Distinguishing the elements of a full product basis set needs only projective measurements and classical communication

Ping-Xing Chen[1,2,*] and Cheng-Zu Li[2]
[1]*Laboratory of Quantum Communication and Quantum Computation, University of Science and Technology of China, Hefei, 230026, People's Republic of China*
[2]*Department of Applied Physics, National University of Defense Technology, Changsha, 410073, People's Republic of China*
(Received 24 April 2003; revised manuscript received 24 February 2004; published 10 August 2004)

Nonlocality without entanglement is an interesting field. A manifestation of quantum nonlocality without entanglement is the possible local indistinguishability of orthogonal product states. In this paper we analyze the character of operators to distinguish the elements of a full product basis set in a multipartite system, and show that distinguishing perfectly these product bases needs only local projective measurements and classical communication, and these measurements cannot damage each product basis. Employing these conclusions one can discuss local distinguishability of the elements of any full product basis set easily. Finally we discuss the generalization of these results to the locally distinguishability of the elements of incomplete product basis set.

DOI: 10.1103/PhysRevA.70.022306 PACS number(s): 03.67.Hk, 03.65.Ud, 89.70.+c

An important manifestation of quantum nonlocality is entanglement [1]. The entangled states can be used for alternative forms of information processing, such as quantum cryptography [2,3], quantum teleportation [4], and fast quantum computation [5]. However, there also exists nonlocality in nonentangled states [6,7], even in a state of a particle [3]. This is known as nonlocality without entanglement. The nonlocality without entanglement may be an important field just as the entanglement. Closely related to the nonlocality without entanglement is the local distinguishability of nonentangled states [6,7].

Alice, Bob and Charles *et al.* share a quantum system, in one of a known set of possible orthogonal states. They do not, however, know which state they have. These states are locally distinguishable if there are some sequence of local operations and classical communication (LOCC) by which Alice, Bob and Charles *et al.* can always determine which state they own. There are many interesting works on the local distinguishability of orthogonal states [6–15]. These works improve our understanding on nonlocality. The discussion on the local distinguishability of orthogonal product states (OPSs) may enlarge our acknowledge of nonlocality without entanglement. Bennett *et al.* first [6] showed that there are nine OPSs in a $3\otimes 3$ system which are indistinguishable by LOCC. Walgate *et al.* [7] provide a more simple proof of indistinguishability of Bennett's nine OPSs. However few papers discussed the local distinguishability of more general OPSs in a multipartite system.

This paper will focus on the local distinguishability of the states of a set of complete OPSs $\{|\Psi_k\rangle\}$ in a multipartite system. We will show that the states of a set of full OPSs are LOCC perfectly distinguishable if and only if these OPSs are distinguishable by projective measurements and classical communication, and these measurements cannot damage each state $|\Psi_k\rangle$. Using this result we can prove easily that Bennett's nine OPSs [6] are indistinguishable by LOCC, and can provide many new sets of locally indistinguishable OPSs in multipartite systems. Finally we discuss the generalization of these results to the LOCC distinguishability of the states of an incomplete product basis set.

Alice, Bob and Charles *et al.* share a quantum system which may be in one of the possible states $\{|\psi_k\rangle, k=1,\ldots,M\}$. Any protocol to distinguish these possible states can be conceived as successive rounds of measurements and communication by Alice, Bob and Charles *et al.* After $N(N\geqslant 1)$ rounds of measurements and communication, there are many possible outcomes which correspond to many measurement operators $\{A_{iN}\otimes B_{iN}\otimes C_{iN}\otimes\cdots\}$ acting on the Alice, Bob and Charles's Hilbert space. Each of these operators is a product of the positive operators and unitary maps corresponding to Alice's, Bob's and Charles's measurements and rotations, and represents the effect of the N measurements and communication. If the outcome iN occurs, the given state $|\psi_k\rangle$ becomes [7,16]:

$$|\psi_k\rangle \rightarrow A_{iN}\otimes B_{iN}\otimes C_{iN}\otimes\cdots|\psi_k\rangle. \tag{1}$$

Operator A_{iN}, B_{iN}, C_{iN} can be expressed as [17]:

$$A_{iN}=c_1^{iN}|\phi_1'^{iN}\rangle\langle\phi_1^{iN}|+\cdots+c_{n_a^{iN}}^{iN}|\phi_{n_a^{iN}}'^{iN}\rangle\langle\phi_{n_a^{iN}}^{iN}|,$$

$$B_{iN}=d_1^{iN}|\xi_1'^{iN}\rangle\langle\xi_1^{iN}|+\cdots+d_{n_b^{iN}}^{iN}|\xi_{n_b^{iN}}'^{iN}\rangle\langle\xi_{n_b^{iN}}^{iN}|,$$

$$C_{iN}=e_1^{iN}|\eta_1'^{iN}\rangle\langle\eta_1^{iN}|+\cdots+e_{n_c^{iN}}^{iN}|\eta_{n_c^{iN}}'^{iN}\rangle\langle\eta_{n_c^{iN}}^{iN}|, \tag{2}$$

where $\{|\phi_j'^{iN}\rangle, j=1,\ldots,n_a^{iN}\}$, $\{|\phi_j^{iN}\rangle, j=1,\ldots,n_a^{iN}\}$ are Alice's two set of orthogonal vectors; $\{|\xi_l'^{iN}\rangle, l=1,\ldots,n_b^{iN}\}$, $\{|\xi_l^{iN}\rangle, l=1,\ldots,n_b^{iN}\}$ are Bob's two set of orthogonal vectors; $\{|\eta_p'^{iN}\rangle, p=1,\ldots,n_c^{iN}\}$, $\{|\eta_p^{iN}\rangle, p=1,\ldots,n_c^{iN}\}$ are Charles's two set of orthogonal vectors. $0\leqslant c_j^{iN}\leqslant 1$, $j=1,\ldots,n_a^{iN}$; $0\leqslant d_l^{iN}\leqslant 1$, $l=1,\ldots,n_b^{iN}$; $0\leqslant e_p^{iN}\leqslant 1$, $p=1,\ldots,n_c^{iN}$. The operator A_{iN} in Eq. (2) can be regarded as a combination of the following three operators: 1) a projective operator which projects out $|\phi_j^{iN}\rangle$, $j=1,\ldots,n_a^{iN}$; 2) a local filter operator which changes

*Email address: pxchen@nudt.edu.cn

the relative weights of the components $|\phi_j^{iN}\rangle$, $j=1,\ldots,n_a^{iN}$; 3) a local unitary operator which transfer the Alice's vectors from $\{|\phi_j^{iN}\rangle, j=1,\ldots,n_a^{iN}\}$ to $\{|\phi_j'^{iN}\rangle, j=1,\ldots,n_a^{iN}\}$, and similarly for B_{iN} and C_{iN}. The local unitary operator 3) just changes the bases of the subspace projected out by the projective operator 1), and does not affect the distinguishability of the possible states $\{|\psi_k\rangle, k=1,\ldots,M\}$ (since a sequent set of states $A_{iN}\otimes I|\psi_k\rangle s$ are LOCC perfectly distinguishable if and only if states $U_A A_{iN}\otimes I|\psi_k\rangle s$ are LOCC perfectly distinguishable, U_A are local unitary operators). So for the aim to distinguish locally states A_{iN} in Eq. (2) can be replaced by

$$A_{iN} = c_1^{iN}|\phi_1^{iN}\rangle\langle\phi_1^{iN}| + \cdots + c_{n_a^{iN}}^{iN}|\phi_{n_a^{iN}}^{iN}\rangle\langle\phi_{n_a^{iN}}^{iN}|, \qquad (3)$$

and similarly for B_{iN} and C_{iN}.

Definition 1. For each operator $A_{iN}\otimes B_{iN}\otimes C_{iN}\otimes\cdots$ in Eq. (1), if states $\{A_{iN}\otimes B_{iN}\otimes C_{iN}\otimes\cdots|\psi_k\rangle, k=1,\cdots,M\}$ are LOCC distinguishable, we say that operator $A_{iN}\otimes B_{iN}\otimes C_{iN}\otimes\cdots$ *is effective* for distinguishing the states $\{|\psi_k\rangle\}$

Theorem 1. $\{|\Psi_k\rangle, k=1,\ldots,M\}$ is a set of complete orthogonal product states in a multipartite system. The states $\{\Psi_k\}$ are LOCC perfectly distinguishable if and only if the states are distinguishable by projective measurements and classical communication, and these measurements cannot damage each state $|\Psi_k\rangle$.

Proof. We first prove Theorem 1 for the cases of bipartite systems. The sufficiency is obvious. We need only prove the necessity. Suppose that Alice and Bob share an $n\otimes m$ system which has nm possible OPSs $\{|\Psi_k\rangle=|v_k\rangle_A|y_k\rangle_B, k=1,\ldots,nm\}$, where $|v_k\rangle_A$, $|y_k\rangle_B$ is a vector of Alice's and Bob's, respectively. If the set of states $\{\Psi_k\}$ is perfectly distinguishable by LOCC, there must be a complete set of final operators $\{A_{if}\otimes B_{if}\}$ representing the effect of all measurements and communication, such that if every outcome *if* occurs Alice and Bob know with certainty that they were given the state $|\Psi_i\rangle\in\{|\Psi_k\rangle\}$. This means that

$$A_{if}\otimes B_{if}|\Psi_i\rangle \neq 0,$$

$$A_{if}\otimes B_{if}|\Psi_j\rangle = 0,\; j\neq i. \qquad (4)$$

Operators A_{if}, B_{if} have similar forms as A_{iN}, B_{iN} in (3) but for $N\to f$. If Eq. (4) holds, we note that $A_{if}\otimes B_{if}$ can "indicate" $|\Psi_i\rangle$ and only $|\Psi_i\rangle$ [15]. Since the number of the operators satisfying (4) is bigger than 1, a state $|\Psi_i\rangle$ can be "indicated" by more than a operator, in general. By the general expressions A_{if}, B_{if} as shown in Eq. (3) if operator $A_{if}\otimes B_{if}$ can "indicates" $|\Psi_i\rangle$ and only $|\Psi_i\rangle$, i.e., (4) holds, the state $|\Psi_i\rangle$ should contain all or part of orthogonal product vectors in the following (we say a state $1/\sqrt{2}(|00\rangle+|11\rangle)$ includes $|00\rangle$ and $|11\rangle$) [15]:

$$|\phi_1^{if}\rangle|\xi_1^{if}\rangle,\cdots,|\phi_1^{if}\rangle|\xi_{n_b^{if}}^{if}\rangle,\cdots,|\phi_{n_a^{if}}^{if}\rangle|\xi_1^{if}\rangle,\cdots,|\phi_{n_a^{if}}^{if}\rangle|\xi_{n_b^{if}}^{if}\rangle, \quad (5)$$

and $|\Psi_j\rangle$ $(j\neq i)$ do not contain any product vectors in Eq. (5), i.e., each product vector in Eq. (5) is orthogonal to $|\Psi_j\rangle$ $(j\neq i)$. Furthermore, since for an $n\otimes m$ system the vector which is orthogonal to $nm-1$ orthogonal states $|\Psi_j\rangle$ $(j\neq i)$ is alone, if operator $A_{if}\otimes B_{if}$ "indicates" $|\Psi_i\rangle$, then $|\Psi_i\rangle$ contains only one product vector in Eq. (5). Namely, $|\Psi_i\rangle$ should be one of the product vectors in Eq. (5). So all the operators $A_{if}\otimes B_{if}$ "indicating" $|\Psi_i\rangle$ and only $|\Psi_i\rangle$ should project out an intact state $|\Psi_i\rangle$, but not a component of $|\Psi_i\rangle$ and then all the OPS $|\Psi_k\rangle s$ are eigenvectors of each operator of the operators $\{A_{if}\otimes B_{if}\}$ (e.g., in Eq. (5) $|\Psi_i\rangle$ is eigenvectors of $A_{if}\otimes B_{if}$ with nonzero eigenvalue; $|\Psi_j\rangle$ is eigenvectors of $A_{if}\otimes B_{if}$ with zero eigenvalue). For the same reason, we can prove that all the OPSs $|\Psi_k\rangle s$ are eigenvectors of each operator $A_{iN}\otimes B_{iN}$ representing the effect of N round measurements and communication, $N=1,2,\ldots$. In fact, suppose that during the first measure to distinguish the states $\{|\Psi_k\rangle\}$ (suppose Alice do the first measure) if an *effective* operator A_{i1} does not project out one or some intact OPS, e.g., A_{i1} projects out the part $|\Psi_k'\rangle$ of a OPS $|\Psi_k\rangle$ $(|\Psi_k\rangle=|\Psi_k'\rangle+|\Psi_k^{\perp}\rangle)$, then the two orthogonal parts $|\Psi_k'\rangle$ and $|\Psi_k^{\perp}\rangle$ of the OPS $|\Psi_k\rangle$ are orthogonal to $nm-1$ orthogonal states. This is impossible. So if the operators $\{A_{i1}\}$ describing the first measurement are *effective*, then A_{i1} must project out some intact OPSs. On the other hand, after Alice and Bob finished the first *effective* measure and get an outcome, the whole space collapses into a subspace and the OPSs in this subspace form a set of complete bases of the subspace. So the sequent measures have same property as the first measure.

Let us now prove that the set of final operators $\{A_{if}\otimes B_{if}\}$ can be carried out by projective measurements and classical communication. To achieve this, we consider Alice's first measurement described by $\{A_{i1}\}$

$$A_{i1} = c_1^{i1}|\phi_1^{i1}\rangle_A\langle\phi_1^{i1}| + \cdots + c_{n_a^{i1}}^{i1}|\phi_{n_a^{i1}}^{i1}\rangle_A\langle\phi_{n_a^{i1}}^{i1}|. \qquad (6)$$

Operator A_{i1} project out a subspace of Alice spanned by Alice's bases $|\phi_1^{i1}\rangle_A,\cdots,|\phi_{n_a^{i1}}^{i1}\rangle_A$. Since A_{i1} should project out some intact OPSs, this subspace should contain some intact Alice's vectors of the OPSs (we say $|v_k\rangle_A$ is Alice's vector of $|v_k\rangle_A|y_k\rangle_B$). Since all OPSs $\{|\Psi_k\rangle\}$ are eigenvectors of the operator A_{i1}, if $\{A_{i1}\}$ is *effective* for distinguishing the states $\{|\Psi_k\rangle\}$ then so does operators $\{A_{i1}'\}$:

$$A_{i1}' = |\phi_1^{i1}\rangle\langle\phi_1^{i1}| + \cdots + |\phi_{n_a^{i1}}^{i1}\rangle\langle\phi_{n_a^{i1}}^{i1}|. \qquad (7)$$

Operator A_{i1}' projects out a same subspace H_i as A_{i1} does, and all OPSs $\{|\Psi_i\rangle\}$ are the eigenvectors of the operator A_{i1}'. If operators $\{A_{i1}'\}$ are not a set of projective operators, we can find a set of projective operators $\{A_{i1}''\}$ by following protocol such that if $\{A_{i1}'\}$ is *effective* for distinguishing the states $\{|\Psi_k\rangle\}$ then so do operators $\{A_{i1}''\}$. We first choose two operators A_{11}', A_{21}' described by $A_{i1}'(i=1,2)$ in Eq. (7). Operators A_{11}', A_{21}' project out subspace H_1, H_2, respectively. Both H_1 and H_2 should contain intact Alice's vectors of some OPSs $|\Psi_k\rangle$. Suppose H_1 and H_2 contains Alice's vectors of $|\Psi_{k1}\rangle s$ and $|\Psi_{k2}\rangle s$, respectively, $|\Psi_{k1}\rangle s$, $|\Psi_{k2}\rangle s\in\{|\Psi_k\rangle\}$. Alice's vectors of the OPSs belonging to $|\Psi_{k2}\rangle s$ but not to $|\Psi_{k1}\rangle s$ form a subspace H of H_2. Obviously, H is orthogonal to H_1. We note operator A_{21}'' projects out subspace H and only H (In fact, $H_2=H\cup(H_2\cap H_1)$, and H is orthogonal to $(H_2\cap H_1)$. Operator A_{21}' projects out subspace H_2. The effect of operator A_{21}'' is to discard some bases of H_2, and only to project out subspace H). If we replace A_{21}' by A_{21}'' then A_{11}' and A_{21}'' project out some OPSs as same as A'_{11} and A'_{21} do, and these OPSs are distinguishable by LOCC if A'_{11} and A'_{21} are *effective* for distinguishing the states $\{|\Psi_k\rangle\}$. Similarly, operator A'_{31} projects out subspace H_3. We can discard the bases of H_3 which are vectors of subspace H_1 or H_2. The left bases of H_3 span a subspace projected out by operator A''_{31}. By a sequence of similar operations we always find a set of orthogonal projective operators $\{A'_{11},A''_{21},A''_{31},\ldots\}$ such that if operators $\{A'_{i1}\}$ are *effective* for distinguishing the OPSs, so do operators $\{A'_{11},A''_{21},A''_{31},\ldots\}$. Obviously $A'^{+}_{11}A'_{11}+A''^{+}_{21}A''_{21}+A''^{+}_{31}A''_{31}+\cdots=I$. So Alice's first measure can be carried out by a set projective operators $\{A'_{11},A''_{21},A''_{31},\cdots\}$, similar for Bob's first measure. On the other hand, after Alice and Bob finished the first measure and get an outcome, the whole space collapses into a subspace and the OPSs in this subspace form a set of complete bases of the subspace. So the sequent measures have the same property as the first measure. Thus the set of operators $\{A_{if}\otimes B_{if}\}$ can be carried out by projective measurements and classical communication. The whole proof is completely fit to the cases of multipartite systems. This ends the proof.

Theorem 1 characterizes the local distinguishability of the states of full product basis set. To judge the local distinguishability of the product bases conveniently we present a more practical theorem as follows.

Theorem 2. A set of states $\{|v_i\rangle_A|y_i\rangle_B, i=1,\ldots,nm\}$ is nm OPSs in a $n\otimes m$ system. If for every given state $|v_i\rangle_A|y_i\rangle_B$ there are $n-1$ states $|v'_j\rangle_A|y'_j\rangle_B\in\{|v_i\rangle_A|y_i\rangle_B, i=1,\ldots,nm\}, j=1,\ldots,n-1$ such that

$$\langle v'_1|v_i\rangle_A\neq 0,\langle v'_2|v'_1\rangle_A\neq 0,\cdots,\langle v'_{n-1}|v'_{n-2}\rangle_A\neq 0, \quad (8)$$

and $|v_i\rangle_A,|v'_1\rangle_A,\ldots,|v'_{n-1}\rangle$ are linearly independent; and if for every given state $|v_i\rangle_A|y_i\rangle_B$, there are $m-1$ states $|v'_k\rangle_A|y'_k\rangle_B\in\{|v_i\rangle_A|y_i\rangle_B, i=1,\ldots,nm\}, k=1,\ldots,m-1$ such that

$$\langle y'_1|y_i\rangle_B\neq 0,\langle y'_2|y'_1\rangle_B\neq 0,\cdots,\langle y'_{m-1}|y'_{m-2}\rangle_B\neq 0, \quad (9)$$

and $|y_i\rangle_B,|y'_1\rangle_B,\ldots,|y'_{m-1}\rangle_B$ are linearly independent, then states $\{|v_i\rangle_A|y_i\rangle_B, i=1,\ldots,nm\}$ are not LOCC distinguishable.

Proof: Suppose Alice does the first measure (Alice goes first [7]). From Theorem 1 it follows that, to distinguish states $\{|v_i\rangle_A|y_i\rangle_B, i=1,\ldots,nm\}$, these states should be eigenstates of Alice's first measure described as A_{j1} in Eq. (7). This means that $|v_i\rangle_A (i=1,\ldots,nm)$ are eigenstates of A_{j1}. If $|v_i\rangle_A$ is an eigenstate of the operator A_{j1} with non-zero eigenvalue and Eq. (8) holds, then $|v'_1\rangle_A$ should also be an eigenstate of the operator A_{j1} with non-zero eigenvalue, and so does $|v'_j\rangle_A, j=2,\ldots,n-1$. So the rank of the operator A_{j1} is full. A full-rank-operator A_{j1} would project out all OPSs and can do nothing to distinguish states $\{|v_i\rangle_A|y_i\rangle_B, i=1,\ldots,nm\}$, and similarly for Bob's first measure. So the states $\{|v_i\rangle_A|y_i\rangle_B, i=1,\cdots,nm\}$ are not LOCC distinguishable. This ends the proof.

Theorem 2 above can be generalized into multipartite cases, obviously. From Theorem 2 we can get many cases of indistinguishable states. There are three examples in the following.

Case 1. The nine OPSs in a $3\otimes 3$ system in the following are indistinguishable as shown in paper [6] of Bennett *et al.*

$$|\Psi_1\rangle=|1\rangle_A|1\rangle_B;|\Psi_{2,3}\rangle=|3\rangle_A|3\pm 1\rangle_B,$$

$$|\Psi_{4,5}\rangle=|2\rangle_A|1\pm 2\rangle_B;|\Psi_{6,7}\rangle=|3\pm 1\rangle_A|2\rangle_B,$$

$$|\Psi_{8,9}\rangle=|1\pm 2\rangle_A|3\rangle_B. \quad (10)$$

Case 2. The following 16 OPSs in a $4\otimes 4$ system are indistinguishable [18,19]:

$$|\Psi_{1,2}\rangle=|1\rangle_A|1\pm 2\rangle_B,\ |\Psi_{3,4}\rangle=|2\rangle_A|2\pm 3\rangle_B,$$

$$|\Psi_{5,6}\rangle=|3\rangle_A|3\pm 4\rangle_B,\ |\Psi_{7,8}\rangle=|4\rangle\big|_A 1\pm 4\big\rangle_B,$$

$$|\Psi_{9,10}\rangle=|1\pm 2\rangle_A|4\rangle_B,\ |\Psi_{11,12}\rangle=|3\pm 4\rangle_A|2\rangle_B;$$

$$|\Psi_{13,14}\rangle=|2\pm 3\rangle_A|1\rangle_B,\ |\Psi_{15,16}\rangle=|1\pm 4\rangle_A|3\rangle_B. \quad (11)$$

Case 3. The following 64 OPSs in a $4\otimes 4\otimes 4$ system are indistinguishable.

$$|\Psi_i\rangle=|\Psi_i\rangle_{AB}|1\rangle_C,\ |\Psi_{i+16}\rangle=|\Psi_i\rangle_{AB}|2\rangle_C,$$

$$|\Psi_{i+32}\rangle=|\Psi_i\rangle_{AB}|3\rangle_C,\ |\Psi_{i+48}\rangle=|\Psi_i\rangle_{AB}|4\rangle_C,$$

$$i=1,\cdots,16, \quad (12)$$

where $|\Psi_i\rangle$ is a state in case 2.

Employing the above Theorem 2, we can prove the cases 1 and 2 easily. In case 3, Charles can do the first projective measurement by operators $|1\rangle\langle 1|$, $|2\rangle\langle 2|$, $|3\rangle\langle 3|$, $|4\rangle\langle 4|$. But after Charles's first round measurement the states of Alice and Bob's part will collapse into some indistinguishable OPSs $|\Psi_i\rangle_{AB}s$. So the OPSs in case 3 is indistinguishable by LOCC.

Locally distinguishing the states of full OPSs set needs only local projective measurements and classical communication. Can this conclusion be generalized into a set of incomplete OPSs? A set of following OPSs shows it is not always true. Nine OPSs [20]

$$|\Psi_{1,2,3}\rangle=|\Psi_{1,2,3}\rangle_{AB}|x\rangle_C,$$

$$|\Psi_{4,5,6}\rangle=|\Psi_{4,5,6}\rangle_{AB}|y\rangle_C,$$

$$|\Psi_{7,8,9}\rangle=|\Psi_{7,8,9}\rangle_{AB}|z\rangle_C, \quad (13)$$

where $|\Psi_i\rangle (i=1,\ldots,9)$ is a state in Eq. (10), $|x\rangle=|1\rangle$; $|y\rangle=(|1\rangle+\sqrt{3}|2\rangle)/2$; $|z\rangle=(|1\rangle-\sqrt{3}|2\rangle)/2$, can be distinguished by Charles doing the first measure described by operators $C_{i1}(i=1,2,3)$ [20]

$$C_{11}=\sqrt{\frac{2}{3}}|x^*\rangle\langle x^*|,$$

$$C_{21}=\sqrt{\frac{2}{3}}|y^*\rangle\langle y^*|,$$

$$C_{31} = \sqrt{\frac{2}{3}}|z*\rangle\langle z*|, \tag{14}$$

where $\langle x|x*\rangle = \langle y|y*\rangle = \langle z|z*\rangle = 0$ and $\Sigma_{i=1}^{3} C_{i1}^{+} C_{i1} = 1$. After Charles get a outcome, nine OPSs in Eq. (13) collapse into locally distinguishable six OPSs. However, states $|\Psi_i\rangle(i = 1, \ldots, 9)$ in Eq. (13) cannot be distinguished by local projective measurements and classical communication.

In conclusion, we analyze the character of operators to distinguish a set of full OPSs in a multipartite system, and show that to distinguish perfectly the elements of a full basis set needs only local projective measurements and classical communication, and these measurements cannot damage each OPS. Employing these conclusions one can discuss local distinguishability the elements of any full product basis set easily. An open question is that whether these conclusions can be generalized to the local distinguishability of states in a quantum system, the sum of Schmidt number of the states is equal to the dimensions of Hilbert space of the system. Another open question is that which classes of operators can be carried out only local projective measurements, since local projective measurements are easier to be achieved than generalized POV measurements.

Note added. Recently, it came to our attention that De. Rinaldis [21] also obtained results partly similar to ours through different methods.

We would like to thank J. Finkelstein for presenting us a set of special states in Eq. (13) and Guangcan Guo for his help with this work.

[1] A. Einstein, B. Podoscky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[3] C. H. Bennett, Phys. Rev. Lett. **68**, 3121 (1992).
[4] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters., Phys. Rev. Lett. **70**, 1895 (1993).
[5] A. Barenco, Contemp. Phys. **37**, 375 (1996).
[6] C. H. Bennett, D. P. DiVincenzo, C. A. Fuchs, T. Mor, E. Rains, P. W. Shor, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **59**, 1070 (1999).
[7] J. Walgate and L. Hardy, Phys. Rev. Lett. **89**, 147901 (2002).
[8] M. Horodecki, A. Sen (De), U. Sen, and R. Horodecki, e-print quant-ph/0301106 (2003).
[9] J. Walgate, A. J. Short, L. Hardy, and V. Vedral, Phys. Rev. Lett. **85**, 4972 (2000).
[10] S. Ghosh, G. Kar, A. Roy, A. Sen, and U. Sen, Phys. Rev. Lett. **87**, 277902 (2001); S. Ghosh, G. Kar, A. Roy, D. Sarkar, A. Sen(De), and U. Sen, e-print quant-ph/0111136 (2001).
[11] S. Virmani, M. F. Sacchi, M. B. Plenio, and D. Markham, Phys. Lett. A **288**, 62 (2001).
[12] M. Horodecki, P. Horodecki, and R. Horodecki, Acta Phys. Slov. **48**, 141 (1998).
[13] L.-M Duan and G.-C Guo, Phys. Rev. Lett. **80**, 4999 (1998).
[14] Y.-X. Chen and D. Yang, Phys. Rev. A **64**, 064303 (2001).
[15] P.-X Chen and C.-Z Li, Phys. Rev. A **68**, 062107 (2003).
[16] V. Vedral and M. B. Plenio, Phys. Rev. A **57**, 1619 (1998).
[17] This can be proved easily by polar decomposition and singular value decomposition in Ref. [18], or see [19].
[18] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge), pp. 78–79
[19] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **80**, 5239 (1998); N. Linden, S. Massar, and S. Popescu, *ibid.* **81**, 3279 (1998); W. K. Wootters, *ibid.* **80**, 2245 (1998).
[20] J. Finkelstein (private communication).
[21] S. De Rinaldis, e-print quant-ph/0304027.