is insignificant when compared to what has been accomplished already for the hydrogen atom[9] (except that it has shown precisely how slowly a perturbation calculation in other than the Coulomb gauge converges).

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[1]T. Murota, M.-T. Noda, and F. Tanaka, Prog. Theor. Phys. **41**, 1251 (1969).

[2]J. Gammel, M. Menzel, and W. Wortman, Phys. Rev. D **3**, 2175 (1971).

[3]We do not call this atom "positronium" because we do not allow annihilation.

[4]M. Levine, J. Tjon, and J. Wright, Phys. Rev. Lett. **16**, 962 (1966).

[5]G. C. Wick, Phys. Rev. **96**, 1124 (1954).

[6]N. Kemmer and A. Salam, Proc. R. Soc. A **230**, 266 (1955).

[7]H. Partovi and E. L. Lomon, Phys. Rev. D **2**, 1999 (1970); see their Eq. (3.3).

[8]A. H. Stroud and Don Secrest, *Gaussian Quadrature Formulas* (Prentice-Hall, Englewood Cliffs, N.J., 1966).

[9]H. Grotch and D. R.Yennie, Rev. Mod. Phys. **41**, 350 (1969).

[10]E. E. Salpeter, Phys. Rev. **87**, 328 (1952); see Eqs. (1) and (2) and remarks at top of the right-hand column on p. 329.

[11]J. A. Campbell and A. C. Hearne, J. Comput. Phys. **5**, 280 (1970).

[12]J. S. R. Chisholm, Proc. Camb. Philos. Soc. **48**, 300 (1952).

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Transfer of Optical Information by Collective States of Electron Beams

L. D. Favro and P. K. Kuo

Department of Physics, Wayne State University, Detroit, Michigan 48202

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We show that all the coherence properties of the field in an optical cavity can be transferred to a second cavity by a monoenergetic electron beam. The energy supplied to the second cavity comes from the electrons's kinetic energy, as in the klystron.

I. INTRODUCTION AND DISCUSSION OF PREVIOUS THEORY

Recently, there has been much theoretical interest in the possibility of using electrons as carriers of optical information. This was stimulated by a report of an unusual experiment in which an electron beam appeared to be able to transport light from one place to another.[1] If the same experiment were done in a much lower frequency range one could readily understand the transfer of frequency and phase information by an electron beam within the context of classical physics. This is the situation that occurs in a conventional klystron. There, the oscillation of the electromagnetic field is sufficiently slow and the wavelength sufficiently long that the entire wave packet of the electron may be considered as a point in both the space and time dimensions. Therefore, the electrons can be described by classical trajectories and the information is carried by variations in the particle density in the beam.

In the optical range the situation is quite differ ent. First, with any readily attainable optical field intensities the average number of quanta absorbed by an electron in a Schwarz-type experiment[1] is less than or of the order of unity. Thus the result of the interaction with the field cannot be described as a simple acceleration. Second, the period and wavelength of the radiation can be long compared to the coherence time and coherence length of an electron's wave packet. As a result, upon interaction with an electromagnetic field the wave packet undergoes an intrinsic change in structure. In addition, the classical picture of particle bunching breaks down because the bunching length is now smaller than the wave-packet size. Thus, it is important to obtain a complete quantum-mechanical understanding of the processes which may be involved.

Existing "quantum-mechanical" theories fall into two general categories. The first category contains all theories which do not consider particle-particle correlations and hence are essentially one-particle in nature.[2] In the second category of theories, the whole effect is ascribed to electron-electron correlations. Theories in the first category typically treat the interaction with the applied field in terms of a single quantum-mechanical electron's interaction with a classical radiation mode. The re-

sulting electron charge-current density is then treated as a classical source of radiation. Since the interaction with the applied field causes the current density to be modulated at the applied frequency, the subsequent radiation is also characterized by the same frequency. While these theories produce believable results in some cases, the practice of using a one-electron quantum charge-current density as a classical source conflicts with the principles of quantum mechanics. The conflict arises because this practice produces radiation intensities which are *quartic* in the initial electron wave function. If a quantum-mechanical calculation were to give this kind of dependence it would have to have a transition amplitude which was quadratic in the initial state, and hence, would violate the superposition principle. One can avoid this difficulty by utilizing the usual semiclassical approximation which uses *matrix elements* instead of expectation values as a source but this does not produce radiation which is preferential at the modulation frequency. Thus on the one hand we have incorrect theories in which an electron appears to be able to carry light and on the other hand a correct (at least in an approximate way) theory which predicts no such effect. The resolution of this dilemma lies in the second (many-particle) category of theories.

It was independently recognized by three groups of workers[3] that the modulation information is not contained in the one-electron radiation but in the many-electron radiation resulting from electron-electron correlations. In these theories the *beam* is treated as a single quantum system, and its interaction with the radiation field is considered. Two groups,[4] using a product wave function for the many-electron system, have shown that the collective radiation intensity has the structure

$$\sim \sum_{m,n} A_m^* A_n \ ,$$

where A_m is essentially the expectation value of the current density (or some other operator) for the mth electron. For a large number of electrons, say N, emitting collectively this gives a result similar to the "one-particle" theories but is proportional to $N(N-1)$ instead of N. Since in this result the wave function of *each* electron appears only quadratically, it is not in conflict with the superposition principle. Also this fits in with the classical klystron theory in which it is correlations (in position) of the electrons which carry the information. With a somewhat different approach, Kondo[3] has shown that when an electron beam interacts with a radiation field in an energy eigenstate the current density does *not* become modulated, but the subsequent collective radiation from the beam nevertheless contains the modulation frequency. This result further emphasizes the inadequacy of theories based on the modulation of one-particle current densities since here the current density is not responsible for carrying the information. These theories indicate that there is more than one way to code information on a beam and hence pose the problem of finding a more general description of the mechanism of information transmission by a beam.

In this paper, we use operator techniques to give an integral treatment of the collective theories and to extend them to include fields of arbitrary coherence. In addition, we take account of exchange effects by using a fully second quantized formalism. The model we use consists of a beam of electrons passing through two optical cavities each of which is described by a single-mode function. The excitation in the second cavity is calculated for an arbitrary state of the first cavity by a perturbation expansion. Some of the qualitative results can be listed here. (i) The power level in the second cavity is proportional to the power level in the first cavity and is independent of its coherence properties or any charge-current modulation. (ii) The phase and amplitude of the expectation value of the electric field in the second cavity are determined by the corresponding quantities in the first cavity. Moreover, all of the coherence properties (or lack thereof) in the first field are transferred to the collective part of the radiation in the second cavity. (iii) The transmission of information is not vitiated by exchange effects.

II. TRANSFER OF COHERENCE

We consider two optical cavities A and B communicating via an electron beam which first passes through cavity A and then through B. For simplicity we assume that each cavity is described by a single-mode function [$\vec{f}_A(\vec{r})$ or $\vec{f}_B(\vec{r})$] and that they have the same frequency ω. Let $a^\dagger$ and $b^\dagger$ be the photon creation operators of the cavities A and B, respectively, and $\eta_{\vec{q}}^\dagger$ be the creation operator of an electron in the momentum state $\vec{p} = \hbar\vec{q}$.[5] The free Hamiltonian of the electron-beam-cavities system is given by

$$H_0 = \hbar\omega(a^\dagger a + b^\dagger b) + \sum_{\vec{q}} \hbar\omega_{\vec{q}} \eta_{\vec{q}}^\dagger \eta_{\vec{q}} \ . \tag{1}$$

In the usual form the interaction between the ith electron and the radiation field is taken to be

$$-\frac{e}{2mc}\left[\vec{p}_i \cdot \vec{A}(\vec{r}_i) + \vec{A}(\vec{r}_i) \cdot \vec{p}_i\right] + \frac{e^2}{2mc^2} A^2(\vec{r}_i) \ . \tag{2}$$

The term quadratic in A will henceforth be ignored because it is of no consequence for subsequent calculations. In terms of the mode functions, the vector potential has the form

$$\vec{A}(\vec{r}) = a\vec{f}_A(\vec{r}) + b\vec{f}_B(\vec{r} - \vec{R}) + \text{H. c.} \ , \tag{3}$$

where we have taken cavity A to be centered at the origin and cavity B at $\vec{R}$. When expressed in second-quantized form the interaction is then given by

$$H' = \hbar \sum_{\vec{q}\vec{q}'} (A_{\vec{q}\vec{q}'} a^\dagger + B_{\vec{q}\vec{q}'} b^\dagger) \eta^\dagger_{\vec{q}} \eta_{\vec{q}'} + \text{H.c.} \tag{4}$$

with

$$A_{\vec{q}\vec{q}'} = -\frac{e}{2mcL^3} (\vec{q}+\vec{q}') \cdot \int e^{i(\vec{q}'-\vec{q})\cdot\vec{r}} \vec{f}_A(\vec{r})\, d^3r\,, \tag{5}$$

$$B_{\vec{q}\vec{q}'} = -\frac{e}{2mcL^3} (\vec{q}+\vec{q}') \cdot \int e^{i(\vec{q}'-\vec{q})\cdot(\vec{r}+\vec{R})} \vec{f}_B(\vec{r})\, d^3r\,. \tag{6}$$

Our general method of calculation will be to calculate the time-dependent expectation values of various operators in the interaction picture by relating them to expectation values in the initial state. For instance the energy in the second cavity, E_B, at time t is given by

$$E_B = \hbar\omega \langle t | b^\dagger b | t \rangle\,, \tag{7}$$

where

$$|t\rangle = T\exp\left(-\frac{i}{\hbar}\int_{t_0}^{t} H'(t')\,dt'\right) |t_0\rangle\,, \tag{8}$$

and $H'(t)$ is the interaction Hamiltonian in the interaction picture and is given by (4) with $A_{qq'}$ and $B_{qq'}$ replaced by $A_{qq'}(t) = A_{qq'} e^{i(\omega+\omega_q-\omega_{q'})t}$ and $B_{qq'}(t) = B_{qq'} e^{i(\omega+\omega_q-\omega_{q'})t}$, respectively. The initial state $|t_0\rangle$ will be taken to be one with a beam of N electrons about to enter the first cavity and with no radiation field (vacuum state) in the second cavity. The initial state of the field in the first cavity will be left arbitrary. The energy spread of the electron beam is taken to be much less than $\hbar\omega$ and the number N much greater than unity.

The lowest-order nonvanishing contribution to (7) is the second-order term

$$\begin{aligned} E_B^{(2)} &= \frac{\hbar\omega}{\hbar^2}\int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \langle t_0 | H'(t') b^\dagger b H'(t'') | t_0 \rangle \\ &= (2\pi)^2 \sum_{\text{all } \vec{q}} \delta(\omega-\omega_1+\omega_2)\delta(\omega+\omega_3-\omega_4) \\ &\quad \times \langle t_0 | \eta^\dagger_{\vec{q}_1}\eta_{\vec{q}_2}\eta^\dagger_{\vec{q}_3}\eta_{\vec{q}_4} | t_0 \rangle B^*_{\vec{q}_2\vec{q}_1} B_{\vec{q}_3\vec{q}_4} \\ &= (2\pi)^2 \sum_{\vec{q}_1,\vec{q}_4} \delta(\omega_1-\omega_4) \langle t_0 | \eta^\dagger_{\vec{q}_1}\eta_{\vec{q}_4} | t_0 \rangle \\ &\quad \times \sum_{\vec{q}_2} \delta(\omega-\omega_1+\omega_2) B^*_{\vec{q}_2\vec{q}_1} B_{\vec{q}_2\vec{q}_4} \\ &\quad - (2\pi)^2 \sum_{\text{all } \vec{q}} \delta(\omega-\omega_1+\omega_2)\delta(\omega+\omega_3-\omega_4) \\ &\quad \times \langle t_0 | \eta^\dagger_{\vec{q}_1}\eta^\dagger_{\vec{q}_3}\eta_{\vec{q}_2}\eta_{\vec{q}_4} | t_0 \rangle B^*_{\vec{q}_2\vec{q}_1} B_{\vec{q}_3\vec{q}_4}\,, \end{aligned} \tag{9}$$

where $t - t_0$ has been assumed to be large compared to the period of any natural oscillation in the problem. The last term is a two-particle cooperative term which is present only if the spread in energy of the beam is wider than $\hbar\omega$. (The energy δ functions require the simultaneous presence of energies differing by $\hbar\omega$ in the initial state $|t_0\rangle$.) It does not contribute here because of our assumptions about the energy spread of the incoming beam. The first term is a one-particle term which is just N times the energy radiated into the second cavity by a single electron and has nothing to do with the presence of radiation in the first cavity. Also, unlike the second term, the δ function here makes this term insensitive to the details of the energy distribution of the beam and independent of the the modulating frequency.

All third-order terms vanish if the beam energy spread is less than $\hbar\omega$ so that the next nonvanishing terms in the perturbation expansion are at least fourth order. One might be tempted to conclude, therefore, that information transferral effects would be quite small compared to the uniform background of the second-order term. However, this fails to take account of the fact that any information transfer is a cooperative effect which depends on higher powers of N. Also it turns out that the energy transferred is proportional to the energy density in the source cavity which can be quite large. Thus, among the fourth-order terms there are terms of the order $(e^2/\hbar c)^2 N(N-1)\langle t_0 | a^\dagger a | t_0 \rangle$ which is to be contrasted with $(e^2/\hbar c)N$ for the second-order background. Even though $e^2/\hbar c$ is a small number, the ratio of these two terms can be large.

The nonvanishing fourth-order terms in $\hbar\omega \langle t | b^\dagger b | t \rangle$ are

$$\begin{aligned} E_B^{(4)} &= \frac{\hbar\omega}{\hbar^4}\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t} dt_3 \int_{t_0}^{t_3} dt_4 \langle t_0 | H'(t_2)H'(t_1)b^\dagger b H'(t_3)H'(t_4) | t_0 \rangle \\ &\quad - \frac{\hbar\omega}{\hbar^4}\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_3 \int_{t_0}^{t} dt_4 \langle t_0 | [H'(t_3)H'(t_2)H'(t_1)b^\dagger b H'(t_4) + H'(t_4)b^\dagger b H'(t_1)H'(t_2)H'(t_3)] | t_0 \rangle\,. \end{aligned} \tag{10}$$

We handle these terms by first commuting the electron creation and annihilation operators into normal order, i.e., with all $\eta^\dagger$'s to the left of all η's. This produces four classes of terms. The first two are a four-particle class with four remaining pairs of $\eta^\dagger$'s and η's and a three-particle

class with three remaining pairs of operators. All of these terms vanish if the initial electron beam energy spread is less than $\hbar\omega$. The next class is the two-particle terms which contain the effects we wish to describe and which dominate the last, one-particle class. There appear to be many two-particle terms when Eq. (10) is put in normal-ordered form but after the time integrations most of them contain energy δ functions which cause them to vanish. The remaining terms can be put in the form

$$E_B^{(4)} = \hbar\omega(2\pi)^2 \sum_{\text{all } \vec{q}\text{'s}} \delta(\omega_1-\omega_4)\delta(\omega_2-\omega_3) \times\langle t_0|\eta^\dagger_{\vec{q}_1}\eta^\dagger_{\vec{q}_2}a^\dagger a\eta_{\vec{q}_3}\eta_{\vec{q}_4}|t_0\rangle C_{\vec{q}_1\vec{q}_4}C^*_{\vec{q}_3\vec{q}_4}, \quad (11)$$

where

$$C_{\vec{q}_1\vec{q}_4} \equiv -i\sum_{\vec{q}'}\left(\frac{B_{\vec{q}_1\vec{q}'}A^*_{\vec{q}_4\vec{q}'}}{\omega_1-\omega'+\omega+i\epsilon}+\frac{A^*_{\vec{q}'\vec{q}_1}B_{\vec{q}'\vec{q}_4}}{\omega_1-\omega'-\omega-i\epsilon}\right). \quad (12)$$

This result is completely general, subject only to the conditions that the second cavity is initially unexcited and that the electron beam energy spread is less than $\hbar\omega$. In order to exhibit the properties of this expression we will specialize to a particular initial state,

$$|t_0\rangle = \left[\sum_{\text{all }\vec{q}} f(\vec{q}_1,\vec{q}_2,\ldots,\vec{q}_N)\eta^\dagger_{\vec{q}_1}\eta^\dagger_{\vec{q}_2}\cdots\eta^\dagger_{\vec{q}_N}|0\rangle_{\text{electrons}}\right] \times|A_0,0\rangle_{\text{rad}}, \quad (13)$$

where $f(\vec{q}_1,\vec{q}_2,\ldots,\vec{q}_N)$ is the wave function of the N electrons in momentum space and $|A_0,0\rangle_{\text{rad}}$ represents a still unspecified state A_0 of the first cavity and the vacuum state of the second cavity, and where the normalization of f is given by

$$1 = \sum_{\text{all }\vec{k},\vec{k}'} f^*(\vec{k}'_1,\ldots,\vec{k}'_N)f(\vec{k}_1,\ldots,\vec{k}_N) \times\langle 0|\eta_{\vec{k}'_N}\cdots\eta_{\vec{k}'_1}\eta^\dagger_{\vec{k}_1}\cdots\eta^\dagger_{\vec{k}_N}|0\rangle. \quad (14)$$

We also assume that the wave fronts are essentially planar at the entrance of the first cavity. This means that $f(\vec{k}_1,\ldots,\vec{k}_N)$ contains Kronecker δ's in the transverse components of all momenta.

With this state we can now compute $E_B^{(4)}$:

$$E_B^{(4)} = (2\pi)^2\hbar\omega \sum_{\text{all }\vec{q},\vec{k},\vec{k}'} \delta(\omega_1-\omega_4)\delta(\omega_2-\omega_3)\langle A_0|a^\dagger a|A_0\rangle C_{\vec{q}_1\vec{q}_1}C^*_{\vec{q}_2\vec{q}_2} \times f^*(\vec{k}'_1,\ldots,\vec{k}'_N)f(\vec{k}_1,\ldots,\vec{k}_N)\langle 0|\eta_{\vec{k}'_N},\ldots,\eta_{\vec{k}'_1}\eta^\dagger_{\vec{q}_1}\eta^\dagger_{\vec{q}_2}\eta_{\vec{q}_2}\eta_{\vec{q}_1}\eta^\dagger_{\vec{k}_1},\ldots,\eta^\dagger_{\vec{k}_N}|0\rangle_{\text{electrons}}, \quad (15)$$

where we have used the fact that the combination of the explicit energy δ functions and the implicit Kronecker δ's on the transverse components of the k's [contained in $f(\vec{k}_1,\ldots,\vec{k}_N)$] effectively require $\vec{q}_4=\vec{q}_1$, $\vec{q}_3=\vec{q}_2$. We observe that one can rewrite the second expectation value in $E_B^{(4)}$ as

$$\langle 0|\eta_{\vec{k}'_N}\cdots\eta_{\vec{k}'_1}\eta^\dagger_{\vec{q}_1}\eta^\dagger_{\vec{q}_2}\eta_{\vec{q}_2}\eta_{\vec{q}_1}\eta^\dagger_{\vec{k}_1}\cdots\eta_{\vec{k}_N}|0\rangle = \left(\sum_{\vec{k}_i}\delta_{\vec{q}_1\vec{k}_i}\right)\left[\left(\sum_{\vec{k}_j}\delta_{\vec{q}_2\vec{k}_j}\right)-\delta_{\vec{q}_1\vec{q}_2}\right] \times\langle 0|\eta_{\vec{k}'_N},\ldots,\eta_{\vec{k}'_1}\eta^\dagger_{\vec{k}_1},\ldots,\eta^\dagger_{\vec{k}_N}|0\rangle. \quad (16)$$

Now, if we further assume that the functions $C_{\vec{q}_1\vec{q}_1}C_{\vec{q}_2\vec{q}_2}$ are slowly varying over the energy spread of the beam (to be examined later) then the factor containing these functions can be written as

$$\sum_{\text{all }\vec{q}}\delta(\omega_1-\omega_4)\delta(\omega_2-\omega_3) \times C_{\vec{q}_1\vec{q}_1}C^*_{\vec{q}_2\vec{q}_2}\left(\sum_{\vec{k}_i}\delta_{\vec{q}_1\vec{k}_i}\right)\left[\left(\sum_{\vec{k}_j}\delta_{\vec{q}_2\vec{k}_j}\right)-\delta_{\vec{q}_1\vec{q}_2}\right] = N(N-1)|C_{kk}|^2\left(\frac{L}{2\pi}\right)^2\left(\frac{m}{\hbar k}\right)^2, \quad (17)$$

where k is a typical value of the wave vector of the beam. By making use of this result and the normalization condition Eq. (14) we obtain[6]

$$E_B^{(4)} = \hbar\omega\, N(N-1)|C_{kk}|^2\langle A_0|a^\dagger a|A_0\rangle\frac{m^2L^2}{\hbar^2k^2}. \quad (18)$$

Thus, the two-particle contribution to the energy in the second cavity depends only on the energy $\langle A_0|a^\dagger a|A_0\rangle$ of the first cavity and is independent of any coherence which may be present. Furthermore, since it can be shown that charge-current modulation occurs only if there is coherence in the first cavity,[7] the semiclassical theory's implication that modulation is necessary for preferential energy transfer is therefore incorrect. If, on the other hand, we were to use the same method to calculate $\langle t|b|t\rangle$ instead of $\langle t|b^\dagger b|t\rangle$ we would obtain, to second order,

$$\langle t|b(t)|t\rangle = e^{-i\omega t}\langle t|b|t\rangle = 2\pi e^{-i\omega t}\sum_{\vec{q},\vec{q}'}\langle t_0|a\eta^\dagger_{\vec{q}}\eta_{\vec{q}'}|t_0\rangle\delta(\omega-\omega')C_{\vec{q}\vec{q}'} = Ne^{-i\omega t}\langle A_0|a|A_0\rangle\frac{Lm}{\hbar k}C_{kk} \quad (19)$$

subject to the same assumptions as in the previous case. Hence, if the field in the first cavity is coherent ($|\langle a\rangle|^2=\langle a^\dagger a\rangle$), and if the two-electron

(N^2) contribution to $\langle b^\dagger b\rangle$ is the dominant term, the second cavity has the same coherence as the first one,

$$\langle t|\, b^\dagger b\,|t\rangle = |\langle t|\, b\,|t\rangle|^2[1+O(N^{-1})]\ . \tag{20}$$

In fact, it can be shown that for the leading term in N (the collective part in the radiation) one obtains the general result, to $2(i+j)$th order,

$$\langle t|\,(b^\dagger)^i b^j\,|t\rangle = \left(\frac{NLm}{\hbar k}\right)^{i+j}(C_{kk}^*)^i\,(C_{kk})^j \times\langle 0|\,(a^\dagger)^i a^j\,|0\rangle[1+O(N^{-1})]\ . \tag{21}$$

III. DEPENDENCE ON THE INTERCAVITY DISTANCE

The function C_{kk} which appears in the expectation values exhibits an interesting dependence on the intercavity distance R. This function can be calculated from the definitions (5), (6), and (12). After integrating over the intermediate states, we obtain

$$C_{kk}=\frac{ie^2\pi^2}{mc^2(2\pi L)^3\hbar}\int d^3r\,d^3r'\,[\vec{f}_B^{\,*}(\vec{r})\cdot(-i\vec{\nabla}_R+2\vec{k})\vec{f}_A(\vec{r}')\cdot(-i\vec{\nabla}_R+2\vec{k})e^{-i\vec{k}\cdot(\vec{R}+\vec{r}-\vec{r}')}e^{ik_+|\vec{R}+\vec{r}-\vec{r}'|}$$
$$+\vec{f}_B^{\,*}(\vec{r})\cdot(i\vec{\nabla}_R+2\vec{k})\vec{f}_A(\vec{r}')\cdot(i\vec{\nabla}_R+2\vec{k})e^{i\vec{k}\cdot(\vec{R}+\vec{r}-\vec{r}')}e^{-ik_-|\vec{R}+\vec{r}-\vec{r}'|}]R^{-1}\ , \tag{22}$$

where $k_\pm^2\equiv k^2\pm 2m\omega/\hbar$. In order to simplify the integrations over $\vec{r}$ and $\vec{r}'$ we will assume that R is large compared to the size of either cavity so that $|\vec{R}+\vec{r}-\vec{r}'|$ can be replaced by R except in the phase factor where we use the expansion

$$|\vec{R}+\vec{r}-\vec{r}'|\approx R\left(1+\frac{|\vec{r}-\vec{r}'|}{R}\cos\theta+\frac{|\vec{r}-\vec{r}'|}{2R^2}\sin^2\theta\right), \tag{23}$$

where θ is the angle between $(\vec{r}-\vec{r}')$ and $\vec{R}$. The expansion is terminated after the second-order term because the essential features of the integrals are unchanged by the subsequent terms. We now introduce the Fourier transforms of the mode functions

$$\vec{f}(\vec{q})\equiv(2\pi)^{-3/2}\int e^{-i\vec{q}\cdot\vec{r}}\vec{f}(\vec{r})\,d^3r \tag{24}$$

and perform three of the resulting integrations to arrive at the following expression for C_{kk}:

$$C_{kk}=\frac{\pi e^2}{2mc^2\hbar L^3}\int d^3q\vec{f}_B^{\,*}(\vec{q})\cdot\hat{z}\,\vec{f}_A(\vec{q})\cdot\hat{z}e^{iq_zR}$$
$$\times\left(\frac{(k_++k)^2}{k_+}\,\delta(k_+-k-q_z)e^{-i(R/2k_+)(q_x^2+q_y^2)}\right.$$
$$\left.-\frac{(k_-+k)^2}{k_-}\,\delta(k-k_--q_z)e^{i(R/2k_-)(q_x^2+q_y^2)}\right). \tag{25}$$

Here we have taken $\vec{k}$ and $\vec{R}$ to be in the z direction.

The remaining integration can be performed in two limits. The first is what we will call the Fresnel limit in which the intercavity distance R is small compared with the Fresnel length[8] (the transverse area of a cavity divided by the electron wavelength). In this limit the physical size of the cavity is so large that the functions $f(\vec{q})$ have a width which is so narrow that the phase factors $\exp[\mp iR(q_x^2+q_y^2)/2k_\pm]$ can be regarded as unity. Hence we get

$$C_{kk}=\frac{4i\pi e^2k}{mc^2\hbar L^3}\left[\int dq_x\,dq_y\vec{f}_B^{\,*}(\vec{q})\cdot\hat{z}\,\vec{f}_A(\vec{q})\cdot\hat{z}\right]_{q_z=(k_+-k_-)/2}$$
$$\times e^{i(k_+-k_-)R/2}\sin\tfrac{1}{2}(k_++k_--2k)R\ , \tag{26}$$

where we have neglected the small differences between k_+, k_-, and k in factors which are insensitive to these differences. In the Fraunhofer limit in which R is large compared to the Fresnel length, the variation of the phase factors dominates the variation of the $f(\vec{q})$. This means that the first cavity looks like a point source to the second cavity and vice versa. In this limit we have

$$C_{kk}=\frac{8i\pi^2e^2k^2}{mc^2\hbar L^3R}\,[\vec{f}_B^{\,*}(\vec{q})\cdot\hat{z}\,\vec{f}_A(\vec{q})\cdot\hat{z}]_{\vec{q}=(k_+-k_-)\hat{z}/2}$$
$$\times e^{i(k_+-k_-)R/2}\cos\tfrac{1}{2}(k_++k_--2k)R\ . \tag{27}$$

These results give the R dependence of the energy transfer through $|C|^2$,

$$E_B^{(4)}\sim\begin{cases}\sin^2\tfrac{1}{2}(k_++k_--2k)R\ , & \text{Fresnel limit}\\ R^{-2}\cos^2\tfrac{1}{2}(k_++k_--2k)R\ , & \text{Fraunhofer limit}\ .\end{cases} \tag{28}$$

The wavelength of the periodicity in both cases is given by

$$\Lambda=2\pi|\,k_++k_--2k\,|^{-1}=2\pi\left(\left|\frac{\partial^2k}{\partial E^2}\right|\hbar^2\omega^2\right)^{-1}=\frac{2\pi p^3}{\hbar^2m^2\omega^2}\ , \tag{29}$$

which is correct for relativistic electrons with m taken to be the rest mass. This wavelength varies, for example, with a photon energy $\hbar\omega=2.5$ eV, from 1 cm at an electron energy of 50 keV to 1 m at an electron energy of 2 MeV. This periodicity in R is the most striking feature of the cooperative energy transfer by particle beams. It can be thought of as resulting from a sort of second-order interference between the interference terms between wave numbers k_+ and k and between wave numbers k_- and k. This effect would not be pres-

ent if the electron's mass were zero since in that limit $(k_+ - k) = (k - k_-)$.

In deriving Eq. (17) we made the assumption that C_{kk} is a slowly varying function in k over the range of the energy spread of the beam ΔE. The most rapidly varying factor in C_{kk} is $\exp[\frac{1}{2}i(k_+ - k_-)R]$. If this phase factor varies by an amount comparable with unity over the spread ΔE, the sums in Eq. (17) are essentially zero due to the random phase introduced into the left-hand side. Since the spread of $\frac{1}{2}(k_+ - k_-)$ is just $|\partial^2 k/\partial E^2|(\hbar\omega)(\Delta E)$, this means that the energy spread of the beam must be restricted to be less than $[R|\partial^2 k/\partial E^2|(\hbar\omega)]^{-1}$, or

$$\Delta E < p^3/m^2\omega R \tag{30}$$

if Eq. (17) is to be valid. If one wishes to see the details of how this energy restriction affects the intensity one can assume a Gaussian distribution of electron energies (with a width ΔE) in the sum in Eq. (17) and obtain an exponential cutoff factor[9]

$$\exp[-\tfrac{1}{2}m^4\omega^2 p^{-6}(\Delta E)^2 R^2] . \tag{31}$$

It is interesting to note that, provided the Fresnel length is greater than Λ, the damping factor at the first maximum ($R = \frac{1}{2}\Lambda$) of the periodicity factor is independent of everything except the energy spread and the photon energy,

$$\exp[-\tfrac{1}{2}(\pi\Delta E/\hbar\omega)^2] . \tag{32}$$

Therefore, since this is perhaps the shortest practical distance at which an experiment can be done, this represents the minimum damping factor one can obtain and provides a reason to make $\Delta E/\hbar\omega$ as small as possible.

IV. DISCUSSION

We have demonstrated that the coherence properties of an optical cavity can be transferred to a second cavity by an electron beam provided the beam energy spread is much less than the photon energy. The fermion statistics of the electrons have been taken into account explicitly by the use of a fully second-quantized formalism. The coherence transfer is a general property of quantum-mechanical waves and does not depend on the statistics (i.e., commutation or anticommutation relations) of the particles involved. In addition, our results confirm earlier calculations for the special case of the energy transfer by an electron beam in which exchange effects were not included. It is interesting to note in both the quantum domain (where the wave nature of the particles is dominant) and in the classical domain (where the particle nature dominates—e.g., in the klystron) the coherence information is carried by the many-particle terms. Of course, one should bear in mind the fact that there is also a background of incoherent radiation whenever an electron beam interacts with a cavity. This background is contained in terms we have dropped from our results. Also we have not included higher-order terms in the perturbation expansion. The neglected terms contain contributions of relative orders $(e^2/\hbar c)\langle A_0|a^\dagger a|A_0\rangle$ and $Ne^2/\hbar c$ but terms of the relative order $(Ne^2/\hbar c)\langle A_0|a^\dagger a|A_0\rangle$ do *not* arise. Thus, our approximation allows the last parameter to be large compared to unity while the first two parameters are required to be small. In this situation the coherent radiation dominates the incoherent radiation. However, even if the first two parameters were not small so that our numerical predictions would be unreliable (i.e., the perturbation expansion fails), one would still expect the same sort of general structure of the radiation, but a correct calculation would require the use of the T matrix rather than our perturbation expansion.

Finally, we would like to emphasize that the *energy* (unlike the coherence information) supplied to the second cavity does not come from the first cavity. It is supplied by the electron beam exactly as in the case of the klystron. This suggests the possibility of using an electron beam as a coherent pump for a laser by first letting the beam interact with another laser.[10] If the energy supplied by the beam were greater than the loss in the laser, an optical amplifier (or klystron) would result.

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[1]H. Schwarz and H. Hora, Appl. Phys. Lett. **15**, 349 (1969); H. Schwarz, Trans. N.Y. Acad. Sci. **33**, 150 (1970).

[2]See, for example, L. L. Van Zandt and James W. Meyer, J. Appl. Phys. **41**, 4470 (1970); A. D. Varshalovich and M. I. D'yakonov, Zh. Eksp. Teor. Fiz. Pis'ma Red. **11**, 594 (1970) [JETP Lett. **11**, 411 (1970)]; A. R. Hutson, Appl. Phys. Lett. **17**, 343 (1970); D. Marcuse, J. Appl. Phys. **42**, 2255 (1971); J. Appl. Phys. **42**, 2259 (1971).

[3](a) L. D. Favro, D. M. Fradkin, and P. K. Kuo, Nuovo Cimento Lett. **4**, 1147 (1970); Phys. Rev. D **3**, 2934 (1971); (b) C. Becchi and G. Morpurgo, Phys. Rev. D **4**, 288 (1971); (c) Jun Kondo, J. Appl. Phys. **42**, 4458 (1971).

[4]References 3(a) and 3(b).

[5]We normalized the electron state in a cubic box of dimension L. The spin of the electron is neglected since it is of no consequence here.

[6]The apparent occurrrence of the normalization length L here is, in fact, spurious. The quantity C_{kk} contains a factor L^{-3}, so that the result depends only on the physical number NL^{-2}, which is the total number of particles per unit area that have passed through the system.

[7]This was demonstrated by Kondo, Ref. 3(c).

[8]The significance of this length in this context was brought to our attention by H. J. Lipkin and M. Peshkin (private

communication).

[9]L. D. Favro, D. M. Fradkin, and P. K. Kuo, Phys. Rev. D **3**, 2934 (1971); L. D. Favro, D. M. Fradkin, P. K. Kuo, and W. B. Rolnick, Appl. Phys. Lett. **19**, 378 (1971).

[10]Probably the simplest method of providing the necessary feedback would be to use a *single* laser in the reflex configuration.

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Quenching of the $2\,^1S_0$ Metastable State of Helium by an Electric Field*

Charles E. Johnson†

University of California, Lawrence Berkeley Laboratory, Berkeley, California 94720

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A time-of-flight technique has been used to measure the quenching of the $2\,^1S_0$ metastable state of helium by a static electric field. Neutral ground-state helium atoms effusing from a cooled source slit are immediately excited to the $2\,^1S_0$ and $2\,^3S_1$ metastable states by a pulse of antiparallel magnetically focused electrons. The metastable beam is collimated before passing through a uniform electric field region 0.5 m long and is then preferentially detected at the end of the time-of-flight region, 1.825 m from the electron gun. The time-of-flight distribution for the $2\,^1S_0$ state is separated from that of the $2\,^3S_1$ state by illuminating the beam before the electric field region with an rf-discharge helium lamp. The $2\,^1S_0$ state is quenched by resonant absorption of a 20 581-Å photon, raising the atom to the $2\,^1P_1$ state, which then decays to the $^1S_0$ ground state; the $2\,^3S_1$ state remains unaffected because it is the ground state for the triplet system. The $2\,^1S_0$ time-of-flight distribution is therefore obtained from the difference between the full beam and the quenched beam. The number of $2\,^1S_0$ atoms arriving at the detector in specific velocity intervals with the electric field off is compared to the number with the field on to determine the quenching rate ($=kE^2$). The result for the quenching constant k for both He^3 and He^4 with E in kV/cm is $k=0.933\pm0.005$; this value is in good agreement with theory and with an earlier less accurate experiment. The error in the present experiment arises from a 0.5% uncertainty in the effective length of the electric field region.

I. INTRODUCTION

A time-of-flight technique[1] has been used to measure the quenching of the $2\,^1S_0$ metastable state of helium by a static electric field. The $2\,^1S_0$ state usually decays by spontaneous two-photon emission[2]; however, the presence of a sufficiently strong electric field may, through admixtures of $n\,^1P$ states, quench the $2\,^1S_0$ state and yield a single photon. This quenching process may be viewed as the zero frequency limit of stimulated two-photon emission.[3]

The theoretical value[4,5] of 19.5 msec for the two-photon radiative lifetime of the $2\,^1S_0$ state of helium is in good agreement with the experimental value of 19.7 ± 1.0 msec. Other heliumlike two-photon lifetime measurements[6] in Li^+ and Ar^{16+} are also in good agreement with theory. An additional test of the calculations is provided by a measurement of the quenching rate ($=kE^2$) of the $2\,^1S_0$ state in a static electric field E; as shown in Sec. II, the matrix element used to calculate the quenching constant k is identical to the matrix element required in the calculation of the two-photon decay rate, but with one photon frequency equal to zero.

The motional electric field quenching of the $2\,^2S_{1/2}$ metastable state of Li^{2+}, C^{5+}, and O^{7+} has been measured[7] and then used in conjunction with the Bethe–Lamb theory[8] of quenching to obtain the Lamb shifts of these hydrogenlike heavy ions. Although apparently adequate for hydrogenlike systems, the Bethe–Lamb theory, as previously noted,[9] predicts a quenching constant for the $2\,^1S_0$ state of helium that is 25% greater than a time-independent perturbation calculation,[10,11] and the perturbation calculation agrees with an earlier measurement[11] of the $2\,^1S_0$ quenching constant. Thus an accurate measurement of the quenching constant for the $2\,^1S_0$ state of helium can serve as an independent confirmation of the theory of electric field quenching of a metastable state. Using the Heitler–Ma formalism, Fontana and Lynch[12] have investigated two excited levels coupled by an external perturbation (electric field) and have shown that the phenomenological Bethe–Lamb theory is justified in this case. Using a nonperturbative time-dependent approach, Holt and Sellin[13] have obtained a result for three excited levels which reduces to the Bethe–Lamb theory when applied to hydrogenlike systems. For the quenching of the $2\,^1S_0$ state of helium, however, an accurate comparison of theory and experiment requires the treatment of all excited discrete $n\,^1P$ levels as well as the continuum.

Following a second-order time-dependent perturbation theory of quenching, this paper outlines