Decoherence of trapped-ion internal and vibrational modes: The effect of fluctuating magnetic fields

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The effect of ambient magnetic fields on the dynamics of a single trapped ion is studied analytically. We consider two electronic states with energy separation stochastically varied by magnetic-field perturbations and coupled to the vibrational modes by a laser. For Gaussian white noise, a master equation is derived and solved. The results obtained reveal how the effective Rabi frequencies and the decay rates depend on the noise strength and on the number state index. The detection of features specific to magnetic-field noise in the evolution of states that can be prepared under standard experimental conditions is discussed. A comparison with the effects of laser intensity and phase fluctuations is presented. Implications for the realization of logic gates are analyzed.

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I. INTRODUCTION

The control of decoherence is crucial for the applicability of trapped ions in quantum computation [1–3]. Recent theoretical and experimental research has focused on different decoherence sources relevant to the internal or the external dynamics, like radiative decay [3], magnetic-field noise [3–5], stray electric fields [6,7], fluctuating trap parameters [8–12], or random variations in the phase and intensity of the laser fields used to implement the logic operations [13]. The diverse origin of the noise sources and the variety of properties that they can present make it difficult to give a complete characterization of their role in the experimental realization of the logic gates. In fact, although important advances have been made in their study, further theoretical research is needed on significant questions, like the description of technical noise, the relative importance of the various decohering mechanisms, the influence of the noise statistics on the emergence of different decoherence features, or the design of error-correction schemes.

Here, we will focus on the effect of magnetic-field fluctuations on the dynamics of a single trapped ion. Previous theoretical studies [3,5] have tackled relevant aspects of this problem. Specifically, a two-level system, the two electronic states that form the "qubit," with an energy splitting stochastically varied by slow magnetic-field fluctuations was studied [3], and analytical results for the induced dephasing were obtained. On the other hand, the effect of rapidly varying deterministic magnetic fields on laser-induced coupling of electronic and vibrational states was analyzed [3]. Recently, we generalized the stochastic two-level approach by including general noise statistics [5]. Now, we extend the analysis by studying the influence of a random magnetic driving on the dynamics of the coupled internal and external modes. In particular, for Gaussian white noise, we give a complete analytical characterization of the effective Rabi frequencies and decay rates. Furthermore, in order to make a comparison with the characteristics of different decohering mechanisms, we generalize the results of previous work on the effects of laser phase and intensity fluctuations [13]. The emergence of the analyzed decoherence features in the evolution of initial states that can be prepared in standard experimental setups is discussed. Apart from applicability in the identification of the characteristics of ambient fields from the experimental data, the variety of analytical results presented can have more general implications: they clarify how fundamental aspects of the decoherence phenomenology are rooted in specific properties of the noise sources. Moreover, they show that, through the introduction of controlled stochastic fields [14], trapped ions can be an appropriate testing ground for predictions on effects distinctive of various decoherence mechanisms.

The outline of the paper is as follows. The system is described in Sec. II. In Sec. III, our approach to dealing with decoherence induced by random magnetic fields is presented. The stochastic Liouville-von Neumann equation [13], which gives the evolution of the density matrix for each particular noise history, is derived; subsequently, for Gaussian white noise, the random terms are averaged to obtain an effective master equation, which is solved analytically. In Sec. IV, the effects of magnetic perturbations are compared with those of other decohering mechanisms which have been recently studied: noise in the phase and intensity of the laser are considered, and some results of previous work are generalized [13]. Finally, in Sec. V our conclusions are summarized.

II. DESCRIPTION OF THE SYSTEM

We model the trapped ion as a two-level system, the two electronic states, $|g\rangle$ and $|e\rangle$, which form the "qubit," with an energy splitting $\hbar[\omega_0+\delta\omega_0(t)]$ stochastically varied by the effect of magnetic-field perturbations: ω_0 is the transition frequency that corresponds to the average magnetic field B_0, and $\delta\omega_0(t)$ is the frequency shift induced by the magnetic fluctuations $B-B_0$. Additionally, we consider coupling of the internal and motional modes through the application of a laser field with frequency ω_L and phase $\phi-\pi/2$. Specifically, we focus on the Hamiltonian [3]

$$H = \frac{\hbar}{2}[\omega_0 + \delta\omega_0(t)]\sigma_z + \hbar\nu a^\dagger a + \left[\frac{\hbar}{2}\Omega\sigma_+ e^{i\eta(a+a^\dagger)+i\omega_L t+i(\phi-\pi/2)} + \mathrm{H.c.}\right], \quad (1)$$

where $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\sigma_+ = |e\rangle\langle g|$; $a^\dagger(a)$ is the creation (annihilation) operator of the external mode along the laser propagation direction; ν is the trap frequency; Ω is the Rabi frequency; and η is the Lamb-Dicke parameter. As we will focus first on the role of magnetic fluctuations, we initially assume that Ω and ϕ take fixed values. (See Sec. IV for a discussion of the effects of random variations in Ω and ϕ.)

In most cases, the random frequency shift $\delta\omega_0(t)$ can be approximated as [3]

$$\delta\omega_0(t) = \left[\frac{\partial\omega}{\partial B}\right]_{B_0}(B - B_0) + \frac{1}{2}\left[\frac{\partial^2\omega}{\partial B^2}\right]_{B_0}(B - B_0)^2. \quad (2)$$

Typical values for the coefficients $[\partial\omega/\partial B]_{B_0}$ and $[\partial^2\omega/\partial B^2]_{B_0}$ and for the noise strength in systems where quantum gates have been realized are given in Ref. [3]. A strong sensitivity of the logic operations to magnetic noise can be anticipated from some of those values. Note that, because of the nonlinear term, $\delta\omega_0(t)$ can present a non-Gaussian probability distribution even for a normal noisy input $B - B_0$ [15]. The relative magnitude of the linear and quadratic contributions to $\delta\omega_0(t)$, which depends on both the noise amplitude and the coefficients $[\partial\omega/\partial B]_{B_0}$ and $[\partial^2\omega/\partial B^2]_{B_0}$, can be controlled by varying B_0.

A simplified description of the system is given in the rotating frame defined by the unitary transformation $U_1(t) = \exp(-i\omega_0\sigma_z t/2 - i\nu a^\dagger a t)$. To first order in η, and for the detuning corresponding to the first red sideband excitation [13], i.e., for $\omega_0 - \omega_L = \nu$, the transformed Hamiltonian reads

$$H = \frac{\hbar}{2}\delta\omega_0(t)\sigma_z + \frac{\hbar}{2}\Omega\eta(\sigma_+ a e^{i\phi} + \sigma_- a^\dagger e^{-i\phi}), \quad (3)$$

which, except for the stochastic term $\hbar\delta\omega_0(t)\sigma_z/2$, corresponds to the standard Jaynes-Cummings Hamiltonian [16]. Although, for simplicity, we will present results only for this Hamiltonian, parallel studies can be carried out for different transitions of the coupled electronic-vibrational spectrum. Namely, results for the carrier, blue sideband ("anti-Jaynes-Cummings" term), and second red sideband transitions, which can be excited by a proper choice of the detuning [13], are straightforwardly obtained following the same scheme.

III. DECOHERENCE INDUCED BY RANDOM MAGNETIC FIELDS

The properties of the different sources of magnetic perturbations that can be relevant to the dynamics of a trapped ion are diverse. Actually, important components of the stochastic magnetic driving are known to be broadband fluctuations and line noise, which can both be present in standard experimental realizations of the system. The dephasing effects of broadband noise on the internal dynamics were described in Ref. [5]. On the other hand, the effect of high-frequency line noise on the coupling of electronic and motional modes was characterized in Refs. [3,5] as a mere renormalization of the effective coupling constant Ω. Here, we aim at extending those previous studies by analyzing the influence of broadband fluctuations on the dynamics of coupled internal and vibrational modes. Specifically, we consider fluctuations, $\xi \equiv B - B_0$, with Gaussian white-noise characteristics, i.e., defined by $\langle\xi(t)\rangle_f = 0$ and $\langle\xi(t)\xi(t')\rangle_f = 2D\delta(t-t')$. Obviously, this approximation is valid provided that the noise correlation time is much smaller than any other relevant time scale in the problem. Additionally, we assume that the main contribution to $\delta\omega_0(t)$ corresponds to the linear term in Eq. (2); namely, we take $\delta\omega_0(t) = [\partial\omega/\partial B]_{B_0}(B - B_0)$. The contribution of the nonlinear term in Eq. (2) can be straightforwardly included in our study through a Gaussian approximation for the probability distribution of $\delta\omega_0(t)$. On this point, it is worth recalling that, in Ref. [5], where it was discussed how the internal-state dephasing is affected by a finite correlation time and by strong departures of the probability distribution for $\delta\omega_0(t)$ from a Gaussian function, it was shown that the features distinctive of the noise probability density tend to disappear as the correlation time decreases.

Applying the methodology presented in Ref. [13] to analyze the decohering effects of laser intensity and phase fluctuations, first we derive the stochastic Liouville-von Neumann equation, which gives the evolution of the density matrix for particular noise realizations; then, averaging the noise terms, we obtain the following master equation

$$\dot{\rho} = -\frac{i}{\hbar}[G,\rho] - \frac{\Gamma}{2}[\sigma_z,[\sigma_z,\rho]], \quad (4)$$

with

$$G \equiv \frac{\hbar}{2}\Omega\eta(\sigma_+ a + \sigma_- a^\dagger), \quad (5)$$

and where $\Gamma \equiv \frac{1}{2}[\partial\omega/\partial B]^2_{B_0} D$. Without loss of generality, we have set $\phi = 0$. The validity of this equation is restricted to the case of white noise. The equations corresponding to the carrier, blue sideband, and second red sideband excitations present the same dissipative term, $(\Gamma/2)[\sigma_z,[\sigma_z,\rho]]$; only the Hamiltonian part, in particular the functional form of G, is specific to each transition. Here, a comment is in order on the parallelism existent between our approach, based on incorporating classical fluctuations in the microscopic dynamics, and studies of decoherence based on system-plus-reservoir models. Whereas our method is valid irrespective of the magnitude of the fluctuations, standard treatments of systems coupled to dissipative environments [16], based on a Born-Markov approach, are valid only in regimes of weak interaction.

An appropriate basis to solve the master equation is that formed by the eigenstates of G, given by

$$|e_n^\pm\rangle = \frac{1}{\sqrt{2}}(|g,n\rangle \pm |e,n-1\rangle), \quad n = 1,2..., \quad (6)$$

$$|e_0\rangle = |g,0\rangle, \tag{7}$$

where n is the index of a Fock state of the external mode. The associated eigenenergies are, respectively, $e_n^{\pm} = \pm(\hbar/2)\eta\Omega\sqrt{n}$ and $e_0=0$. In this representation, Eq. (4) leads to

$$\langle e_n^{\pm}|\dot\rho(t)|e_m^{\pm}\rangle = [-i(e_n^{\pm}-e_m^{\pm})/\hbar - \Gamma]\langle e_n^{\pm}|\rho(t)|e_m^{\pm}\rangle + \Gamma\langle e_n^{\mp}|\rho(t)|e_m^{\mp}\rangle, \tag{8}$$

$$\langle e_n^{\pm}|\dot\rho(t)|e_0\rangle = [-ie_n^{\pm}/\hbar - \Gamma]\langle e_n^{\pm}|\rho(t)|e_0\rangle + \Gamma\langle e_n^{\mp}|\rho(t)|e_0\rangle, \tag{9}$$

$$\langle e_0|\dot\rho(t)|e_0\rangle = 0. \tag{10}$$

Note that, because of the dissipative term in Eq. (4), the elements $\langle e_n^{\pm}|\rho(t)|e_m^{\pm}\rangle$ $[\langle e_n^{\pm}|\rho(t)|e_0\rangle]$ are coupled to $\langle e_n^{\mp}|\rho(t)|e_m^{\mp}\rangle$ $[\langle e_n^{\mp}|\rho(t)|e_0\rangle]$. Actually, apart from the trivial Eq. (10), we have a set of systems of two equations. Equations with the same functional form are found for the carrier, blue sideband, and second red sideband transitions in the representation of the eigenstates of their associated Hamiltonians. The following analytical solutions are readily found:

$$\langle e_n^{\pm}|\rho(t)|e_m^{\pm}\rangle = \langle e_n^{\pm}|\rho(0)|e_m^{\pm}\rangle\left[\cos(\omega_{n,m}^{\pm,\pm}t) - i\frac{e_n^{\pm}-e_m^{\pm}}{\hbar\omega_{n,m}^{\pm,\pm}}\sin(\omega_{n,m}^{\pm,\pm}t)\right]e^{-\Gamma t} + \langle e_n^{\mp}|\rho(0)|e_m^{\mp}\rangle\frac{\Gamma}{\omega_{n,m}^{\pm,\pm}}\sin(\omega_{n,m}^{\pm,\pm}t)e^{-\Gamma t}, \tag{11}$$

$$\langle e_n^{\pm}|\rho(t)|e_0\rangle = \langle e_n^{\pm}|\rho(0)|e_0\rangle\left[\cos(\omega_{n,0}^{\pm}t) - i\frac{e_n^{\pm}-e_0}{\hbar\omega_{n,0}^{\pm}}\sin(\omega_{n,0}^{\pm}t)\right]e^{-\Gamma t} + \langle e_n^{\mp}|\rho(0)|e_0\rangle\frac{\Gamma}{\omega_{n,0}^{\pm}}\sin(\omega_{n,0}^{\pm}t)e^{-\Gamma t}, \tag{12}$$

$$\langle e_0|\rho(t)|e_0\rangle = \langle e_0|\rho(0)|e_0\rangle, \tag{13}$$

where $\omega_{n,m}^{\pm,\pm} = \sqrt{(e_n^{\pm}-e_m^{\pm})^2/\hbar^2-\Gamma^2}$ and $\omega_{n,0}^{\pm} = \sqrt{(e_n^{\pm}-e_0)^2/\hbar^2-\Gamma^2}$. The analysis of these equations uncovers the characteristics of the decoherence process. (i) The coherences are found to decay and asymptotically reach a zero value. Moreover, it is apparent that a transfer of population takes place between the eigenstates $|e_n^{+}\rangle$ and $|e_n^{-}\rangle$ (with the same index n) until both states are equally populated. Therefore, the asymptotic solution corresponds to a statistical mixture of eigenstates with populations that can be different from the initial ones. This behavior can be qualitatively anticipated from the form of the Hamiltonian in Eq. (3): since the stochastic term, $(\hbar/2)\delta\omega_0(t)\sigma_z$, does not commute with G, the decoherence process cannot be a mere dephasing in the basis of eigenstates of G. As there are no transitions between states with different n, there is no energy relaxation. (ii) The decay of the coherences is purely exponential with rate Γ scaling with the noise variance. At this point, we must recall that we are dealing with Gaussian white noise (see Sec. IV and Ref. [5] for an analysis of the role played by noise color in the emergence of nonexponential decay). (iii) In contrast with the results of the study of intensity noise (see Sec. IV), the damping rates are independent of the number state indexes. (Note the relevance of the form in which fluctuations enter the system to the dependence of the damping rate on n). Furthermore, the oscillation frequencies of the coherences, $\omega_{n,m}^{\pm,\pm}$, and, in particular, the effective Rabi frequencies, $\omega_n \equiv \omega_{n,n}^{+,-} = \sqrt{4\eta^2\Omega^2 n - \Gamma^2}$, depend on the damping coefficient Γ, and, consequently, on the noise strength D. As no restrictions on D have been imposed to derive the master equation, the form obtained for the noise-induced frequency shifts is valid irrespective of the noise magnitude.

It is important to stress that, since the time for a single operation τ_{op} in the implementation of the logic gates is given by $\tau_{op} \simeq \omega_n^{-1}$ [3] and the decoherence time is $\tau_d = \Gamma^{-1}$, the range for the noise strength in which the system can have applicability in quantum computation is defined by the condition $\omega_n \gg \Gamma$. In this regime, the noise-induced shifts of the Rabi frequencies are negligible. Additionally, one should remark that a certain degree of control over the decoherence process can be achieved by changing the mean field B_0, and, consequently, the coefficient $[\partial\omega/\partial B]_{B_0}$, which determines, along with the noise amplitude, the decoherence time τ_d. It is even possible to work with a first-order field-independent transition [3]. In this case, magnetically induced decoherence still enters the dynamics through the second-order shift in $\delta\omega_0(t)$ [see Eq. (2)] and through the effect of the fluctuations on the transitions to auxiliary levels, which may be needed for the implementation of the logic gates and are not considered in our study. Obviously, it is the minimum global effect of noise that must be sought when choosing the value of B_0.

The experimental characterization of the noise sources is usually made by recording and analyzing the probability $P_g(t) \equiv \Sigma_n\langle g,n|\rho|g,n\rangle$ of finding the atom in the ground state for different initially prepared states [3]. Then, it is worth discussing how the decoherence features show up in $P_g(t)$. We consider the initial state $|\psi(t=0)\rangle = |g\rangle|\alpha\rangle$, where $|\alpha\rangle$ is a coherent state of the vibrational mode, which can be prepared by different methods under standard experimental conditions. The evolution of this state is trivially obtained by inverting the sequence of unitary transformations. In particular, $P_g(t)$ is given by

$$P_g(t) = \frac{1}{2}\left\{1 + e^{-\Gamma t}\sum_n p_n\left[\cos(\omega_n t) + \frac{\Gamma}{\omega_n}\sin(\omega_n t)\right]\right\}, \tag{14}$$

where $p_n \equiv |\langle n|\alpha\rangle|^2 = (|\alpha|^{2n}/n!)e^{-|\alpha|^2}$ is the Poisson distribution of the coherent state over the number states [16]. In the regime of practical interest, i.e., for $\omega_n \gg \Gamma$, the contribution of the terms $(\Gamma/\omega_n)\sin(\omega_n t)$ is negligible. A similar functional structure of $P_g(t)$, namely, a sum of oscillating terms modulated by damping factors, is found for the carrier, blue sideband, and second red sideband transitions; only the form of the effective Rabi frequencies ω_n is specific to each case. Actually, this structure of $P_g(t)$ corresponds to that introduced phenomenologically to account for experimental results obtained for the blue sideband transition [3]. We point out that the nontrivial dependence of the damping rates on n found in the experiments cannot be rooted in magnetic noise, which, in fact, has been shown to lead to the uniform damping factor $e^{-\Gamma t}$. (See Refs. [17–19] for a discussion of different aspects of this problem.)

The result corresponding to the *deterministic* Jaynes-Cummings Hamiltonian is trivially recovered by taking $\Gamma=0$ in Eq. (14). Typical of the noiseless system are the collapses and revivals of the population for the considered initial state [16]. These features are rooted in the dependence of the effective Rabi frequencies on the number index: the dephasing of the different oscillating terms leads to a collapse; the recovery of an approximately constant phase difference induces a partial revival. The persistence of the revivals in the noisy system can be anticipated to depend crucially on the relative magnitude of the revival and decoherence times. The time of revival $t_r \sim 2\pi|\alpha|/\eta\Omega$ [16] is determined by the mean occupation number of $|\alpha\rangle$ and by the effective coupling constant $\eta\Omega$ of the Jaynes-Cummings Hamiltonian; the decoherence time has been shown to scale with the inverse of the noise variance. Although the appearance of collapses and revivals could be affected by the noise-induced decrease of ω_n, this effect is unimportant for noise strengths in the range of practical interest.

We close this section with a comment on the limit of high noise amplitude. This regime, which would prevent the applicability of the system in quantum information processing, is not expected to be relevant to standard experimental setups. However, its analysis is of interest, as it gives additional insight into the decoherence mechanism and shows the consistency of our approach. From our results, a nontrivial behavior is noticeable: in the limit of high noise strength, the populations in the basis $\{|g,n\rangle,|e,n\rangle\}$ become stable, and, consequently, the collapse in $P_g(t)$ for an initial state $|\psi(t=0)\rangle=|g\rangle|\alpha\rangle$ is inhibited. More specifically, for $4\eta^2\Omega^2 n<\Gamma^2$, a purely imaginary ω_n is obtained, which implies that the corresponding coherence does not oscillate, but simply decays. These features are understood by taking into account that for the Rabi oscillations to take place, the coherent evolution of a superposition of states $|e_n^+\rangle$ and $|e_n^-\rangle$ is needed. Therefore, the oscillations cannot be apparent if the energy splitting $e_n^+-e_n^-=\hbar\eta\Omega\sqrt{n}$ is smaller than its associated coherence decay rate $\hbar\Gamma$. Indeed, for large D, the behavior corresponds to an effective decoupling of internal and motional states which approaches that existent in the system in the absence of the exciting laser: in the basis $\{|g,n\rangle,|e,n\rangle\}$, the populations are fixed and the coherences decay, the form of the decay being an exponential function for Gaussian white noise.

IV. A COMPARISON WITH OTHER DECOHERING MECHANISMS

As previously mentioned, different sources of decoherence can be relevant to the dynamics of trapped ions. Actually, the experimental results are expected to reflect the combined effect of the various noise sources. Especially important are, apart from magnetic noise, the fluctuations in the phase and intensity of the exciting laser. We compare now the most prominent features of those decohering mechanisms, which were analyzed in previous work [13], with those of random magnetic perturbations.

A. Phase fluctuations

First, we will show that there is a partial analogy between the effects of laser-phase fluctuations (see Ref. [13]), and the influence of stochastic magnetic fields. To this end, we consider the system defined by Eq. (1) with $\delta\omega_0(t)=0$, i.e., no magnetic-field fluctuations, and a random phase $\phi(t)$. The stochastic unitary transformation $U_2(t)=\exp[i\phi(t)\sigma_z/2]$, which leaves the populations in the basis $\{|g,n\rangle,|e,n\rangle\}$ invariant, leads to the transformed Hamiltonian

$$H=\frac{\hbar}{2}\zeta(t)\sigma_z+\frac{\hbar}{2}\Omega\,\eta(\sigma_+a+\sigma_-a^\dagger), \qquad (15)$$

which has the same form as the Hamiltonian that describes magnetic-field noise [see Eq. (3)]. The fluctuations in $\phi(t)$ have been converted into an effective random variation in the energy, $\hbar\zeta(t)$, characterized by $\zeta dt=d\phi$. Therefore, the populations in the initial representation evolve as if the system were driven by magnetic fluctuations with $\delta\omega_0(t)=\zeta(t)$. In particular, a Wiener process in $\phi(t)$ [15] has the same effects on the populations as a white-noise input in $\delta\omega_0(t)$. In contrast, given that the coherence operators do not commute with U_2, differences in the effects of both mechanisms on the coherences can be anticipated. One should notice that the effect of phase noise on the coherences cannot be studied by deriving a master equation from Eq. (15) as previously done for magnetic fluctuations: because of the previous random transformation U_2, the stochastic averaging performed in the derivation of a master equation is not complete if the evolution of the coherences in the initial representation is concerned. Similar arguments were presented in Ref. [13].

B. Intensity fluctuations

Now, in order to compare the effects of laser-intensity noise with those of random magnetic fields, we consider the system described by Eq. (3) with $\delta\omega_0(t)=0$, a fixed phase $\phi=0$, and a noisy intensity $\Omega(t)=\Omega_0+\delta\Omega(t)$, where $\delta\Omega(t)$ denotes fluctuations around the mean value Ω_0. In the basis formed by the eigenstates of G, given by Eqs. (6) and (7), the Hamiltonian is expressed as

$$H=\frac{\hbar}{2}\eta\Omega(t)\sum_{n=1}^{\infty}\sqrt{n}\sigma_z^{(n)}, \qquad (16)$$

where $\sigma_z^{(n)}\equiv|e_n^+\rangle\langle e_n^+|-|e_n^-\rangle\langle e_n^-|$. The change of representation allows us to describe the fluctuations in intensity as random variations in effective energy separations. Therefore, we can anticipate that in this case, decoherence corresponds to a pure dephasing process in the representation of eigenstates of G. Applying the methodology presented in Refs. [5,6], we can obtain the exact time propagator without solving a master equation. An evident advantage of this approach is that the restriction of considering only Gaussian white noise, imposed to derive the master equation, can be removed. This is not a minor advantage taking into account that the intensity noise actually relevant to the experimental arrangements can present a finite bandwidth and high-frequency contributions [3]. For Gaussian colored noise [15], i.e., for $\delta\Omega(t)$ defined by $\langle \delta\Omega(t)\rangle_f=0$ and $\langle \delta\Omega(t)\delta\Omega(t')\rangle_f=(D/\tau_c)e^{-|t-t'|/\tau_c}$, we have obtained analytical results for the populations and coherences. In particular, for the initial state $|\psi(t=0)\rangle=|g\rangle|\alpha\rangle$, the probability of finding the atom in the ground state is given by

$$P_g(t)=\frac{1}{2}[1+\sum_n p_n \cos(\eta\Omega_0\sqrt{n}t)e^{-\eta^2 nD[t+\tau_c(e^{-t/\tau_c}-1)]}], \tag{17}$$

where qualitative differences with the result found for random magnetic fields are evident. The effective Rabi frequencies are independent of the noise strength. Additionally, the exponents of the damping factors depend on the number state index, the dependence being linear for the considered red sideband transition. Furthermore, the effect of noise color is apparent: a finite value of the correlation time τ_c implies nonexponential time decay. More specifically, for $t\ll\tau_c$, the decay has a Gaussian form; subsequently, there is an intermediate regime distinctive of the noise spectrum; finally, for $t\gg\tau_c$, exponential decay sets in. It is important to remark that there is no sharp crossover from Gaussian to exponential decay, but a transient period, which can be analytically characterized for different types of Gaussian noise; its identification in the experimental data can serve to define the color properties of the noise source. In the white-noise limit, $\tau_c\to 0$, purely exponential decay is found at any time, in agreement with the results of Ref. [13].

V. CONCLUDING REMARKS

We have studied the effect of ambient magnetic fields on laser-induced coupling of electronic and vibrational states of trapped ions. Magnetic white noise has been shown to induce pure exponential decay of the Rabi oscillations, the damping rates being independent of the number state indexes and scaling with the noise variance. Additionally, we have found that the fluctuations shift the effective Rabi frequencies. The applicability of the system in quantum information processing requires that the time scale for the logic operations, determined by the effective Rabi frequencies, must be much smaller than the decoherence time. In this range for the noise strength, the noise-induced frequency shifts are negligible. Although we have focused on the case of having a dominant linear contribution in the magnetically induced stochastic frequency shift, our results can also serve to describe qualitatively a quadratic Zeeman effect, provided that a Gaussian approximation for the frequency shift is valid.

A comparison with the effect of other relevant sources of fluctuations has been presented. Whereas there is a partial analogy with decoherence induced by laser-phase noise, qualitative differences with the effect of laser-intensity fluctuations are found. Moreover, we have extended a previous analysis [13] of the influence of intensity noise on the dynamics by considering fluctuations with a finite bandwidth. Our results uncover features distinctive of the noise spectrum in the evolution of the coherences.

Apart from the characterization of the effect of uncontrolled noise sources, the study provides an understanding of how fundamental aspects of the phenomenology of dissipation are rooted in the properties of the decohering mechanisms. We recall that in previous work [14], different controlled environments, realized by incorporating random variations in the trap frequency and by including stochastic electric fields in the system, served to test predictions on the scaling of decoherence rates. In a similar way, the introduction of controlled stochastic variations in the intensity and phase of the laser, which is feasible under the current experimental conditions, can be used to “engineer” reservoirs with particular properties. Therefore, it is possible to test, with trapped ions, crucial aspects of decoherence, like the role of noise color in the appearance of different functional forms for the decay, or the relevance of the form in which dissipation enters the system to the emergence of diverse dynamical properties.

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