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# Conditional production of superpositions of coherent states with inefficient photon detection

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It is shown that a linear superposition of two macroscopically distinguishable optical coherent states can be generated using a single photon source and simple all-optical operations. Weak squeezing on a single photon, beam mixing with an auxiliary coherent state, and photon detecting with imperfect threshold detectors are enough to generate a coherent state superposition in a free propagating optical field with a large coherent amplitude ($\alpha > 2$) and high fidelity ($F > 0.99$). In contrast to all previous schemes to generate such a state, our scheme does not need photon number resolving measurements nor Kerr-type nonlinear interactions. Furthermore, it is robust to detection inefficiency and exhibits some resilience to photon production inefficiency.

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Since Schrödinger suggested his famous cat paradox [1], there has been great interest in generating and observing a quantum superposition of a macroscopic system. The component states composing such a superposition state should be macroscopically distinguishable, i.e., they should give macroscopically distinct measurement outcomes [2]. Two coherent states can be discriminated by a homodyne measurement, which can be considered a macroscopic measurement, when they are well separated in the phase space. Therefore, a superposition of two optical coherent states with sufficiently large amplitudes and with a $\pi$ phase difference between these amplitudes is considered a realization of such a macroscopic superposition.

Recently, the coherent state superposition (CSS) in a free propagating optical field has been studied for application to quantum information processing including quantum teleportation [3,4], quantum computation [5–7], entanglement purification [8] and error correction [9]. In particular, it was found that quantum computation can be realized using only linear optics and photon counting, given pre-arranged CSSs [6,7]. In this framework, initial CSSs of amplitude $\alpha \geqslant 2$ are required for efficient quantum computation with simple optical networks [7].

Unfortunately, it is extremely demanding to generate a free propagating CSS using current technology. It is well known that the CSS can be generated from a coherent state by a nonlinear interaction in a Kerr medium [10]. However, Kerr nonlinearity of currently available nonlinear media is extremely small and attenuation is not negligible compared with the required level to generate a CSS [11].

Some alternative methods have been studied to generate a superposition of macroscopically distinguishable states based upon conditional measurements [12,13]. A crucial drawback of these schemes is that a highly efficient detector which can discriminate photon numbers is necessary. Cavity quantum electrodynamics has been studied to enhance nonlinearity [14]. Some success has been reported in creating such superposition states within high $Q$ cavities in the microwave [15] and optical [16] domains. However, all the suggested schemes for quantum information processing with coherent states [3–9] require a *free propagating* CSS.

In this letter, we show that a free propagating optical CSS can be generated with a single photon source and simple optical operations. A CSS with a small coherent amplitude ($\alpha \leqslant 1.2$) and high fidelity ($F > 0.99$) can be deterministically generated by squeezing a single photon. A large CSS ($\alpha > 2$) with high fidelity ($F > 0.99$) can be obtained in a nondeterministic way from small CSSs. Weak squeezing, beam mixing with an auxiliary coherent field and photon detecting with threshold detectors are enough to generate such large CSSs given a single photon source. Remarkably, neither discrimination of photon numbers nor $\chi^{(3)}$ nonlinear interactions are required in our scheme. Furthermore, our scheme is robust to detection inefficiency and somewhat resilient to photon production inefficiency. In a more general sense, our examples reveal some previously unrealized relations between the quantum states of harmonic oscillators: we learn that the first excited energy eigenstates can be converted to superpositions of large coherent states by linear operations and projections.

A CSS can be defined as $|\mathrm{CSS}_\varphi(\alpha)\rangle = N_\varphi(\alpha)(|\alpha\rangle + e^{i\varphi}|-\alpha\rangle)$, where $N_\varphi(\alpha)$ is a normalization factor, $|\alpha\rangle$ is a coherent state of amplitude $\alpha$, and $\varphi$ is a real local phase factor. The amplitude $\alpha$ is assumed to be real for simplicity without loss of generality. In this paper we refer to the magnitude of $\alpha$ as the size of the CSS. Note that CSSs such as $|\mathrm{CSS}_\pm(\alpha)\rangle = N_\pm(\alpha)(|\alpha\rangle \pm |-\alpha\rangle)$ are called even and odd CSSs, respectively, because the even (odd) CSS, $|\mathrm{CSS}_+(\alpha)\rangle$ ($|\mathrm{CSS}_-(\alpha)\rangle$), always contains an even (odd) number of photons.

*An arbitrarily large CSS can be produced out of arbitrarily small CSSs using the simple experimental setup depicted in Fig. 1.* Let us first illustrate this procedure with a simple example. Suppose that one has a collection of identical small odd CSSs with known amplitude $\alpha_i$. Two of the small CSSs are selected and incident onto a 50:50 beam splitter BS1 as

$$|\mathrm{CSS}_-(\alpha_i)\rangle_a|\mathrm{CSS}_-(\alpha_i)\rangle_b \xrightarrow{\mathrm{BS1}} |0\rangle_f(|\sqrt{2}\alpha_i\rangle_g + |-\sqrt{2}\alpha_i\rangle_g) - (|\sqrt{2}\alpha_i\rangle_f + |-\sqrt{2}\alpha_i\rangle_f)|0\rangle_g, \tag{1}$$

where the normalization factor is omitted on the right-hand side. One can then say that if one could condition on detect-

 

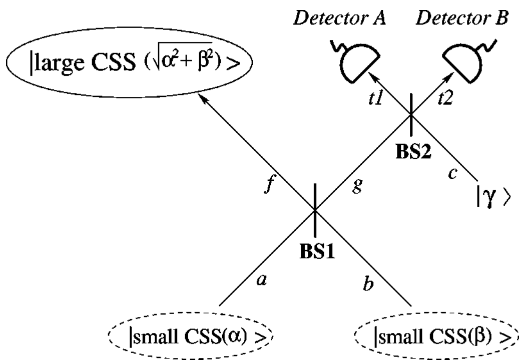


FIG. 1. A schematic of the non-deterministic CSS-amplification process. See the text for details.

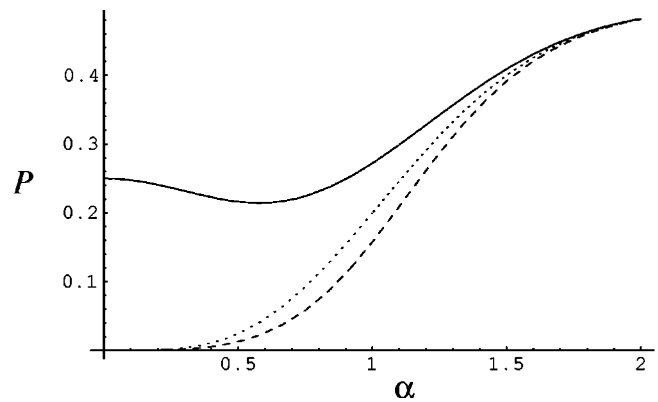


FIG. 2. The success probabilities of the CSS-amplifying process in Fig. 1 for the input fields of two identical odd CSSs (solid line), two identical even CSSs (dashed line), and even and odd CSSs (dotted line).

ing $|0\rangle_g$, a larger CSS with amplitude $\sqrt{2}\alpha_i$ would be obtained at mode $f$. An additional step is therefore needed to unambiguously discriminate between the vacuum and coherent states $|\pm\sqrt{2}\alpha_i\rangle_g$ with inefficient detectors. Another 50:50 beam splitter, BS2, mixes the field at mode $g$ and an auxiliary coherent state $|\sqrt{2}\alpha_i\rangle_c$ as

$$|\mathrm{BS1}\rangle_{f,g}|\sqrt{2}\alpha_i\rangle_c \xrightarrow{\mathrm{BS2}} |0\rangle_f(|2\alpha_i\rangle_{t1}|0\rangle_{t2}+|0\rangle_{t1}|2\alpha_i\rangle_{t2}) - (|\sqrt{2}\alpha_i\rangle_f + |-\sqrt{2}\alpha_i\rangle_f)|\alpha_i\rangle_{t1}|-\alpha_i\rangle_{t2}, \tag{2}$$

where $|\mathrm{BS1}\rangle_{f,g}$ represents the right-hand side of Eq. (1) and the normalization factor is omitted. Finally, photodetectors $A$ and $B$ are set to detect photons at modes $t1$ and $t2$. The remaining state at mode $f$ is selected only when both the detectors detect any photon(s) at the same time. In this case, it is obvious that the right-hand side of Eq. (2) is reduced to larger CSSs. If either of the detectors fails to click, the resulting state is discarded. This process can be recursively applied until a sufficiently large CSS is obtained. Suppose that an even CSS with amplitude $\alpha \geqslant 2$ is required while the initial amplitude of small odd CSSs is $\alpha_i = 1/\sqrt{2}$. After a sufficient number of CSSs of the amplitude $\sqrt{2}\alpha_i$ are obtained, the second step will be taken with the same experimental setup and another auxiliary coherent state $|2\alpha_i\rangle$. In this second stage, larger even CSSs of amplitude $2\alpha_i$ will be gained from pairs of even CSSs of $\sqrt{2}\alpha_i$. Eventually, the amplitude will reach the required value by four recursive applications of the process, i.e., $\alpha = 4\alpha_i \approx 2.83$.

The process described above can be generalized for arbitrarily small CSSs with known amplitudes as shown in Fig. 1. Suppose two small CSSs, $|\mathrm{CSS}_\varphi(\alpha)\rangle$ and $|\mathrm{CSS}_\phi(\beta)\rangle$, with amplitudes $\alpha$ and $\beta$. The reflectivity $r$ and transmitivity $t$ of BS1 are set to $r=\beta/\sqrt{\alpha^2+\beta^2}$ and $t=\alpha/\sqrt{\alpha^2+\beta^2}$, where the action of the beam splitter is represented by $\hat{B}_{a,b}(r,t)|\alpha\rangle_a|\beta\rangle_b = |t\alpha+r\beta\rangle_f|-r\alpha+t\beta\rangle_g$. The other beam splitter BS2 is a 50:50 beam splitter $(r=t=1/\sqrt{2})$ regardless of the conditions and the amplitude $\gamma$ of the auxiliary coherent field is determined as $\gamma = 2\alpha\beta/\sqrt{\alpha^2+\beta^2}$. The resulting state for mode $f$ then becomes $|\mathrm{CSS}_{\varphi+\phi}(\mathcal{A})\rangle \propto |\mathcal{A}\rangle + e^{i(\varphi+\phi)}|-\mathcal{A}\rangle$, whose coherent amplitude $\mathcal{A}=\sqrt{\alpha^2+\beta^2}$ is larger than both $\alpha$ and $\beta$. The relative phase of the resulting CSS is the sum of the relative phases of the input CSSs. The success probability $P_{\varphi,\phi}(\alpha,\beta)$ for a single iteration is

$$P_{\varphi,\phi}(\alpha,\beta) = \frac{e^{-4\alpha^2\beta^2/(\alpha^2+\beta^2)}(e^{2\alpha^2\beta^2/(\alpha^2+\beta^2)}-1)^2[1+\cos(\varphi+\phi)e^{-2(\alpha^2+\beta^2)}]}{2(1+\cos\varphi e^{-2\alpha^2})(1+\cos\phi e^{-2\beta^2})},$$

which is plotted for a number of different combinations in Fig. 2. The success probability approaches 1/2 as the amplitudes of initial CSSs become large. Note that the probabilities depend on the type of CSSs (odd or even) used. The effect of detector inefficiency is just to decrease this success probability.

*We now show that a small odd CSS with $\alpha \leqslant 1.2$ is surprisingly well approximated by a squeezed single photon.* The single mode squeezing operator is $\hat{S}(r) = e^{-(r/2)(\hat{a}^2-\hat{a}^{\dagger 2})}$, where $r$ is the squeezing parameter and $\hat{a}$ is the annihilation operator. This operator reduces quantum noise of a vacuum state in the phase quadrature by a factor of $e^{-2r}$. When the squeezing operator is applied to a single photon the resultant state can be expanded in terms of photon number states as

$$\hat{S}(r)|1\rangle = \sum_{n=0}^{\infty} \frac{(\tanh r)^n}{(\cosh r)^{3/2}} \frac{\sqrt{(2n+1)!}}{2^n n!}|2n+1\rangle. \tag{3}$$

The state contains only odd photon numbers and has coefficients decaying exponentially as $n$ increases in a similar fashion to an odd CSS. The fidelity of this state with an odd CSS is

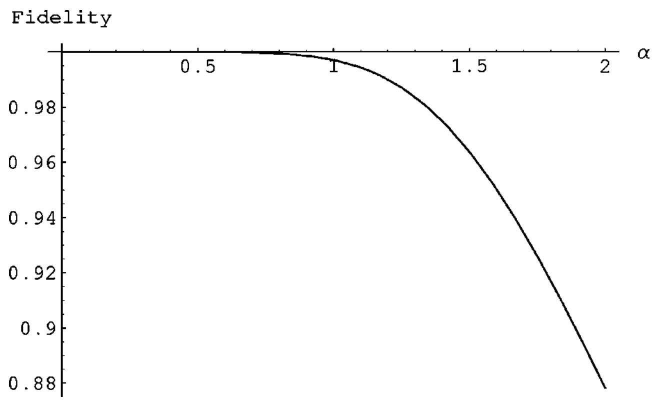


FIG. 3. The fidelity between an odd CSS and squeezed single photon. The odd CSS is extremely well approximated by the squeezed single photon for a small coherent amplitude, $\alpha \leqslant 1.2$.

$$F(r,\alpha) = |\langle \mathrm{CSS}_{-}(\alpha)|S(r)|1\rangle|^2 = \frac{2\alpha^2 \exp[\alpha^2(\tanh r - 1)]}{(\cosh r)^3(1-\exp[-2\alpha^2])}.$$

Figure 3 shows the maximized fidelity on the $y$ axis plotted against a range of possible values for $\alpha$ for the desired odd CSS. Some example values are: $F=0.999\,99$ for amplitude $\alpha=1/2$, $F=0.9998$ for $\alpha=1/\sqrt{2}$, and $F=0.997$ for $\alpha=1$, where the maximizing squeezing parameters are $r=0.083$, $r=0.164$, and $r=0.313$, respectively. First note that for $\alpha$ very close to zero the fidelity approaches unity. When $\alpha \to 0$, $r \to 0$ and hence the squeezing operator $\hat{S}(r)$ approaches the identity transformation. An odd CSS with $\alpha$ very close to zero has a significant contribution from a single photon and very little from higher odd photon numbers. This is the reason for the high fidelity as $\alpha$ tends to zero. The fidelity remains high for $\alpha$ near zero as one can match the three photon contribution to the CSS by the squeezing operator whilst still being able to neglect higher order photon number terms. Eventually as $\alpha$ increases, higher photon numbers cannot be matched and so as $\alpha$ tends to infinity, the fidelity tends to zero.

As the fidelity between a squeezed single photon and an ideal small CSS is extremely high, it can be conjectured that a large CSS distilled from squeezed single photons by our scheme will also be very close to an ideal large CSS. In what follows, we will show that this conjecture is true for $\alpha \leqslant 2.5$.

In order to calculate multiple iterations we need to use numerical techniques. We are using coherent states of some bounded coherent amplitude and superpositions there of. Provided the coherent amplitudes are not too large, the most significant contributions to these states are Fock states of low number. For computations here the lowest thirty Fock states were used. This provides a very good approximation for coherent states with $\alpha \leqslant 2.5$. All 29 possible "click" events are included for all detectors.

If one wished to create a CSS with a particular $\alpha$ with $n$ CSS-amplification steps, then initial CSSs with $\alpha_i=\alpha/\sqrt{2}^n$ are required. As the number of steps increases the required $\alpha_i$ decreases. When generating a large CSS out of the squeezed single photon states the fidelity maximizes for a particular number of iterations. Figure 4 shows the maximum possible fidelity using this process in (a) and the number of steps in (b) against the desired $\alpha$ in the CSSs. For example, four iterations starting from the initial amplitude $\alpha_i=1/2$ are required to gain the maximum fidelity $F=0.995$ for $\alpha=2$. It is evident from Fig. 4 that high fidelity, $F>0.99$, can be obtained up to $\alpha=2.5$. The error rate for discrimination between coherent states with $\alpha=\pm 2.5$ via a classical measurement (homodyne detection) is only $3\times 10^{-7}$.

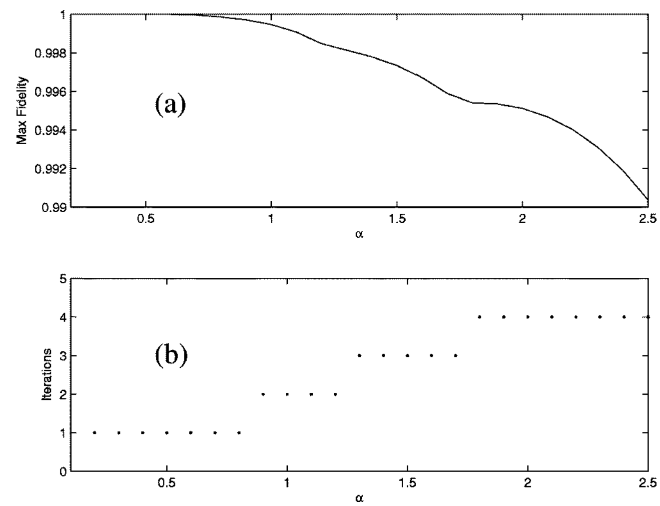


FIG. 4. (a) The maximum fidelity obtained in our scheme vs the coherent amplitude. (b) The number of iterations which gives the maximum fidelity vs the coherent amplitude.

Current technology does not produce pure single photon states; the single photon is always in a mixture with the vacuum as $p|0\rangle\langle 0|+(1-p)|1\rangle\langle 1|$, where $p$ is the inefficiency of the photon production. Hence the squeezed single photon state will also be a mixture with a squeezed vacuum. However, an interesting aspect of our scheme is that it may be somewhat resilient to the photon production inefficiency because its first iteration purifies the mixed CSSs while amplifying them. The initial input states for the CSS amplification process from the imperfect single photon source are

$$\begin{aligned}\rho_{a,b,c} = [&(1-p)^2|S_1\rangle\langle S_1| \otimes |S_1\rangle\langle S_1| + p^2|S_0\rangle\langle S_0| \otimes |S_0\rangle\langle S_0| \\ &+ p(1-p)(|S_0\rangle\langle S_0| \otimes |S_1\rangle\langle S_1| + |S_1\rangle\langle S_1| \otimes |S_0\rangle\langle S_0|)]_{a,b} \\ &\otimes (|\gamma\rangle\langle\gamma|)_c, \end{aligned} \qquad (4)$$

where $|S_0\rangle=\hat{S}(r)|0\rangle$ and $|S_1\rangle=\hat{S}(r)|1\rangle$. Here, the terms with $p^2$ and $p(1-p)$ are undesired error terms where either (or both) of the single photons is missing. Note that the initial amplitude is required to be small to produce a large CSS with high fidelity. Provided such a small amplitude, input states incident onto the beam splitters in our experimental setup contain approximately only two (or slightly more than two) photons. In such cases the probability of simultaneous clicks at detectors $A$ and $B$ in Fig. 1 will significantly decrease when any of the single photons is missing. In other words, the undesired cases will rarely be selected for the next iteration of the amplification process. We have obtained numerical results for the initial amplitude $\alpha_i=1/2$ as follows by the methods that we have already explained. If $p=0.4$, the fidelity of the initial CSS, which is a mixture with a squeezed vacuum, is $F=0.60$ but it will become $F=0.89$ by the first iteration. Thus a larger CSS of significantly high fidelity is produced. If $p=0.25$ ($p=0.05$), the fidelity of the initial CSS is $F=0.750$ ($F=0.950$) but becomes $F=0.941$ ($F=0.990$) by the first iteration. Such purification effects for multiple iterations are beyond the scope of this letter but deserve further investigations.

In the CSS amplification process, the zero amplitude coherent states that occur in the detection modes in Eq. (2) may be slightly different from zero because of imperfect mode matching at beam splitters. This will lead to a small probability of accepting the wrong state. Good mode matching is a requirement in any linear optical network where one wishes to measure manifestly quantum mechanical effects. Highly efficient mode matching of a single photon from parametric down conversion and a weak coherent state from an attenuated laser beam at a beam splitter has been experimentally demonstrated using optical fibers [19]. Such techniques could be employed for the implementation of our scheme.

The dark count rate of photodetectors will affect the fidelity of the CSSs. Currently, highly efficient detectors have relatively high dark count rates while less efficient detectors have very low dark count rates [18]. We emphasize again that our scheme does not require highly efficient detectors because the inefficiency of the detectors does not affect the quality of CSSs even though it decreases the success probability. Silicon avalanche photodiodes operating at the visible wavelength have relatively high efficiency and a small dark count rate, which is preferred in our proposal.

The single photons required for our scheme could be generated conditionally from a down-converter [17]. This is a $\chi^{(2)}$ process (like squeezing) and does not require photon number resolving detection. Once free propagating CSSs are generated, they can be detected by homodyne measurements, which can be highly efficient in quantum optics experiments.

Our scheme non-deterministically generates large CSSs. However, a non-deterministic CSS source is useful enough for quantum information processing [6,7]. Efficient gate operations for coherent-state quantum computation [7] are based on teleportation via an entangled coherent state [3,4]. Entangled coherent states can be simply generated from CSSs using a beam splitter and can be used as off-line resources for quantum computation. We note that such entanglement of macroscopically distinguishable states is perhaps more closely aligned with Schrödinger's original concept [1].

In conclusion, we have proposed a simple all-optical scheme to generate a linear superposition of macroscopically distinguishable coherent states in a propagating optical field. We have found a previously unrealized connection between squeezed number states and superpositions of coherent states as well as the interesting additive properties of the latter. In stark contrast to all previous schemes our scheme requires neither $\chi^{(3)}$ nonlinearity nor photon number resolving detection to generate a macroscopic superposition state.

*Note added.* The first two authors contributed equally to this work.

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