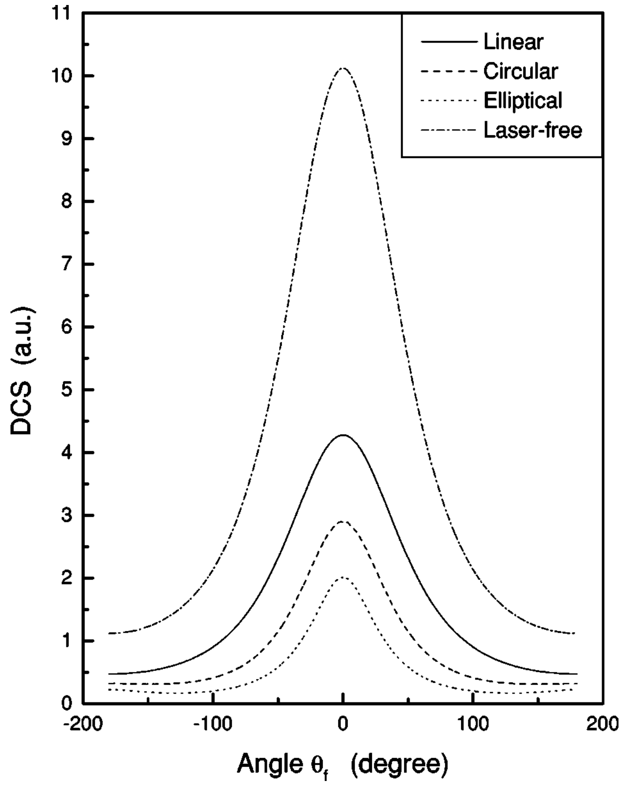

FIG. 2. The summed spin-unpolarized cross sections for an exchange of ± 150 photons scaled in 10^{-5}. As in Fig. 1, $\mathcal{E}=0.05$ (a.u.) and $w=0.043$ (a.u.). The parameters of the geometry are $\theta_i=60°$, $\phi_i=0°$, and $\phi_f=90°$.

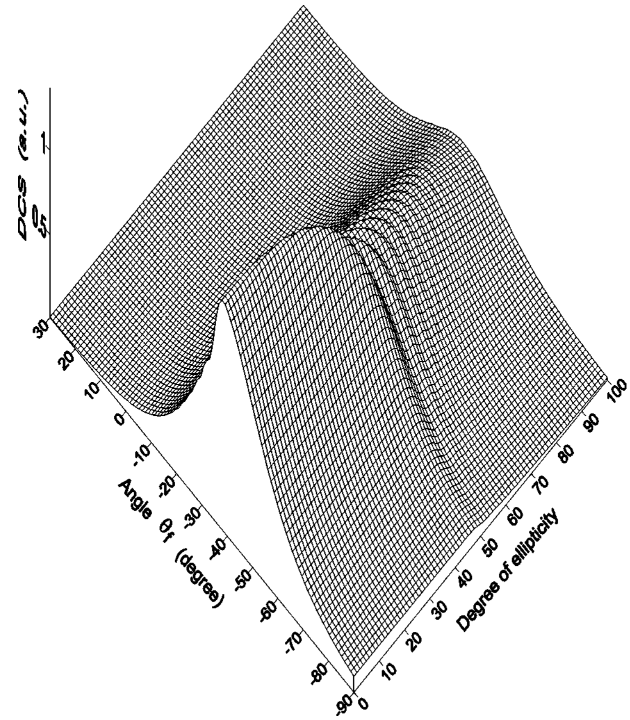

FIG. 3. The summed spin-unpolarized cross section for an exchange of ± 40 photons scaled in 10^{-3}, and for a degree of ellipticity varying from $0 \rightarrow 100$ but scaled in $\pi/200$, $\mathcal{E}=0.025$ (a.u.) and $w=0.043$ (a.u.). The parameters of the geometry are $\theta_i=45°$, $\phi_i=0°$, and $\phi_f=135°$.

when one finds the value for the convergence corresponding to the linear polarization whereas it is much more difficult to infer from the previous results for which value of the number of photons exchanged, the DCS for the elliptical polarization will converge to the laser-free DCS. However, depending on the value of η and in the nonrelativistic regime we have chosen, this number is ± 1250 photons. When the incident electron relativistic parameter is increased from $\gamma=1.0053$ to $\gamma=(1-\beta^2)^{-1/2}=2$, the previous mentioned results remain valid but the corresponding DCSs are very small indicating a small probability that the very fast projectile electron will exchange photons with the radiation field. As for the behavior of the DCSs with respect to the degree of ellipticity, the elliptically and circularly polarized laser modified cross sections become more sharply peaked around the angle $\theta_f=0°$. A similar result has been reported [5].

IV. CONCLUSIONS

In this work, we have extended the study of the Mott scattering process of an electron by a charged nucleus to the case of a general polarization. We have shown that the Mott scattering geometry as well as the key parameters such as the electric field strength and the incident electron kinetic energy influence the behavior of the DCSs. Moreover, the degree of ellipticity η is also a key parameter for the description of the Mott scattering process particularly in the region of large momentum transfer and for a number of photons exchanged lower than that for which the DCSs tend to the laser-free one.

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