Extended theory of Kapitza-Dirac scattering

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The standard theoretical treatment of the Kapitza-Dirac effect—that is, the scattering of an electron passing through a standing-wave laser field—is extended here through the use of the Pauli equation to account for the interaction of the electron spin with the magnetic field of the standing wave. Prescriptions for determining unitarity-preserving approximations for the transition probabilities for scattering both with and without rotation of the electron spin direction are provided. This formalism is used to develop a perturbation theory for the spin-flip probability which, in the strong-field limit of interest here, reduces to a fairly simple relation between *S*-matrix elements for scattering with and without change in spin orientation, each expressed in terms of a Bessel function. A similar perturbative procedure is applied to estimate corrections to the standard theory for scattering in the absence of spin-flip processes, in which interactions that change the net number of photons in the field are ignored.

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I. INTRODUCTION

The recent observation [1] of the diffraction of an electron beam passing through a standing wave, first discussed by Kapitza and Dirac [2], has had the effect of reinvigorating the subject, opening the door to a variety of new experiments; some proposals are discussed in Ref. [1]. This experimental development also provides the impetus to test the limits of validity of current calculational methods and to consider an extension of the range of applications. Regarding earlier theoretical work on the Kapitza-Dirac effect, we mention specifically the high-intensity limit studied by Fedorov [3] and approximations based on expansions of the wave function in terms of Mathieu functions [4,5]. It was pointed out in [1] that the feasibility of observing electron spin flip in its passage through an intense standing-wave field remains in doubt. This suggests that an extension of the current theory to account for spin interactions along with diffraction might be of interest. Here we develop a theoretical treatment that allows for an examination of strong-field effects in a systematic way.

The theory of the Kapitza-Dirac effect is developed, in Sec. II (with some details recorded in the Appendix), in the context of the Pauli equation. A formal procedure is established for determining the probabilities for momentum shifts of the electron due to interaction with the standing-wave field, both with and without rotation of electron spin direction through 180°. Since the spin-flip process is nonresonant, the effect is expected to be small. Whether it builds up to observable magnitudes for sufficiently intense fields is not clear at the outset. For this reason we have applied the formalism to the development of a perturbation theory for the determination of the spin-flip amplitude; as described in Sec. III, it takes on a particularly simple form in the strong-field limit, making the dependence of that amplitude on field parameters quite explicit. A similar perturbative approach is applied to account for corrections to the standard calculation [4], in which spin interactions are ignored and which involves a time average of the electron-field interaction. In this way we are able to assess the validity of that approximation when laser fields of very high intensity are considered. Results are discussed in Sec. IV.

II. EFFECT OF ELECTRON SPIN

The Pauli equation describing the motion of an electron in an external field has the form, in units with $\hbar=1$:

$$i\frac{\partial}{\partial t}|\Psi\rangle = H|\Psi\rangle. \tag{2.1}$$

The Hamiltonian is written, in the occupation-number representation, with $\boldsymbol{\sigma}$ representing the triplet of Pauli spin matrices, as

$$H = \frac{(\mathbf{p}_{\rm op} - e\mathbf{A}/c)^2}{2m_{\rm e}} - \frac{e}{2m_{\rm e}c}\boldsymbol{\sigma}\cdot\mathbf{B} + H_{\rm F}, \tag{2.2}$$

where $H_{\rm F}$ is the field energy, $\mathbf{B}=\boldsymbol{\nabla}\times\mathbf{A}$ is the magnetic intensity and $\mathbf{A}$, the vector potential in the Coulomb gauge, is $\mathbf{A}=\mathbf{A}_1+\mathbf{A}_2$, with components

$$\mathbf{A}_i = A_0\hat{\mathbf{x}}\{a_i\exp[i\mathbf{k}_i\cdot\mathbf{r}] + a_i^\dagger\exp[-i\mathbf{k}_i\cdot\mathbf{r}]\}, \quad i=1,2. \tag{2.3}$$

The field eigenstates are denoted as $|n_1,n_2\rangle$, with energies $(n_1+n_2)\omega$ and momenta $n_1\mathbf{k}_1+n_2\mathbf{k}_2$. We take $\mathbf{k}_1\equiv\mathbf{k}$ to be directed along the z axis, with $\mathbf{k}_2=-\mathbf{k}_1$. The electron enters the field, at time $t=0$, with momentum in the y-z plane; the y component is taken to be fixed during passage through the field and reference to it is suppressed in the following. The z component has the initial value $\mathbf{p}+m\mathbf{k}$. We write $\mathbf{p}=-s\mathbf{k}$ with $-1\leqslant s\leqslant 1$ and m an integer. The quantization axis for spin is taken to be the z axis, the electron entering with positive spin projection.

With the spin interaction ignored for the moment, we introduce the approximation, standard in treatments of the Kapitza-Dirac effect, in which contributions to the A^2 term in the interaction energy that change the net number of photons in the field are removed. (This is analogous to the rotating-

wave approximation in a semiclassical description, in which terms that vary rapidly in time are averaged to zero. We return in Sec. III to consider corrections to this approximation.) A suitable set of basis functions, accounting for those interactions (expected to be dominant owing to their near-resonant character) in which a photon is absorbed from one mode and emitted into the other, may be taken as

$$|\chi_j^{(u)}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} |n+j, n-j\rangle |\mathbf{p} - 2j\mathbf{k}\rangle, \quad j = 0, \pm 1, \pm 2, \ldots \tag{2.4a}$$

(The superscript u reminds us that the photon number is unchanged in this basis.) The photon number n, amplitude A_0, and field intensity I satisfy the relation $nA_0^2 = 2\pi Ic/\omega^2$.

Now with the spin interaction included, processes are allowed in which spin flip is accompanied by either the absorption or emission of a photon. To account for absorption we introduce the basis set

$$|\chi_j^{(a)}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} |n+j-1, n-j\rangle |\mathbf{p} - (2j-\mathbf{1})\mathbf{k}\rangle, \tag{2.4b}$$

while emission processes require introduction of the set

$$|\chi_j^{(e)}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} |n+j+1, n-j\rangle |\mathbf{p} - (2j+1)\mathbf{k}\rangle. \tag{2.4c}$$

The wave function is expanded in a complete set of eigenstates satisfying

$$(H - E^{(v)})|\Theta^{(v)}\rangle = 0. \tag{2.5}$$

Here v is a "channel" index that specifies the initial state of the system. We require three different channels to form a complete set. Consider first an initial state given by Eq. (2.4a) with $j = -m/2$ and m an even integer—that is, one with spin up and longitudinal momentum $\mathbf{p} + m\mathbf{k}$. With the channel labeled v_u we look for a solution in the form

$$|\Theta^{(v_u)}\rangle = \sum_{j=-\infty}^{\infty} \{|\chi_j^{(u)}\rangle \xi_{2j+m}^{(v_u)} + |\chi_j^{(a)}\rangle \xi_{2j-1+m}^{(v_u)} + |\chi_j^{(e)}\rangle \eta_{2j+1+m}^{(v_u)}\}. \tag{2.6}$$

As indicated, the initial state, prior to the sudden onset of the interaction, is $|\chi_{-m/2}^{(u)}\rangle$, and this requires that in the limit of vanishing coupling strength $\xi_{2j+m}^{(v_u)}$ reduce to $\delta_{j,-m/2}$; the coefficients $\xi_{2j-1+m}^{(v_u)}$ and $\eta_{2j+1+m}^{(v_u)}$ must vanish in that limit. As will be seen, this behavior is consistent with the recurrence relations that are satisfied by the expansion coefficients. These relations are obtained by inserting the expansion (2.6) into the eigenvalue equation (2.5) and projecting the result successively onto the basis states defined in Eqs. (2.4), making use of the orthonormality relations satisfied by these states. The recurrence relations thus derived are written out in the Appendix. The dimensionless strength parameters d and q (they supply measures of the A^2 and spin-flip interactions, respectively) that appear in these relations are defined according to

$$d\frac{k^2}{2m_e} = n\frac{(eA_0)^2}{m_e c^2}, \tag{2.7}$$

$$q\frac{k^2}{2m_e} = \sqrt{n}(eA_0)\frac{\omega}{2m_e c^2}. \tag{2.8}$$

The energy eigenvalue is represented, with $\omega = kc$, as

$$E^{(v_u)} = \frac{(\mathbf{p} + m\mathbf{k})^2}{2m_e} + 2n\omega + \lambda^{(v_u)}\frac{k^2}{2m_e}. \tag{2.9}$$

The three recursion relations, Eqs. (A1)–(A3), for this channel may be combined into a single matrix eigenvalue equation, which, after truncation, may be solved by standard means. This procedure becomes increasingly difficult as the field strength is increased owing to the size of the matrix required for convergence. Fortunately, in the intense-field limit a simpler perturbative procedure may be derived, based on the formalism developed here. We turn to that below, in Sec. III.

To obtain a complete set of states two additional eigenstates $|\Theta^{(v_a)}\rangle$ and $|\Theta^{(v_e)}\rangle$ must be defined. In the absence of the electron-field interaction the former reduces to $|\chi_j^{(a)}\rangle$ with $2j-1+m=0$ and the latter to $|\chi_j^{(e)}\rangle$ with $2j+1+m=0$, with m an odd integer. These states may be expanded in terms of the basis states, as in Eq. (2.6), each eigenstate assigned its own set of expansion coefficients. The associated energy eigenvalues are written as

$$E^{(v_a)} = \frac{(\mathbf{p} + m\mathbf{k})^2}{2m_e} + 2n\omega - \omega + \lambda^{(v_a)}\frac{k^2}{2m_e} \tag{2.10a}$$

and

$$E^{(v_e)} = \frac{(\mathbf{p} + m\mathbf{k})^2}{2m_e} + 2n\omega + \omega + \lambda^{(v_e)}\frac{k^2}{2m_e}. \tag{2.10b}$$

The recursion relations take the same form as those shown in Eqs. (A1)–(A3), but [accounting for the frequency shifts in Eqs. (2.10)] with $\lambda^{(v_a)}$ replaced by $\lambda^{(v_a)} - \bar{\omega}$ for channel v_a and $\lambda^{(v_e)}$ replaced by $\lambda^{(v_e)} + \bar{\omega}$ for channel v_e. The dimensionless frequency $\bar{\omega}$ is defined as $\bar{\omega} = \omega(2m_e/k^2) = 2m_e c^2/\omega$.

The expansion of the wave function takes the form

$$|\Psi_i(t)\rangle = \sum_v \exp[-iE^{(v)}t]|\Theta^{(v)}\rangle\langle\Theta^{(v)}|\Psi_i(0)\rangle. \tag{2.11}$$

The completeness property ensures that $|\Psi_i(t)\rangle$ reduces to $|\Psi_i(0)\rangle$ at time $t=0$ when the electron enters the field in the state $|\chi_i^{(u)}\rangle$. With the replacement of the eigenstates $\Theta^{(v)}$ in Eq. (2.11) by their expansions in basis states, that relation becomes

$$|\Psi_i(t)\rangle = \sum_j \{|\chi_j^{(u)}\rangle S_{uj;ui}(t) + |\chi_j^{(a)}\rangle S_{aj;ui}(t) + |\chi_j^{(e)}\rangle S_{ej;ui}(t)\}. \tag{2.12}$$

The probability amplitude for scattering into state $|\chi_j^{(u)}\rangle$ after a time t in the field is

$$S_{uj;ui}(t) = \sum_{m\ \text{even}} \exp[-iE^{(v_u)}t]\zeta^{(v_u)}_{2j+m}\zeta^{(v_u)}_{2i+m} + \sum_{m\ \text{odd}} \exp[-iE^{(v_a)}t]\zeta^{(v_a)}_{2j+m}\zeta^{(v_a)}_{2i+m} + \sum_{m\ \text{odd}} \exp[-iE^{(v_e)}t]\zeta^{(v_e)}_{2j+m}\zeta^{(v_e)}_{2i+m}. \quad (2.13)$$

The S-matrix element $S_{aj;ui}(t)$ has a similar expansion but with coefficient $\zeta^{(v_c)}_{2j+m}$ replaced by $\xi^{(v_c)}_{2j-1+m}$ for channels $c=u,a$, and e. Similarly, $S_{ej;ui}(t)$ is obtained by replacing $\zeta^{(v_c)}_{2j+m}$ with $\eta^{(v_c)}_{2j+1+m}$ in the expression on the right-hand side of Eq. (2.13). We note for future reference that, according to these representations, the scattering matrices satisfy the symmetry and time-reversal properties $S_{cj;c'i}(t)=S_{ci;c'j}(t)$ and $S_{cj;c'i}(-t)=S^*_{cj;c'i}(t)$.

The initial condition on the wave function may be expressed in terms of the behavior of the S matrix at the time $t=0$, when the field is turned on suddenly; explicitly, we have $S_{uj;ui}(0)=\delta_{ji}$, $S_{aj;ui}(0)=0$, and $S_{ej;ui}(0)=0$. With the S matrix expanded as in Eq. (2.13) and in its analogs for the other two channels, these initial conditions take the form of sum rules satisfied by the expansion coefficients. Another set of sum rules is obtained by identifying $v_c=\{m,c\}$ and $v'_{c'}=\{m',c'\}$ and writing the orthonormality property of the eignestates as

$$\langle \Theta^{(v_c)} | \Theta^{(v'_{c'})} \rangle = \delta_{mm'}\delta_{c,c'}. \quad (2.14)$$

Insertion of the eigenfunction expansions then provides us with the relations

$$\sum_j \{\zeta^{(v_c)}_{2j+m}\zeta^{(v'_{c'})}_{2j+m'} + \xi^{(v_c)}_{2j-1+m}\xi^{(v'_{c'})}_{2j-1+m'} + \eta^{(v_c)}_{2j+1+m}\eta^{(v'_{c'})}_{2j+1+m'}\} = \delta_{mm'}\delta_{c,c'}. \quad (2.15)$$

These sum rules can provide a useful check on numerical solutions. Taken together they may be used to verify the unitarity relation

$$\sum_j \{|S_{uj;ui}(t)|^2 + |S_{aj;ui}(t)|^2 + |S_{ej;ui}(t)|^2\} = 1. \quad (2.16)$$

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III. PERTURBATION THEORY

We may anticipate a relatively small probability for transitions with a change of spin orientation and this suggests the use of perturbation theory. In lowest order the scattering matrix is determined by ignoring the spin-dependent interaction—that is, by setting q equal to zero in the recurrence relation (A1) [6]. The coefficients that are then determined (for example, by converting the recurrence relation to an eigenvalue problem involving a finite, tridiagonal matrix) provide a first approximation,

$$S_{uj;ui}(t) \cong \sum_{m\ \text{even}} \exp[-iE^{(v_u)}t]\zeta^{(v_u)}_{2j+m}\zeta^{(v_u)}_{2i+m}, \quad (3.1)$$

for the scattering matrix. From the wave equation (2.1) we have, writing the spin interaction operator as $h'_{\text{op;spin}}$,

$$\langle \chi^{(a)}_j | \left(i\frac{\partial}{\partial t} - H_0 - h'_{\text{op;spin}} \right) | \Psi_i(t) \rangle = 0. \quad (3.2)$$

With the wave function represented as shown in Eq. (2.12) and with the use of the orthogonality property of the basis functions, Eq. (3.2) becomes

$$\langle \chi^{(a)}_j | \left(i\frac{\partial}{\partial t} - H_0 \right) \sum_{j'} |\chi^{(a)}_{j'}\rangle S_{aj';ui}(t) = \langle \chi^{(a)}_j | h'_{\text{op;spin}} \sum_{j'} |\chi^{(u)}_{j'}\rangle S_{uj';ui}(t), \quad (3.3a)$$

where the relation

$$\langle \chi^{(a)}_j | \left(i\frac{\partial}{\partial t} - H_0 \right) \sum_{j'} |\chi^{(u)}_{j'}\rangle S_{uj';ui}(t) = 0, \quad (3.3b)$$

valid in lowest order, has been used. We introduce some notation for the matrix elements that appear here. The tridiagonal matrix $E^{(a)}$ is defined as $E^{(a)}_{jj'}=\langle \chi^{(a)}_j | H_0 | \chi^{(a)}_{j'} \rangle$. Diagonal elements are given by

$$E^{(a)}_{jj} = \frac{[\mathbf{p}-(2j-1)\mathbf{k}]^2}{2m_e} + 2n\omega - \omega + 2d\frac{k^2}{2m_e} \quad (3.3c)$$

and elements $d(k^2/2m_e)$ appear just above and just below the main diagonal. The spin interaction matrix h'_{spin} is given explicitly as

$$(h'_{\text{spin}})_{jj'} = q(k^2/2m_e)(\delta_{jj'} - \delta_{jj'+1}). \quad (3.4)$$

Then, with the notation $(S_{cc'})_{ji}=S_{cj;c'i}$, the matrix form of Eq. (3.3a) becomes

$$\left(i\frac{\partial}{\partial t} - E^{(a)} \right) S_{au} = h'_{\text{spin}} S_{uu}. \quad (3.5)$$

The solution to Eq. (3.5) may be expressed as

$$S_{au}(t) = \int_0^t dt' G_0(t,t') h'_{\text{spin}} S_{uu}(t'), \quad (3.6)$$

with the Green's function, describing propagation of the electron in the spin-down state, represented in matrix form as

$$[G_0(t,t')]_{jj'} = -i\theta(t-t') \sum_i \langle \chi^{(a)}_j | \Psi^{(a)}_i(t) \rangle \langle \Psi^{(a)}_i(t') | \chi^{(a)}_{j'} \rangle. \quad (3.7)$$

Here θ is the step function and the states appearing in the sum satisfy the wave equation

$$\left(i\frac{\partial}{\partial t} - H_0 \right) |\Psi^{(a)}_i(t)\rangle = 0, \quad (3.8)$$

with initial condition $|\Psi^{(a)}_i(0)\rangle = |\chi^{(a)}_i\rangle$, and S-matrix elements are defined through the expansion

$$|\Psi^{(a)}_i(t)\rangle = \sum_j |\chi^{(a)}_j\rangle S_{aj;ai}(t). \quad (3.9)$$

The closure relation satisfied by the state vectors implies the sum rule

$$\sum_i S_{aj;ai}(t)S^*_{aj';ai}(t) = \delta_{jj'}; \tag{3.10}$$

the symmetry and time-reversal properties mentioned earlier in connection with Eq. (2.13) apply to the matrix $S_{aa}(t)$ as well. The solution (3.6) can then be written (to within a phase) as

$$S_{au}(t) = \int_0^t dt' S_{aa}(t)S^*_{aa}(t')h'_{\text{spin}}S_{uu}(t'). \tag{3.11}$$

In consideration of the strong-field limit, of relevance here, it is fortunate that an approximation to the S matrix defined in Eq. (3.1) is available [3], one that does not require explicit solution of the recurrence relation for the expansion coefficients and eigenvalues. The desired result is obtained, in the context of the present formalism, by looking for a solution of Eq. (3.3b) in the form

$$S_{uj;ui}(t) = (-i)^{j-i}\exp[-i(s^2 + 2d + 2n\bar{\omega})\tau]F_j(i;\tau), \tag{3.12}$$

where we have introduced the dimensionless time variable $\tau = t(k^2/2m_e)$ and have again set $\mathbf{p} = -s\mathbf{k}$. It then follows that F_j satisfies

$$\frac{\partial}{\partial \tau}F_j + i4j(j+s)F_j + d[F_{j-1} - F_{j+1}] = 0. \tag{3.13}$$

Let us take the intensity parameter d to be large enough so that the second term on the left may be ignored [7]; we have the solution

$$F_j(i;\tau) = J_{j-i}(-2d\tau). \tag{3.14}$$

The S matrix obtained by combining this result with Eq. (3.12) satisfies the required initial condition as well as the correct symmetry and time-reversal properties. Since the Bessel function $J_j(x)$ behaves as $x^{-1/2}$ for large x, it follows that the approximate treatment of Eq. (3.13) is consistent, in that the neglected term is of order d^{-1} relative to the terms retained. The matrix $S_{aa}(t)$, which by virtue of Eqs. (3.8) and (3.9) satisfies the wave equation

$$\left(i\frac{\partial}{\partial t} - E^{(a)}\right)S_{aa}(t) = 0 \tag{3.15}$$

and initial condition $S_{aj;ai}(0) = \delta_{ji}$, may, after a similar analysis, be approximated as

$$S_{aj;ai}(t) = (-i)^{j-i}\exp[-i(s^2 + 2d + 2n\bar{\omega} - \bar{\omega})\tau]J_{j-i}(-2d\tau). \tag{3.16}$$

An addition formula for the Bessel function now allows us, within this high-intensity approximation, to extend Eq. (3.10) in the matrix form

$$S_{aa}(t)S^*_{aa}(t') = S_{aa}(t-t') \tag{3.17}$$

and to replace Eq. (3.11) with

$$S_{au}(t) = \int_0^t dt' S_{aa}(t-t')h'_{\text{spin}}S_{uu}(t'). \tag{3.18}$$

The time-reversal property $S_{uu}(t') = S^*_{uu}(-t')$ along with a second application of the Bessel-function addition formula provides us, after an integration over time, with the form

$$S_{au}(t) = [1 - \exp(i\omega t)]\omega^{-1}h'_{\text{spin}}S_{uu}(t) \tag{3.19a}$$

or, more explicitly,

$$S_{aj;ui}(t) = (-i)^{j-i}(q/\bar{\omega})[1 - \exp(i\bar{\omega}\tau)]\exp[-i(s^2 + 2d + 2n\bar{\omega})\tau] \times [J_{j-i}(-2d\tau) - iJ_{j-i-1}(-2d\tau)]. \tag{3.19b}$$

With the introduction of the large-argument approximation for the Bessel functions and with the factor $|(1-\exp(i\bar{\omega}\tau))|^2$ replaced by 2, its average over many cycles, the probability of spin flip accompanied by absorption of a photon becomes

$$|S_{aj;ui}|^2 = \frac{1}{\pi\tau}\left(\frac{\omega}{2mc^2}\right)^2. \tag{3.20}$$

Noting that the dimensionless time variable τ will be of order unity under typical experimental conditions [4], it follows that with a photon energy of 1 eV the spin-flip probability is indeed negligible, of order 10^{-12}.

The neglect of contributions to the A^2 term in the electron-field interaction that change the net number of photons in the field may be justified using a perturbation approach very similar to that described above. That is, calculating the perturbation to the no-spin-flip S-matrix element that results from inclusion of such previously omitted contributions, one arrives at an approximation similar in form to that shown in Eq. (3.19a), but with the spin interaction replaced by the previously omitted part of the A^2 interaction. To illustrate the perturbation procedure in somewhat more detail, it will be sufficient to consider modifications in the matrix $S_{uu}(t)$ arising from two-photon absorption process. The relevant perturbation is

$$h'_{2a} \equiv \frac{e^2A_0^2}{2m_ec^2}(a_1^2e^{2i\mathbf{k}\cdot\mathbf{r}} + a_2^2e^{-2i\mathbf{k}\cdot\mathbf{r}} + 2a_1a_2). \tag{3.21}$$

The last term in this expression will play no role if, for simplicity, we further restrict our attention to perturbations in which both photons are absorbed from the same mode; this requires the introduction of additional basis states of the form

$$|\chi_j^{(2a)}\rangle = \begin{pmatrix}1\\0\end{pmatrix}|n+j-2, n-j\rangle|\mathbf{p} - (2j-2)\mathbf{k}\rangle. \tag{3.22}$$

The state vector may now be expanded as

$$|\Psi_i(t)\rangle = \sum_j \{|\chi_j^{(u)}\rangle S_{uj;ui}(t) + |\chi_j^{(2a)}\rangle S_{2aj;ui}(t)\}, \tag{3.23}$$

with the aid of which the wave equation, to first order in h'_{2a}, becomes

$$\langle \chi_j^{(2a)}| \left(i\frac{\partial}{\partial t} - H_0 \right) \sum_{j'} |\chi_{j'}^{(2a)}\rangle S_{2aj';ui}(t) = \langle \chi_j^{(2a)}| h'_{2a} \sum_{j'} |\chi_{j'}^{(u)}\rangle S_{uj';ui}(t). \tag{3.24}$$

The similarity of this form with what shown in Eq. (3.3a) suggests that the solution may be worked out in a similar manner. Omitting details, we pass directly to the analog of Eq. (3.19a), which is written as

$$S_{2aj;ui}(t) = [1 - \exp(2i\omega t)](2\omega)^{-1} \sum_j (h'_{2a})_{jj'} S_{uj';ui}(t), \tag{3.25a}$$

with

$$(h'_{2a})_{jj'} = \frac{d}{2}\left(\frac{k^2}{2m_e} \right)(\delta_{jj'} + \delta_{jj'+2}). \tag{3.25b}$$

IV. DISCUSSION

An extension of the theory of Kapitza-Dirac scattering has been derived that accounts quite generally for spin interactions and also provides the basis for a perturbative treatment of the spin-flip process. An explicit, perturbative, expression for the transition probability for spin flip is given in Eq. (3.20), obtained with the aid of Eq. (3.18) and a strong-field approximation for scattering without spin interactions. It follows that the spin-flip probability is independent of field strength in the intense-field domain. One might have hoped that by raising the laser intensity sufficiently the spin-flip process would be enhanced to observable levels [1]. We must conclude, however, that within the range of validity of the nonrelativistic approximation adopted here, effects of spin play no role in Kapitza-Dirac scattering. A relativistic generalization of the present approach may be relevant in an analysis of possible future experiments done with ultrastrong laser fields.

In the standard treatment of Kapitza-Dirac scattering based on the occupation-number representation transitions which change the net number of photons in the field are ignored since they lack the resonant enhancement accompanying scattering events that preserve photon number. As a second application of the perturbation theory developed here, we have obtained, in Eq. (3.25), a correction to this standard approximation in which a dimensionless strength parameter $g \equiv d/\overline{\omega}$ appears. The identification of this parameter is in agreement with the conclusion obtained in Ref. [4] based on the use of a time-averaged interaction in a semiclassical formulation. As observed in Ref. [4], the parameter g need not be small compared to unity under realizable laboratory conditions; thus, for an intensity $I = 10^{11}$ W/cm^2 and a wavelength of 10 μm one does find g to be of order unity [7]. However, in the more explicit estimate obtained from the perturbation theory presented here we may include the behavior of the unperturbed S-matrix element S_{uu}. As noted, this brings in the additional factor $(\pi d\tau)^{-1/2}$ arising from the Bessel function, so that with the same intensity and frequency parameters and τ of order unity the estimate of the perturbation to the S-matrix element is of order 10^{-4}. These considerations validate the standard treatment in which corrections of the type just considered are ignored.

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APPENDIX

The recurrence relation for channel v_u is found to be

$$[(2j)^2 - m^2 + 2s(2j+m) + 2d - \lambda^{(v_u)}]\xi_{2j+m}^{(v_u)} + d[\xi_{2j+2+m}^{(v_u)} + \xi_{2j-2+m}^{(v_u)}] + q[\xi_{2j-1+m}^{(v_u)} + \eta_{2j-1+m}^{(v_u)}] - q[\xi_{2j+1+m}^{(v_u)} + \eta_{2j+1+m}^{(v_u)}] = 0. \tag{A1}$$

Recognizing that the parameters d, q, and $\lambda^{(v_u)}$ vanish in the limit of vanishing interaction, one sees that relation (A1) is consistent with the initial condition placed on the expansion coefficients [as specified just below Eq. (2.6)], with the index m taken to be an even integer. Writing $\omega = \overline{\omega}(k^2/2m_e)$, we find the remaining two recurrence relations for channel v_u in the form

$$[(2j-1)^2 - m^2 + 2s(2j-1+m) - \overline{\omega} + 2d - \lambda^{(v_u)}]\xi_{2j-1+m}^{(v_u)} + d[\xi_{2j+1+m}^{(v_u)} + \xi_{2j-3+m}^{(v_u)}] + q[\zeta_{2j+m}^{(v_u)} - \zeta_{2j-2+m}^{(v_u)}] = 0 \tag{A2}$$

and

$$[(2j+1)^2 - m^2 + 2s(2j+1+m) + \overline{\omega} + 2d - \lambda^{(v_u)}]\eta_{2j+1+m}^{(v_u)} + d[\eta_{2j+3+m}^{(v_u)} + \eta_{2j-1+m}^{(v_u)}] + q[\zeta_{2j+2+m}^{(v_u)} - \zeta_{2j+m}^{(v_u)}] = 0. \tag{A3}$$

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[6] As a rough estimate of the relative importance of the spin-dependent and spin-independent interactions, we may form the ratio of interaction energies $(\overline{q^2}/\omega)/\overline{d}$. With $\overline{q}$ and $\overline{d}$ given by the right-hand sides of Eqs. (2.8) and (2.7), respectively, this ratio is $\omega/4m_ec^2$ and is of order 10^{-7} for photon energies of order 1 eV. Another error, even larger though still quite small, enters with the use of a time-averaged interaction that elimi-

nates those transitions generated by the A^2 term that change the net number of photons in the field. (This approximation was introduced earlier in the second paragraph of Sec. II.) Inclusion of such terms leads to the correction shown in Eq. (3.25), below; it is estimated numerically at the end of Sec. IV.

[7] For a field intensity $I=10^{11}\ \mathrm{W/cm^2}$ and a wavelength of 10 μm d is of order $2m_e c^2/\omega$, large compared to unity for frequencies in the optical range.