Leptonic annihilation in hydrogen-antihydrogen collisions

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We consider the question of competition between leptonic and hadronic annihilation in matter-antimatter interaction. The rate of direct positron-electron annihilation in cold hydrogen-antihydrogen collisions has been calculated. The presence of leptonic annihilation introduces an absorptive, imaginary component to the hydrogen-antihydrogen scattering length; this component is 1.4×10^{-4} a.u. for the singlet state of the leptonic spins, and 1.2×10^{-7} a.u. for the triplet state. Leptonic annihilation is shown to be about 3 orders of magnitude slower than proton-antiproton annihilation.

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The recent success in the synthesis of antihydrogen atoms at CERN [1,2] and the prospects of experiments with cold and trapped antihydrogen have drawn attention to the physics of atom-antiatom interactions.

When we think about the encounter of an antiatom with an atom we tend to imagine only one possible outcome: annihilation. However, atom-antiatom scattering can result in a number of collisional reactions. In the case of ultracold hydrogen-antihydrogen collisions, we have previously calculated the cross sections for elastic scattering, rearrangement collisions, proton-antiproton in-flight annihilation, and formation of the hydrogen-antihydrogen molecule *via* radiative association [3–7]. Our calculations show that, in a certain energy range, the rate of the rearrangement reaction $(\mathrm{H}+\overline{\mathrm{H}}\rightarrow\mathrm{Pn}+\mathrm{Ps})$ is comparable to the rate of proton-antiproton annihilation in flight $(\mathrm{H}+\overline{\mathrm{H}}\rightarrow e^{+}+e^{-}+$ other products). This finding conforms with other calculations of the rearrangement process [8–10].

Since the electromagnetic interaction is much weaker than the strong nuclear force, one might expect that, in an atom-antiatom collision, proton-antiproton annihilation will be the dominant process. However, the strong force is characterized by a very short interaction range. The electromagnetic interaction between the atoms, on the other hand, is characterized by the effective long-range van-der-Waals interactions. Processes of an electromagnetic nature, such as rearrangement into positronium and protonium, can therefore occur over a much larger range of internuclear distances. Indeed, our previous results show that the cross section for the rearrangement collision is surprisingly large and only slightly smaller than that for proton-antiproton annihilation in flight. The present paper is devoted to the question: how fast is the positron-electron annihilation?

Large rearrangement implies strong interparticle correlation, meaning that during the slow $\overline{\mathrm{H}}$-H collision the leptons might have plenty of time "to find each other." In addition, the rate constant for e^{+}-e^{-} annihilation is larger than that for the $\overline{p}$-p annihilation. Could that be so that during the slow hydrogen-antihydrogen encounter, the leptons will annihilate *before* the hadrons do?

The cross section for proton-antiproton annihilation has been obtained in the distorted wave approximation, using the adiabatic internuclear interaction in the initial channel [3]. This means that the approaching nuclei are assumed to move in the field of the surrounding leptons. This assumption would turn out to be a wishful thinking in case the leptons, instead of creating the interaction potential, would directly annihilate each other as the atoms come close. As a consequence, one would have bare-nuclei interaction without any leptonic screening. Most importantly, if direct leptonic annihilation would turn out to occur with large probability, this would be an important loss channel for antihydrogen when interacting with ordinary matter. In the present work we show that this is *not* the case, i.e., during the collision the leptons annihilate on a very different time scale compared with the hadrons. Therefore, direct leptonic annihilation is a negligible loss channel in cold hydrogen-antihydrogen collisions and the leptonic potential is a useful concept in considering atom-antiatom collisions.

The direct leptonic annihilation in hydrogen-antihydrogen collisions occurs according to

$$H+\overline{H}\rightarrow p+\overline{p}+2\gamma \text{ or } p+\overline{p}+3\gamma. \tag{1}$$

If this process occurs during the collisional approach, the hadrons get stripped from their leptons and start to interact as bare nuclei *via* the Coulomb and strong interactions.

The probability of leptonic annihilation in flight may be obtained in the contact approximation, i.e., assuming that electron-positron annihilation occurs at the exact point of coalescence of the two leptons. The rate of annihilation is obtained as a product of the leptonic probability density for coalescence and the positron-electron annihilation rate constant, integrated over all space

$$\lambda_a^{e\overline{e}}=\langle\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}_e,\mathbf{r}_{\overline{e}},\mathbf{R})|A^{e\overline{e}}\delta(\mathbf{r}_e-\mathbf{r}_{\overline{e}})|\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}_e,\mathbf{r}_{\overline{e}},\mathbf{R})\rangle, \tag{2}$$

where $\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}_e,\mathbf{r}_{\overline{e}},\mathbf{R})$ is the four-body scattering wave function in the initial channel. The latter is an eigenfunction of the total Hamiltonian H

$$H\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}_e,\mathbf{r}_{\bar{e}},\mathbf{R}) = E_i\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}_e,\mathbf{r}_{\bar{e}},\mathbf{R}), \tag{3}$$

and asymptotically describes two atoms colliding with energy $\epsilon_i = k_i^2/2\mu$. E_i is the total energy of the collisional system and thus equal to the sum of the internal energies of the two atoms (−1.0 a.u. in the case of hydrogen and antihydrogen in their 1*s* ground states) and the kinetic energy ϵ_i. Here and in the following atomic units (hartrees) are used, if not specified otherwise.

The annihilation constant $A^{e\bar{e}}$ is given as the number of annihilation events per unit density and unit time. It is obtained from the life time of positronium, which for the singlet state is $\tau_{2\gamma}=1.25\times 10^{-10}$ s and for the triplet state is $\tau_{3\gamma}=1.42\times 10^{-7}$ s. The positronium decay rate is related to the decay constant *via* $\lambda = 1/\tau = A^{e\bar{e}}|\phi_{100}^{Ps}(0)|^2$, which yields $A_+^{e\bar{e}} = 8\pi/\tau_{2\gamma} = 4.86\times 10^{-6}$ a.u. for para-positronium and $A_-^{e\bar{e}} = 8\pi/\tau_{3\gamma} = 4.28\times 10^{-9}$ a.u. for orthopositronium.

Even though the hydrogen-antihydrogen system is not an eigenstate with respect to charge conjugation of the leptons, the same selection rules as for the positronium ground state apply, i.e., two-photon decays for the molecular singlet state and three-photon decays for the triplet states.

The ratio of triplet collisions to singlet collisions will depend on the experimental conditions. For a statistical mixture the ratio is 3 to 1. In a magnetic trap only the spin-polarized states can be held, i.e., the electron spin will be parallel to the magnetic field, while the positron spin, being parallel to the magnetic moment, will point in the opposite direction. Hence, the spin-polarized $\bar{\mathrm{H}}$-H system corresponds to $M_s = 0$ and the collisions will be in the singlet and triplet states in equal proportions.

To obtain the annihilation cross section, the annihilation rate in Eq. (2) is divided by the flux F of the oncoming atoms

$$\sigma_a^{e\bar{e}} = \frac{\lambda_a^{e\bar{e}}}{F} = \frac{(2\pi)^3}{k_i^2}A^{e\bar{e}}\int |\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{R},\mathbf{r}_e,\mathbf{r}_{\bar{e}})|^2\delta(\mathbf{r}_e-\mathbf{r}_{\bar{e}})d\tau, \tag{4}$$

where the integration $d\tau$ is over all space coordinates.

As is apparent from Eq. (4), in order to calculate the cross section for leptonic annihilation in flight we need the scattering wave function in the initial channel of the colliding system, $\Psi_{\mathbf{k}_i}^{(+)}$. The latter may be obtained by means of the formalism developed in our previous work [3,5].

We solve the hydrogen-antihydrogen scattering problem in a distorted wave approximation based on the separation of the leptonic and hadronic motions. The total wave function of the system is written as

$$\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}_e,\mathbf{r}_{\bar{e}},\mathbf{R}) = \psi_i(\mathbf{r}_e,\mathbf{r}_{\bar{e}};R)\chi_{\mathbf{k}_i}(\mathbf{R}), \tag{5}$$

where the leptonic wave function $\psi_i(\mathbf{r}_e,\mathbf{r}_{\bar{e}};R)$ depends on the interhadronic distance in a parametric way. In this approximation, the solution of the problem separates into two parts. One first calculates the leptonic potential for the hadronic motion by solving the leptonic eigenvalue problem

$$H_i^{\mathrm{lep}}\psi_i = V_i^{\mathrm{lep}}(R)\psi_i, \tag{6}$$

where the leptonic Hamiltonian $\mathrm{H}_i^{\mathrm{lep}}$ is given by

$$H_i^{\mathrm{lep}} = -\frac{1}{2}\nabla_e^2 - \frac{1}{2}\nabla_{\bar{e}}^2 - \frac{1}{|\mathbf{r}_p-\mathbf{r}_e|} - \frac{1}{|\mathbf{r}_{\bar{p}}-\mathbf{r}_{\bar{e}}|} + \frac{1}{|\mathbf{r}_p-\mathbf{r}_{\bar{e}}|} + \frac{1}{|\mathbf{r}_{\bar{p}}-\mathbf{r}_e|} - \frac{1}{|\mathbf{r}_e-\mathbf{r}_{\bar{e}}|}. \tag{7}$$

The leptonic potential $V_i^{\mathrm{lep}}(R)$ is then used for solving the Schrödinger equation describing the nuclear motion

$$\left(-\frac{1}{m_p}\nabla_R^2 + V_i(R)\right)\chi_{\mathbf{k}_i} = E_i\chi_{\mathbf{k}_i} \tag{8}$$

with $V_i(R) = V_i^{\mathrm{lep}}(R) - 1/R$.

The leptonic eigenvalue problem (6) is solved by means of the variational method [5] expanding wave function ψ as a linear combination of (N) basis functions φ,

$$\psi = \sum_{j=1}^{N} c_j\varphi_j. \tag{9}$$

The trial wave functions φ_j are expressed in prolate spheroidal coordinates, properly symmetry adapted, and of the same explicitly correlated form as introduced by Kołos *et al.* [11]

$$\varphi_j = \left(\frac{2r_{e\bar{e}}}{R}\right)^{\mu_j}\left(e^{-\alpha_1\xi_e-\alpha_2\xi_{\bar{e}}+\beta_1\eta_e+\beta_2\eta_{\bar{e}}}\xi_e^{p_j}\xi_{\bar{e}}^{\bar{p}_j}\eta_e^{q_j}\eta_{\bar{e}}^{\bar{q}_j} + (-1)^{(q_j+\bar{q}_j)}e^{-\alpha_2\xi_e-\alpha_1\xi_{\bar{e}}+\beta_2\eta_e+\beta_1\eta_{\bar{e}}}\xi_{\bar{e}}^{p_j}\xi_e^{\bar{p}_j}\eta_{\bar{e}}^{q_j}\eta_e^{\bar{q}_j}\right), \tag{10}$$

where the rational numbers α_i and β_i and the positive integers p_j, q_j, and μ_j characterize the basis set. α_i and β_i were optimized variationally as a function of the internuclear distance R.

The factorization of the wave function due to the Born-Oppenheimer approximation in Eq. (5) allows to structure the annihilation rate given in Eq. (2),

$$\lambda_a^{e\bar{e}} = \langle\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{R};\mathbf{r}_e,\mathbf{r}_{\bar{e}})|A^{e\bar{e}}\delta(\mathbf{r}_e-\mathbf{r}_{\bar{e}})|\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{R};\mathbf{r}_e,\mathbf{r}_{\bar{e}})\rangle \tag{11}$$

$$= A^{e\bar{e}}\int \mathrm{d}V_R|\chi_{\mathbf{k}_i}(\mathbf{R})|^2P(R), \tag{12}$$

where $P(R)$ denotes the conditional probability density for the electron and positron to coalesce (at any place in the leptonic coordinate space) while the hadrons are at a distance R apart,

$$P(R) = \int |\psi_i(R;\mathbf{r}_e,\mathbf{r}_{\bar{e}})|^2\delta(\mathbf{r}_e-\mathbf{r}_{\bar{e}})\mathrm{d}V_{\bar{e}}\mathrm{d}V_e. \tag{13}$$

Using expansion (9) in Eq. (13) one gets

$$P(R) = \int \mathrm{d}V_e\int \mathrm{d}V_{\bar{e}}\sum_{j,i} c_j^*c_i\varphi_j^*(R;\mathbf{r}_e,\mathbf{r}_{\bar{e}})\varphi_i(R;\mathbf{r}_e,\mathbf{r}_{\bar{e}})\delta(\mathbf{r}_e-\mathbf{r}_{\bar{e}}). \tag{14}$$

Since the basis functions φ_i are expressed in prolate spheroidal coordinates both the volume element and the delta function must be accordingly transformed.

With the aid of $P(R)$ and $\chi_{\mathbf{k}_i}(\mathbf{R})$ the direct leptonic annihilation rate can be calculated according to Eq. (12). The

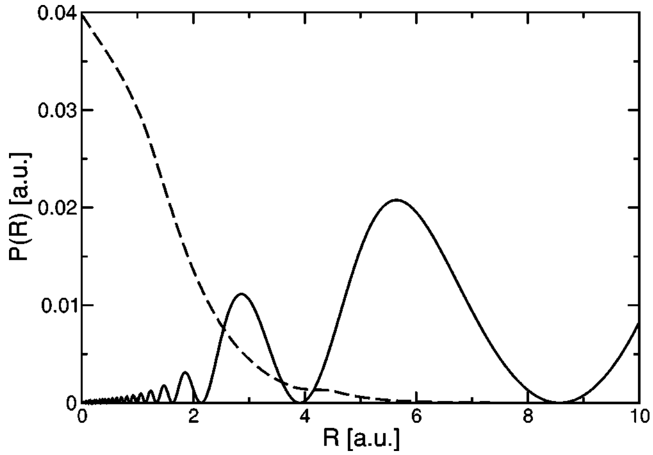

FIG. 1. The leptonic coalescence probability density $P(R)$ in atomic units (dashed) is shown together with the hadronic radial density $|f_0(R)|^2=|\chi_{\mathbf{k}_i}(R)|^2R^2$ (solid, scaled to fit into plot) for a collision energy of 10^{-7} a.u.

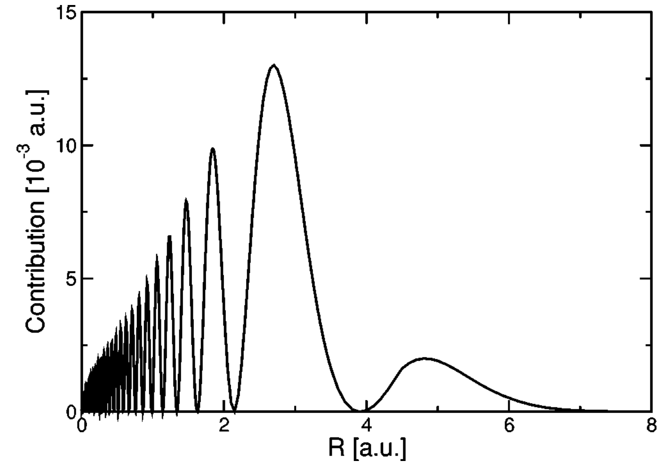

FIG. 2. Contribution to the leptonic annihilation for different interhadronic distances, i.e., the product of the hadronic radial density $|\chi_{\mathbf{k}_i}(R)|^2R^2=|f_0(R)|^2$ and the leptonic coalescence density $P(R)$. Shown is the result for a collision energy of 10^{-7} a.u.

hadronic wave function $\chi_{\mathbf{k}_i}$ in Eq. (8) is obtained numerically after performing a partial-wave decomposition, as described in [5].

The leptonic coalescence density $P(R)$ has been evaluated as a function of the internuclear distance R in the interval $0.8 \leqslant R \leqslant 8.0$ using up to 908 basis functions φ. The $P(R)$ calculated this way goes to zero for large R values (see Fig. 1). This is expected since in the limit $R\to\infty$ the ground-state $H\bar{H}$ system dissociates into separated $H(1s)$ and $\bar{H}(1s)$ atoms. In this case the electron and positron densities are spatially separated and do not overlap.

In the limit $R\to 0$ $P(R)$ is simply given by the coalescence density of positronium. Using the explicit form of the positronium ground-state wave function one obtains $P(0) = 1/(8\pi)$.

For small R distances we might expect inaccuracies in $P(R)$ related to the limitation of the presently employed basis functions to correctly represent positronium [5]. An error of about 5% is found for the positronium energy. Usually, one expects the error in the energy to be quadratic compared to the one of the wave function. Therefore, we might expect an error of about 22% for $P(R)$ at the distances where the positronium character of the $H\bar{H}$ wave function becomes dominant. This happens below the critical distance $R_c \simeq 0.74$ where the $\bar{p}$-p pair does not bind the leptons; the latter can be freed in form of positronium [12]. Because of that, leptonic coalescence density $P(R)$ has been smoothly extrapolated from its value at $R\simeq 0.8$ a.u. to the correct positronium value at $R=0$. The resulting $P(R)$ is shown in Fig. 1.

The extrapolation procedure is justified if the final result does not critically depend on the leptonic coalescence density for $R<0.8$ a.u. Fortunately, the importance of this R range turns out to be small. Even if one disregards the interval $R\leqslant 0.8$ completely there is no more than a 7% difference between this result and the one obtained when using a constant value of the integrand in Eq. (12) (e.g., its value at $R=0.8$ a.u.) for all $R<0.8$ a.u. The reason is the small probability density of the hadronic scattering wave function in this R range.

As an example, the hadronic radial density is shown for a collisional energy $\epsilon_i=10^{-7}$ a.u. in Fig. 1. As is apparent from Eq. (12), the direct leptonic annihilation rate depends on the product of the hadronic density and $P(R)$. This product is shown in Fig. 2. Clearly, the range with $R<0.8$ adds only a minor, though nonnegligible, part to the integral in Eq. (12). According to the test calculation mentioned above we estimate this contribution to be on the order of 10%. This accuracy is sufficient for the purpose of this work, which is to give a reliable order-of-magnitude estimate of the role of the leptonic annihilation channel in the hydrogen-antihydrogen scattering.

The result of the present calculation of the direct leptonic annihilation rate is presented in Fig. 3. This figure shows (on a logarithmic scale) the cross section as a function of the

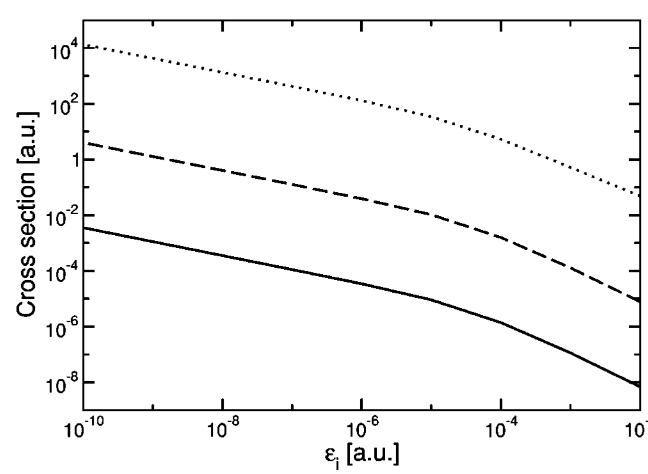

FIG. 3. The cross section for the leptonic annihilation in flight in atomic units. Solid line: leptonic annihilation for the triplet collisions $\sigma_-^{e^+e^-}=(3.5\times 10^{-8})/\sqrt{\epsilon_i}$ a_0^2 (obtained with $A_-^{e\bar{e}}=4.3\times 10^{-9}$ a.u.). Dashed line: leptonic annihilation for the singlet collisions $\sigma_+^{e^+e^-}=(4.0\times 10^{-5})/\sqrt{\epsilon_i}$ a_0^2 (obtained with $A_+^{e\bar{e}}=4.9\times 10^{-6}$ a.u.). For comparison, the upper curve (dotted) represents the cross section for $p\bar{p}$ annihilation in flight, $\sigma^{p\bar{p}}=0.14/\sqrt{\epsilon_i}$ a_0^2.

collisional energy. As one would expect, the cross section increases for decreasing collision energies, since the leptons have more time to interact (and annihilate). In the low-energy limit the cross section for leptonic annihilation shows the behavior characteristic for inelastic collisions and expected from the Wigner's threshold law. The cross section is $\sigma_+^{e^+e^-}=(4.0\times10^{-5})/\sqrt{\epsilon_i}\ \mathrm{a}_0^2$ for the singlet state and $\sigma_-^{e^+e^-}=(3.5\times10^{-8})/\sqrt{\epsilon_i}\ \mathrm{a}_0^2$ for the triplet state. The inelasticity due to leptonic annihilation induces an imaginary component in the hydrogen-antihydrogen scattering length. This component can be obtained as $\beta=4\pi\sigma_a^{e\bar{e}}$ and is $\beta_+=1.4\times10^{-4}\ \mathrm{a}_0$ for the singlet and $\beta_-=1.2\times10^{-7}\ \mathrm{a}_0$ for the triplet case, respectively.

As seen from Fig. 3 the direct hadronic annihilation ($\sigma^{\bar{p}-p}=0.14/\sqrt{\varepsilon_i}$ [3]) is about three orders of magnitude more likely than leptonic annihilation in flight in the entire range of considered energies. This result appears counterintuitive in the sense that the annihilation reaction constant for para-positronium is *larger* than that for protonium ($A_{\bar{p},p}=1.69\times10^{-7}$ a.u.). However, at any interhadronic distance R the leptonic coalescence density has to be weighted by the hadronic probability density at that distance. Because of that, e^+-e^- annihilation occurs mainly within the interval $\Delta R\simeq1$ a.u. around $R\simeq3$ a.u. (see Fig. 2) with the probability proportional to $P(R\simeq3)|f_0(R\simeq3)|^2\Delta R$, whereas the hadrons annihilate basically at $R=0$ with the probability proportional to $|\chi_{\mathbf{k}_i}(R=0)|^2Y_{00}$ (where the spherical harmonic Y_{00} represents the angular part of the hadronic wave function). The relative probability of the two processes should be therefore roughly on the order of

$$\frac{\sigma^{\bar{p}p}}{\sigma^{e^+e^-}}\simeq\frac{A^{\bar{p}p}}{A_-^{e^+e^-}}\frac{|\chi_{\mathbf{k}_i}(0)|^2Y_{00}^2}{|\chi_{\mathbf{k}_i}(3)|^2P(3)\cdot3^2\cdot1}\simeq4.7\times10^3.$$

Since $\chi_{\mathbf{k}_i}(R)$ is considerably enhanced at $R=0$ (see Fig. 5 in Ref. [5]) the $p-\bar{p}$ annihilation dominates over e^+-e^- annihilation. This estimate is confirmed by the full calculation. The cross section for the in flight annihilation of the $e-e^+$ pair according to Eq. (1) has been found to be 3.5×10^3 times smaller than the corresponding cross section for proton-antiproton annihilation. Assuming the accuracy of the present result to be about 10 %, a more precise calculation is not likely to change the basic conclusion of this work, namely that direct leptonic annihilation in flight is a negligible effect in hydrogen-antihydrogen scattering at low temperatures.

The leptonic annihilation is nevertheless observable in hydrogen-antihydrogen collisions. Its main appearance is, however, due to indirect processes, namely the intermediate formation of positronium as a consequence of rearrangement collisions. In that case the leptons are likely to annihilate with a certain time delay after hadronic annihilation. This is because regardless the final state of positronium its lifetime is longer than that of the most probable final state of protonium with $N=23$.

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