[13]H. Kulenkampff, Z. Phys. 30, 513 (1929).
[14]B. Dasannacharya, Phys. Rev. **35**, 129 (1930).
[15]Y. F. Cheng, Phys. Rev. **46**, 243 (1934).
[16]D. S. Piston, Phys. Rev. **49**, 275 (1936).
[17]B. F. Boardman, Phys. Rev. **60**, 163 (1941).
[18]V. W. Slivinsky, Bull. Am. Phys. Soc. **16**, 546 (1971).
[19]W. Duane, Proc. Natl. Acad. Sci. USA **15**, 803 (1929).
[20]H. Kulenkampff, S. Leisegang, and M. Scheer, Z. Phys. **137**, 435 (1954).
[21]J. W. Motz, Phys. Rev. **104**, 557 (1956).
[22]J. W. Motz and R. C. Placious, Phys. Rev. **112**, 1039 (1958).
[23]J. W. Motz and R. C. Placious, Nuovo Cimento **15**, 571 (1960).
[24]H. Kulenkampff and W. Zinn, Z. Phys. **161**, 428 (1961).
[25]F. Huffman, Ph.D. thesis (Johns Hopkins University, 1964) (unpublished).
[26]M. Scheer, E. Trott, and G. Zahs, Z. Phys. **209**, 68 (1968).
[27]R. W. Kuckuck, Lawrence Livermore Laboratory, Report No. UCRL 51188, 1972 (unpublished).
[28]E. Keil, E. Zeitler, and W. Zinn, Z. Naturforsch. A **15**, 1031 (1960) [translated in UCRL-Trans-1036 (1964)].
[29]M. J. Berger and S. M. Seltzer, NAS-NRC Publication No. 1131, 1964 (unpublished); also NASA Report No. NASA SP-3012, 1964 (unpublished).
[30]V. W. Slivinsky and P. J. Ebert, Nucl. Instrum. Methods **71**, 346 (1969).
[31]V. W. Slivinsky and D. E. Smith (private communication).
[32]H. W. Koch and J. W. Motz, Rev. Mod. Phys. **31**, 920 (1959); and Ref. 23.
[33]H. Kulenkampff, M. Scheer, and E. Zeitler, Z. Phys. **157**, 275 (1959).

PHYSICAL REVIEW A VOLUME 7, NUMBER 2 FEBRUARY 1973

Direct Measurement of the Ratio between the Transfer Rates of Muons from μp and μd Atoms to Xenon in a Gaseous Target of Deuterated Hydrogen

A. Bertin, M. Bruno, and A. Vitale

Istituto di Fisica dell'Università di Bologna and Istituto Nazionale di Fisica Nucleare, Sezione di Bologna, Italy

A. Placci and E. Zavattini

CERN, Geneva, Switzerland

(Received 12 September 1972)

The ratio B between the transfer rates $\lambda_{\mu p,\mathrm{Xe}}$ and $\lambda_{\mu d,\mathrm{Xe}}$ of muons from μp and μd muonic atoms to xenon has been directly measured by stopping negative muons in a gaseous target containing deuterated hydrogen and small xenon admixtures at a total pressure of 6 atm abs. and at 293 °K. The results were obtained by analyzing the differential time distribution of the decay electrons coming from muons stopped within the gaseous mixture. In this way one gets $B=1.98\pm0.12$, which supports the dependence of the transfer rates on the mass of the primary muonic atom within 6%. More precise values for $\lambda_{\mu p,\mathrm{Xe}}$ and $\lambda_{\mu d,\mathrm{Xe}}$ are also given, i.e., $\lambda_{\mu p,\mathrm{Xe}}=(4.53\pm0.15)\times10^{11}$ sec^{-1} and $\lambda_{\mu d,\mathrm{Xe}}=(2.30\pm0.17)\times10^{11}$ sec^{-1}. A lower limit for the scattering cross section σ of μd atoms against xenon is obtained, i.e., $\sigma\geq10^{-15}$ cm^2.

I. INTRODUCTION

The atomic and molecular processes which negative muons undergo in hydrogen and deuterium were widely investigated because of their close relation with the muonic catalysis of nuclear reactions[1] and with the nuclear capture of muons by protons[2,3] and deuterons.[4,5]

Among these phenomena, particular interest was devoted to the study of the transfer reactions of muons from μp and μd atoms to other elements $_ZY$ (Z being the atomic number),

$$\mu p + {_ZY} \rightarrow \mu\, {_ZY} + p\,, \qquad (1)$$

$$\mu d + {_ZY} \rightarrow \mu\, {_ZY} + d\,, \qquad (2)$$

which may occur at large rates if the hydrogen or deuterium is contaminated by even a small amount of $_ZY$.

A theoretical analysis of process (1) was carried out by Gershtein.[6] Quite generally, he found that large rates are to be expected for reaction (1) due to the existence of *crossing points* of the molecular terms corresponding to charge exchange in the $(\mu p + {_ZY})$ system. More specifically, he showed that approximate predictions of the algebraic form of the rate $\lambda_{\mu p,Y}$ for reaction (1) can be obtained, provided the kinetic energy T of the μp muonic atom satisfies either one of the following conditions:

$$T \ll 0.12/(M^2Z^2)\,, \qquad (3)$$

$$T \gg 0.12/(M^2Z^2)\,. \qquad (4)$$

The energies are given here in μ-atomic units, and M is roughly equal to the mass of the μp muonic system in units of the muon mass.

Experimental results on the rates of process (1) for several $_ZY$ elements were obtained by different techniques,[7-12] confirming the large rates predicted by Gershtein. However, only a few measurements[7,9,13] were carried out to study process (2). The only systematic investigation on this reaction was performed by Placci *et al.*,[13] who observed the transfer reactions at room temperature in a gaseous target of pure deuterium, contaminated by known admixtures of various $_ZY$ elements.

By comparing the rates $\lambda_{\mu d,Y}$ for process (2) with the corresponding rates $\lambda_{\mu p,Y}$ measured for process (1),[12] these authors noticed that the ratio $B=\lambda_{\mu p,Y}/\lambda_{\mu d,Y}$ is quite close to 2 when reactions (1) and (2) take place at the same kinetic energy T for the μp and μd atoms, if condition (4) is fulfilled. This is actually the case at room temperature for high-atomic-number elements ($T=0.038$ eV at 293 °K).

To explain these features, starting from Gershtein's treatment, Placci *et al.* advanced the suggestion that in this case the rates λ_Y for reactions (1) and (2) can be expressed by the relation[13]

$$\lambda_Y = \text{const} f(Z)/(TM) , \qquad (5)$$

where $f(Z)$ is a function of Z, and T and M are, respectively, the kinetic energy and mass of the μp and μd atoms.

The dependence on the mass of Eq. (5) is verified by the above-mentioned experimental results within 15%. Intending to get a more precise knowledge of the limits within which this approximate relation is valid, we have developed a method which allows a direct determination of the B ratio in the case $_ZY=\text{Xe}$, for which, due to the high value of Z, condition (4) is certainly verified at room temperature.

We report here on an experiment which has been performed at the muon channel of the CERN synchrocyclotron (SC) by applying this method, which, in turn, represents the development of a technique recently used to determine the rate λ_e of the reaction[14]

$$\mu p + d \to \mu d + p + 135 \text{ eV} , \qquad (6)$$

when it takes place at room temperature (see Sec. II).

II. METHOD

A. Short Survey of μ-Atomic and μ-Molecular Processes

When negative muons are slowed down in a hydrogen target contaminated by small amounts of deuterium, it is well known that the μp atoms initially formed may go through the following reactions:

(i) Elastic scattering against hydrogen or deuterium molecules:

$$\mu p + H_2 \to \mu p + H_2 , \qquad (7)$$

$$\mu p + D_2 \to \mu p + D_2 . \qquad (8)$$

As a result of process (7), the μp atoms are quickly slowed down to thermal energies.[15,16]

(ii) Formation of $p\mu p$ molecular ions:

$$\mu p + p \to p\mu p . \qquad (9)$$

(iii) Muon transfer to a deuterium atom, i.e., reaction (6).

The μd atoms released in process (6) can, in turn, undergo the following processes:

(iv) Elastic scattering against hydrogen or deuterium molecules:

$$\mu d + H_2 \to \mu d + H_2 , \qquad (10)$$

$$\mu d + D_2 \to \mu d + D_2 , \qquad (11)$$

through which the μd atoms are slowed down from the initial kinetic energy of 45 eV. It has to be mentioned here, however, that in order to explain the experimental results by Alvarez *et al.*[1] it was necessary to introduce a Ramsauer–Townsend effect in reaction (10); a theoretical analysis by Cohen *et al.*[17] has shown that the cross section for this reaction actually vanishes at a laboratory kinetic energy E_R of the μd atom around 0.45 eV (see Fig. 1). Experimental results consistent with this prediction were given also by Dzhelepov *et al.*[18] and by Bulgarelli *et al.*[19]

(v) Formation of $p\mu d$ and $d\mu d$ molecular ions:

$$\mu d + p \to p\mu d , \qquad (12)$$

$$\mu d + d \to d\mu d , \qquad (13)$$

eventually followed by nuclear fusion reactions.

All of processes (i)–(v) compete with the β decay of the muon. On the other hand, nuclear capture of muons by protons and deuterons occurs at negligible rates compared to the muon decay (see Table I).

If the deuterated hydrogen is contaminated by a further amount of an element $_ZY$, reactions (1) and (2) have to be included among the possible channels. After one of these reactions has taken place, the muon can either decay or be captured by the nucle-

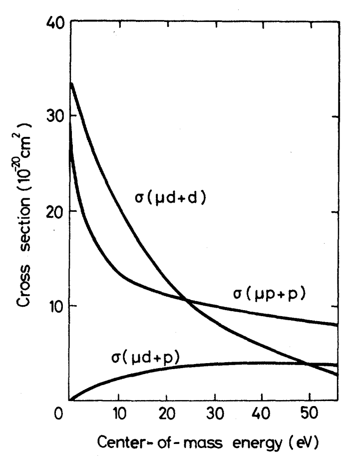

FIG. 1. Theoretical values of the cross sections for the scattering processes $\mu p+p\to\mu p+p$, $\mu d+d\to\mu d+d$, and $\mu d+p\to\mu d+p$ as given in Ref. 17.

TABLE I. Experimental results on the rates of the main processes which negative muons undergo in a target of deuterated hydrogen.

Process	Rate	Experimental result[a] (10^6 sec^{-1})
$\mu p + d \to \mu d + p$	λ_e	$(0.84 \pm 0.13) \times 10^4$; $(1.41 \pm 0.13) \times 10^4$ [b]
$\mu p + p \to p\mu p$	λ_{pp}	(2.55 ± 0.18)[c]; (1.89 ± 0.2)[d]
$\mu d + p \to p\mu d$	λ_{pd}	(6.82 ± 0.25)[c]; (5.8 ± 0.3)[d]
$\mu d + d \to d\mu d$	λ_{dd}	(0.75 ± 0.11)[e]
$\mu p + {}_Z\mathrm{Y} \to \mu_Z\mathrm{Y} + p$	$\lambda_{\mu p, Y}$	10^4–10^5[f]
$\mu d + {}_Z\mathrm{Y} \to \mu_Z\mathrm{Y} + d$	$\lambda_{\mu d, Y}$	10^4–10^5[f]
$\mu^- + p \to n + \nu_\mu$		$(6.51 \pm 0.57) \times 10^{-4}$ [g]
$\mu^- + d \to n + n + \nu_\mu$		$(4.50 \pm 0.70) \times 10^{-4}$ [h]
$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$	λ_0	0.45

[a]The quoted value for λ_{pp} refers to the density of liquid hydrogen. Similarly, the results for λ_e, λ_{pd}, and λ_{dd} refer to the rates at which the corresponding processes take place when the density of deuterium molecules is equal to 2.11×10^{22} molecules cm^{-3}, and the values given for $\lambda_{\mu p, Y}$ and $\lambda_{\mu d, Y}$ refer to a density of the ${}_ZY$ element equal to 2.11×10^{22} molecules cm^{-3}.

[b]See Refs. 14 and 20, respectively.

[c]See Ref. 9.

[d]See Ref. 20. The results reported in the present work are independent of the choice between the different results given in this table for λ_{pp} and λ_{pd}.

[e]See Ref. 21.

[f]See Refs. 7–13.

[g]See Ref. 3.

[h]See Ref. 5.

us of ${}_ZY$ at a rate $\lambda_{c,Y}$ which may be much larger than the muon decay rate for sufficiently high Z (as is the case for ${}_ZY = \mathrm{Xe}$).[22]

As to the scattering processes, the following can now occur in a $(\mathrm{H}_2 + \mathrm{D}_2 + {}_ZY)$ target:

$$\mu p + {}_ZY \to \mu p + {}_ZY \,, \tag{14}$$

$$\mu d + {}_ZY \to \mu d + {}_ZY \,, \tag{15}$$

which must have larger cross section than those of processes (10) and (11), due to the larger atomic dimensions of ${}_ZY$. Very few experimental results are available, however, on this point.[18]

B. Principle of Experiment

The experiment was performed by slowing down negative muons in a gaseous target at a total pressure of 6 atm abs. and a temperature of 293 °K, containing, in most of the cases, a mixture of ultrapure hydrogen (protium) with small amounts of deuterium and xenon, and measuring the differential time distribution dn_e/dt of the decay electrons coming from the muons stopped within the gaseous target.

The various competing processes in this case are summarized in Fig. 2. The time distribution dn_e/dt is obtained by solving the system of equations associated with the scheme of Fig. 2, i.e.,

$$\begin{aligned}
\frac{dN_1}{dt} &= -(\varphi C_{\mathrm{D}}\lambda_e + \lambda_0 + \varphi C_{\mathrm{Xe}}\lambda_{\mu p,\mathrm{Xe}} + \varphi\lambda_{pp})N_1(t)\,,\\
\frac{dN_2}{dt} &= \varphi C_{\mathrm{D}}\lambda_e N_1(t) - (\lambda_0 + \varphi C_{\mathrm{Xe}}\lambda_{\mu d,\mathrm{Xe}} + \varphi\lambda_{pd})N_2(t)\,,\\
\frac{dN_3}{dt} &= \varphi C_{\mathrm{Xe}}\lambda_{\mu p,\mathrm{Xe}}N_1(t) + \varphi C_{\mathrm{Xe}}\lambda_{\mu d,\mathrm{Xe}}N_2(t)\\
&\quad - (\lambda_0 + \lambda_{c,\mathrm{Xe}})N_3(t)\,,\\
\frac{dN_4}{dt} &= \varphi\lambda_{pd}N_2(t) - \lambda_0 N_4(t)\,,\\
\frac{dN_5}{dt} &= \varphi\lambda_{pp}N_1(t) - \lambda_0 N_5(t)\,,
\end{aligned} \tag{16}$$

where $N_i(t)$ gives the population of the corresponding muonic system at time t, and $\lambda_{\mu p,\mathrm{Xe}}$ and $\lambda_{\mu d,\mathrm{Xe}}$ are the rates of processes (1) and (2), respectively (where ${}_ZY = $ xenon, with the xenon density being 2.11×10^{22} molecules cm^{-3}). The other λ's are defined in Table I, and $\lambda_{c,\mathrm{Xe}}$ is the nuclear capture rate of muons by xenon nuclei,[22] φ is the rate between the density of hydrogen at chosen pressure and the density of liquid hydrogen, and $C_{\mathrm{D}}(C_{\mathrm{Xe}})$ is the ratio between the deuterium (xenon) partial pressure and the total pressure of the gaseous mixture.

$$\begin{aligned}
\lambda_1 &= \varphi C_{\mathrm{D}}\lambda_e + \lambda_0 + \varphi C_{\mathrm{Xe}}\lambda_{\mu p,\mathrm{Xe}} + \varphi\lambda_{pp}\,,\\
\lambda_2 &= \lambda_0 + \varphi C_{\mathrm{Xe}}\lambda_{\mu d,\mathrm{Xe}} + \varphi\lambda_{pd}\,,\\
\lambda_3 &= \lambda_0 + \lambda_{c,\mathrm{Xe}}\,,\\
\lambda_4 &= \lambda_5 = \lambda_0\,,
\end{aligned} \tag{17}$$

then dn_e/dt can be expressed as

$$\frac{dn_e}{dt} = A\lambda_0 \sum_i N_i(t) = A\lambda_0 \sum_i C_i\, e^{-\lambda_i t}\,, \tag{18}$$

where A is a constant and the C_i coefficients are determined assuming the initial conditions

$$N_1(0) = 1, \qquad N_2(0) = N_3(0) = N_4(0) = N_5(0) = 0\,.$$

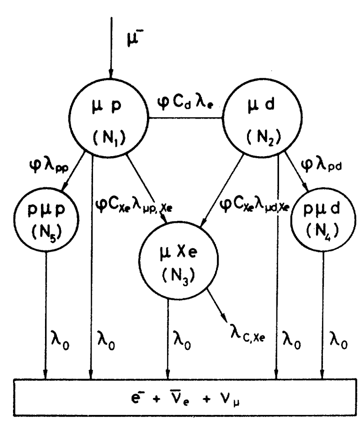

FIG. 2. μ-atomic and μ-molecular processes in a target of deuterated hydrogen contaminated by xenon.

The nuclear capture rates of muons by hydrogen and deuterium nuclei, as well as the formation rate of $d\mu d$ molecular ions, are neglected in the above formulas.

The values of $\lambda_{\mu p,\mathrm{Xe}}$ and $\lambda_{\mu d,\mathrm{Xe}}$ can then be determined by fitting the observed dn_e/dt distributions by Eq. (18), the other rates being assumed as given in Table I. Alternatively, setting in this equation $\lambda_{\mu p,\mathrm{Xe}} = B\lambda_{\mu d,\mathrm{Xe}}$, the best fit to the experimental dn_e/dt directly gives the value of B.

It has now to be emphasized that the values of $\lambda_{\mu p,\mathrm{Xe}}$ and $\lambda_{\mu d,\mathrm{Xe}}$ in the treatment described above are assumed as time independent, as would be the case if both rates did not depend on the kinetic energy T of the μp and μd atoms. However, both theoretical results and experimental evidence suggest an energy-dependent form for the transfer rates, as shown by Eq. (5). Keeping in mind these facts, the following basic remarks now have to be made:

(i) From the measured values of the cross section for reaction (7)[15,16] (see also Fig. 1), it is easily seen that this scattering process is in any case so strong that the μp atoms initially formed are rapidly thermalized in the present experimental conditions. The results which will be obtained for $\lambda_{\mu p,\mathrm{Xe}}$ are then to be referred to an energy $T = 0.038$ eV.

(ii) As for the 45-eV μd atoms from process (6) and for the values which will be obtained for $\lambda_{\mu d,\mathrm{Xe}}$, the situation turns out to be more complicated. If Cohen's treatment[17] of process (10) gives the exact predictions, the cross section for this reaction vanishes, as mentioned, at a laboratory kinetic energy for the μd atoms of $E_R = 0.45$ eV. The only processes, which can contribute to slowing down the μd atoms below this energy are, then, reaction (11) and the scattering of μd's against xenon atoms, i.e.,

$$\mu d + \mathrm{Xe} \rightarrow \mu d + \mathrm{Xe}\ . \tag{19}$$

The behavior of the cross section for process (11) as a function of the kinetic energy T is given in Fig. 1. On the other hand, no data are available on process (19), although one obviously expects a large value for its cross section.

Taking into account this situation, the present measurements have been performed for several deuterium concentrations C_{D} (the hydrogen pressure and the xenon concentration being kept at constant values), starting from a C_{D} value small enough to ensure that, according to the calculation of Cohen *et al.*,[17] the slowing down of the μd atoms through reaction (11) is not too great. On the other hand, if process (11) is still the leading channel to thermalize the μd atoms in the present conditions, i.e., if process (19) is negligible and if the transfer rates are energy dependent [see Eq. (5)], one should observe different values for $\lambda_{\mu d,\mathrm{Xe}}$ for different C_{D} concentrations. On the contrary, under the same previous assumptions, if all the measurements supply the same value for $\lambda_{\mu d,\mathrm{Xe}}$ this will mean the μd atoms are slowed down to 0.038 eV essentially through process (19).

The chosen values for C_{D} range from 2×10^{-2} to 5×10^{-2}. The different measurements will also allow us to evidence out the possible systematic errors due to the assumption of some of the rates listed in Table I.

The xenon concentration was kept at a fixed value around 10^{-4} to get the best sensitivity for the method in the case that $\lambda_{\mu d,\mathrm{Xe}}$ is equal to the value previously obtained at $T = 0.038$ eV.[13]

III. EXPERIMENT

A. Apparatus

The measurements were carried out using an apparatus, the main features of which are shown in Fig. 3.

The gaseous mixture was contained in a stainless-steel tank (T), the wall and entrance window of which were 2.5 and 1.5 mm thick, respectively. Proper degassing of the tank was ensured by keeping it for several days at a static vacuum of 10^{-6} mm Hg.

Deuterium-free hydrogen (protium) was filtered through a palladium purifier before letting it into the tank, to achieve a high level of purity. The injected deuterium was also purified by the palladium filter, whereas the added xenon was pure to $1/10^3$.

The apparatus was described in detail in Ref. 13. We shall only recall here the outlines of its operation, which can be summarized in the two following points:

(i) To supply a signal (MUSTOP signal) when a

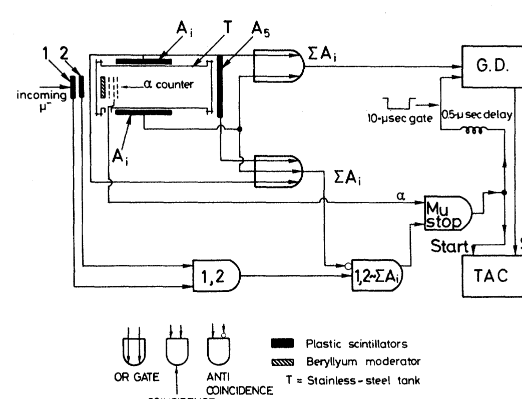

FIG. 3. Simplified scheme of the experimental apparatus and associated electronics (G.D. = gated discriminator).

muon has stopped in the gaseous mixture. The MUSTOP signal was given by a fast anticoincidence circuit (1, 2, $\sim\sum A_i$, α) (here ~ means "not"), where the counters 1 and 2 and all the A_i's were plastic scintillators and α was a special wire-grid proportional counter using the gaseous mixture itself as a working gas.[13] Counter α was working in fast coincidence to define the incoming beam. It was essential to make possible working at the chosen reduced gaseous density and to reduce the accidental counts.

(ii) To measure the time interval Δt between the MUSTOP signal and the detection of the correlated decay electron. The electrons coming from the muon decay were detected by the four plastic scintillators A_1, A_2, A_3, and A_4 within a gate 10 μsec long, which was delayed by about 0.5 μsec with respect to the MUSTOP signal. The Δt intervals, the distribution of which supplied the experimental dn_e/dt, were measured by a time-to-amplitude converter (TAC). The output pulses from the TAC were finally recorded by a 1024 channels pulse-height analyzer.

B. Measurements and Data Analysis

The beam characteristics for the present measurements are listed in Table II, in which the importance of counter α is also pointed out. The corresponding range curve had a width of 3 cm of polyethylene at half-maximum (see Fig. 4). The SC duty cycle was about 35%.

The measurements were performed under the conditions listed in Table III. Run 1 was performed to determine the time distribution of the background events. During this measurement, the muons were stopped in the deuterated-hydrogen target to which a large concentration of xenon had been added (more than 1%). In such condition, the muons are promptly transferred from the μp and μd atoms to xenon atoms, where they undergo nuclear capture. The differential time distribution of the electrons recorded during this run was found to be flat, as expected, at a level of about 3% of the total counting

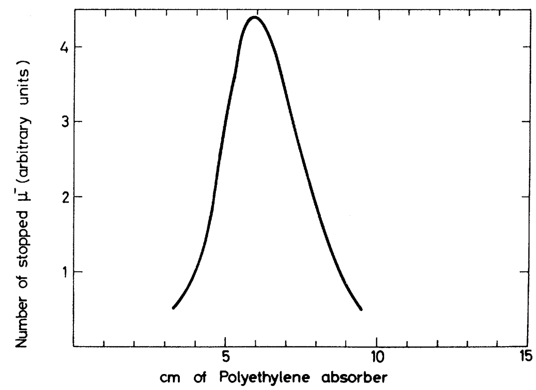

FIG. 4. Typical range curve obtained for the present measurements.

rate in the first time channel considered for runs 2–7.

The results of runs 2–7 were then fitted by an expression made of the sum of Eq. (18) with a constant term representing the background events.

Runs 2 and 7 were carried out by letting into the target ultrapure hydrogen, contaminated only by xenon (C_{Xe} of the order of 10^{-4}), to get two reference measurements for $\lambda_{\mu p,Xe}$. These data were fitted assuming in Eqs. (16)–(18) $C_D = 0$, obtaining the results shown in Table IV.

Runs 3–6 correspond to the main measurements. The experimental dn_e/dt distributions obtained during these runs are given in Fig. 5. The analysis of these runs was carried out through the following steps:

1. Two-Parameter Fits ($\lambda_{\mu p,Xe}$ *and* $\lambda_{\mu d,Xe}$ *or B and* $\lambda_{\mu d,Xe}$)

The dn_e/dt distributions were first fitted leaving $\lambda_{\mu p,Xe}$ and $\lambda_{\mu d,Xe}$ free and looking for the best values of these rates, assuming for the rate λ_e of process (6) the recent result by Bertin *et al.*[14] (see Table I).

The results obtained in this way are listed in Table V. Run 3 supplies a more precise value for $\lambda_{\mu p,Xe}$ due to the low deuterium concentration, which makes this measurement more sensitive to this rate. For the opposite reason, runs 4–6 turn out

TABLE II. Beam and μ^-'s stopping conditions during the present experiment.

Counts of the (1, 2) counter telescope (10×10 cm^2)	15 000 sec^{-1}
Number of muons impinging on the beryllium moderator	6000 sec^{-1}
Number of muons scaled by counter α (after discrimination)	2000 sec^{-1}
Number of muons stopped in the useful thickness (70 cm) of the gas target at the pressure of 6 atm abs.	50 sec^{-1}

TABLE III. Experimental conditions for the measurements reported in the present work.

Run no.	Total pressure of the H_2+D_2+Xe mixture (atm)	Xe concentration (C_{Xe})	D concentration (C_D) ($\times 10^{-2}$)
1	6	1.5×10^{-2}	3
2	6	$(1.65\pm0.08)\times10^{-4}$	0
3	6	$(1.65\pm0.08)\times10^{-4}$	(2.25 ± 0.16)
4	6	$(1.65\pm0.08)\times10^{-4}$	(3.08 ± 0.21)
5	6	$(1.65\pm0.08)\times10^{-4}$	(3.93 ± 0.27)
6	6	$(1.65\pm0.08)\times10^{-4}$	(5.46 ± 0.39)
7	6	$(2.03\pm0.10)\times10^{-4}$	0

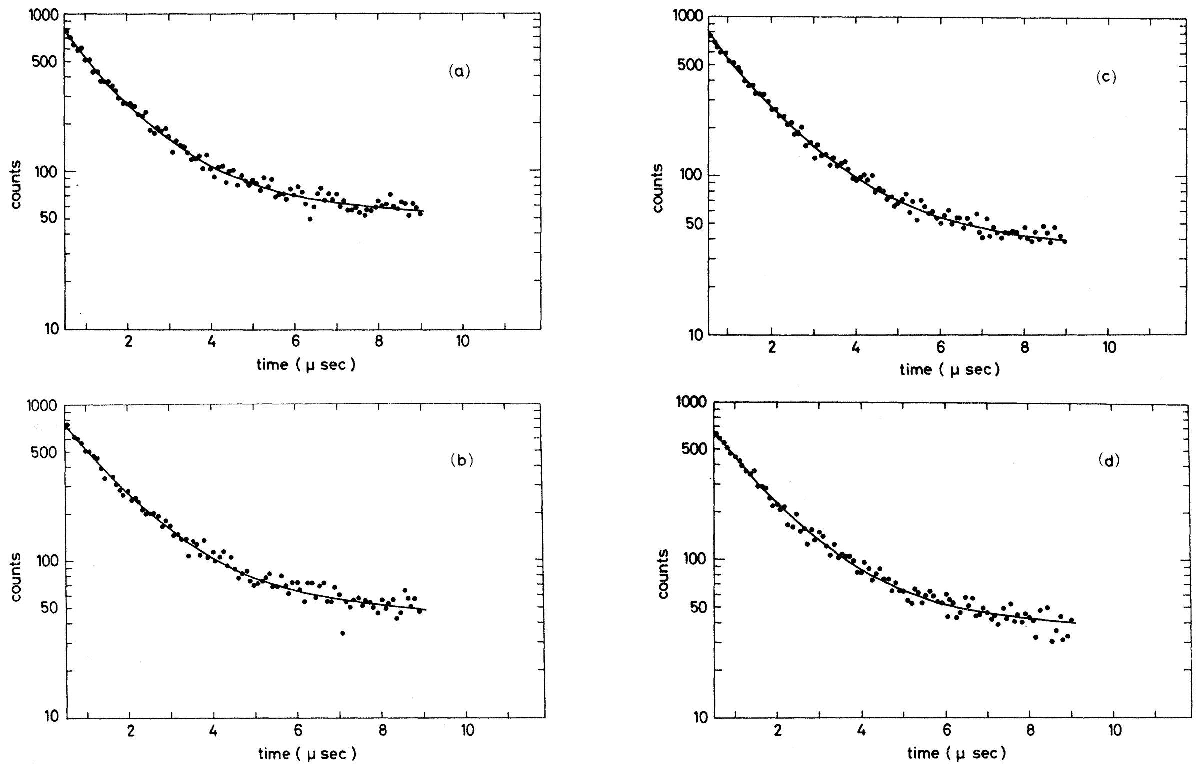

FIG. 5. Differential time distributions dn_e/dt obtained in runs 3–6. (a) Run 3; (b) run 4; (c) run 5; (d) run 6. The continuous lines represent the theoretical fit to the experimental points obtained assuming $\lambda_e = 0.84\times10^{10}$ sec^{-1} and $\lambda_{\mu p,\mathrm{Xe}} = 4.53\times10^{11}$ sec^{-1}. The background counts are not subtracted in the figures.

to be more sensitive to $\lambda_{\mu d,\mathrm{Xe}}$.

The different values for $\lambda_{\mu p,\mathrm{Xe}}$ and $\lambda_{\mu d,\mathrm{Xe}}$ so determined are all equal within the experimental errors and turn out to be in good agreement with the results for the same rates given in Refs. 12 and 13. Within the confidence one gives to Eq. (5), this feature can only be attributed to the fact that the μd atoms released from process (6) are rapidly thermalized through reaction (19), even if the Ramsauer–Townsend effect is present in process (10). It will be seen in the following that a lower limit for the cross section of process (19) can be calculated starting from this result.

To get a direct determination of B, the experimental dn_e/dt distributions were subsequently analyzed setting in Eq. (18) $\lambda_{\mu p,\mathrm{Xe}} = B\lambda_{\mu d,\mathrm{Xe}}$ and still assuming for λ_e the value given in Ref. 14. The results of these fits are summarized in Table VI. The values obtained for B are all compatible with 2, and the values for $\lambda_{\mu d,\mathrm{Xe}}$ are still consistent with the preceding results.

However, the errors given in Tables V and VI do not contain the systematic uncertainty due to the assumption of λ_e, which may be as big as 10%. If this is taken into account, we have to say that the results listed in Table VI support the dependence on mass of Eq. (5) within 15%.

TABLE IV. Summary of the chief experimental results on the rate $\lambda_{\mu p,\mathrm{Xe}}$ for the reaction $\mu p + \mathrm{Xe} \rightarrow \mu\mathrm{Xe} + p$.[a]

Experiment	$\lambda_{\mu p,\mathrm{Xe}}$ (10^{11} sec^{-1})
Present work—run 2	4.50 ± 0.30
Present work—run 7	4.70 ± 0.30
Placci *et al.*[b]	4.41 ± 0.20
Average value	4.53 ± 0.15

[a]The rates reported here and in the following always refer to a density of xenon molecules equal to 2.11×10^{22} molecules cm^{-3}.

[b]See Ref. 12.

TABLE V. Results obtained in the present work by two-parameter fits of the experimental data $\lambda_{\mu p,\mathrm{Xe}}$ and $\lambda_{\mu d,\mathrm{Xe}}$.

Run no.	$\lambda_{\mu p,\mathrm{Xe}}$ (10^{11} sec^{-1})	$\lambda_{\mu d,\mathrm{Xe}}$ (10^{11} sec^{-1})	Number of points	χ^2
3	4.66 ± 0.50	2.38 ± 0.27	93	90
4	4.46 ± 0.70	2.10 ± 0.19	93	100
5	4.60 ± 0.70	2.18 ± 0.19	93	95
6	4.44 ± 0.74	2.22 ± 0.19	93	92

2. *One-Parameter Fits (B only)*

Starting from the conclusion of the preceding point, the data were further analyzed taking for λ_e different values within the error of the result by Bertin *et al.*,[14] and for $\lambda_{\mu p,\mathrm{Xe}}$ several values within the error of the average result given in Table IV. Furthermore, in Eq. (18) we set $\lambda_{\mu d,\mathrm{Xe}} = \lambda_{\mu p,\mathrm{Xe}}/B$.

The results of this treatment are given in Table VII and are summarized in Fig. 6. It can readily be seen from this figure that, although the obtained B values depend very weakly on $\lambda_{\mu p,\mathrm{Xe}}$, they show a more pronounced dependence on λ_e, showing that the uncertainty of this rate represents the biggest source of error for the present results.

Taking into account these systematic uncertainties, one gets from Fig. 6 in correspondence to the average value for $\lambda_{\mu p,\mathrm{Xe}}$ given in Table IV

$$B = 1.98 \pm 0.12\,, \qquad (20)$$

which directly confirms the validity of Eq. (5) within 6% as regards its mass dependence.

IV. CONCLUSIONS

The following conclusions can now be drawn:

(i) If the energy dependence of Eq. (5) is correct, the consistent results obtained in runs 3–6 indicate that—even at the lowest deuterium concentrations chosen for the present measurements—the scattering process [Eq. (19)]

$$\mu d + \mathrm{Xe} \rightarrow \mu d + \mathrm{Xe}$$

is so effective in slowing down the 45-eV μd atoms from reaction (6) that they actually reach the thermal energy of 0.038 eV in a time shorter than 600 nsec. Assuming that the cross section σ for this process is not dependent on the energy of the colliding systems and corresponds to isotropic angular distributions of the scattered μd atoms in the laboratory system,[16] an approximate lower limit for this cross section can be calculated. Then one gets

$$\sigma \geq 10^{-15}\ \mathrm{cm}^2\,. \qquad (21)$$

Such a big value allows us to say that the transfer rates $\lambda_{\mu p,Y}$ and $\lambda_{\mu d,Y}$ found with the present method

TABLE VI. Results obtained in the present work by two-parameter fits of the experimental data: B.[a]

Run no.	B	Number of points	χ^2
3	1.92 ± 0.40	93	90
4	2.30 ± 0.40	93	100
5	2.21 ± 0.40	93	92
6	1.87 ± 0.50	93	102

[a]The results obtained by this type of fit for $\lambda_{\mu d,\mathrm{Xe}}$ are obviously consistent with those presented in Table V.

TABLE VII. Results obtained in the present work for the ratio B (one-parameter fits of the experimental data).

Assumed λ_e (10^{10} sec^{-1})	0.71			0.84			0.97			Number of points for all runs
Assumed $\lambda_{\mu p,\mathrm{Xe}}$ (10^{11} sec^{-1})	4.38	4.53	4.68	4.38	4.53	4.68	4.38	4.53	4.68	93
Run no.										Typical χ^2
3	1.91 ± 0.15	1.98 ± 0.15	2.04 ± 0.17	1.80 ± 0.13	1.90 ± 0.13	1.95 ± 0.14	1.75 ± 0.12	1.84 ± 0.13	1.90 ± 0.13	90
4	2.05 ± 0.15	2.15 ± 0.14	2.22 ± 0.17	1.97 ± 0.14	2.07 ± 0.13	2.12 ± 0.15	1.92 ± 0.13	2.00 ± 0.13	2.06 ± 0.14	100
5	1.99 ± 0.12	2.07 ± 0.13	2.11 ± 0.13	1.91 ± 0.11	2.00 ± 0.12	2.05 ± 0.12	1.89 ± 0.11	1.97 ± 0.11	2.02 ± 0.11	95
6	1.83 ± 0.12	1.92 ± 0.12	1.95 ± 0.13	1.81 ± 0.11	1.87 ± 0.12	1.91 ± 0.12	1.77 ± 0.11	1.85 ± 0.11	1.89 ± 0.11	93
Average value for B	1.95 ± 0.07	2.03 ± 0.07	2.08 ± 0.08	1.87 ± 0.06	1.96 ± 0.06	2.01 ± 0.07	1.83 ± 0.06	1.91 ± 0.06	1.97 ± 0.06	

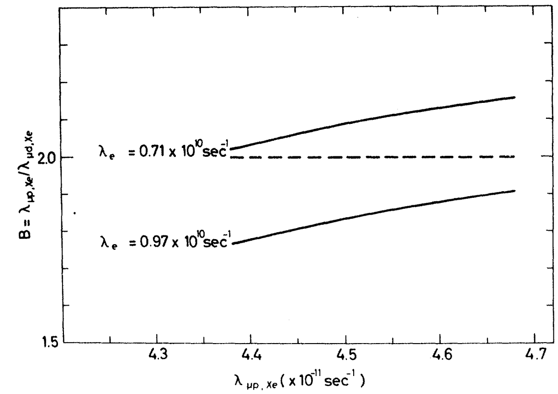

FIG. 6. Summary of the results on the ratio B listed in Table VII (average values over runs 3–6). The full lines represent the results of the present analysis obtained for different values of the rate $\lambda_{\mu p,Xe}$, assuming for the rate λ_e the extreme values compatible with its experimental error. The dotted line indicates the theoretical prediction $B=2$.

refer to a kinetic energy equal to the thermal energy.

(ii) Although Eq. (5) was derived in an approximate way, its validity has now been directly verified by the result (20) within 6%, as far as its dependence on mass is concerned and for a high-Z element like xenon.

(iii) Starting from the results shown in Table VII, the assumed $\lambda_{\mu p,Xe}$ and the obtained B values can be combined to determine a more precise value of $\lambda_{\mu d,Xe}$,

$$\lambda_{\mu d,Xe} = (2.30 \pm 0.17)\times 10^{11}\ sec^{-1}, \tag{22}$$

which is in very good agreement with the previous results by Placci *et al.*[13] The error in Eq. (22) represents the maximum variation of the ratio $\lambda_{\mu p,Xe}/B$ that can be obtained from Table VII.

(iv) Combining the results for $\lambda_{\mu p,Xe}$ obtained in runs 2 and 7 with the corresponding result by Placci *et al.*,[12] one gets (see Table IV)

$$\lambda_{\mu p,Xe} = (4.53 \pm 0.15)\times 10^{11}\ sec^{-1}, \tag{23}$$

which represents the final average value for this rate.

ACKNOWLEDGMENTS

The authors are grateful to R. Schillsott, G. Sicher, and B. Smith for their skillful technical assistance. They are particularly indebted to Professor A. Alberigi Quaranta and Professor P. Bassi for their encouragement and support.

[1] L. W. Alvarez, H. Bradner, F. S. Crawford, J. A. Crawford, P. Falk-Variant, N. L. Good, J. D. Gow, A. H. Rosenfeld, F. T. Solmitz, M. L. Stevenson, H. K. Ticho, and R. D. Tripp, Phys. Rev. **105**, 1127 (1957); M. Cresti *et al.*, University of California Radiation Laboratory, Report No. UCRL3782, 1957 (unpublished).

[2] R. Hildebrand, Phys. Rev. Lett. **8**, 34 (1962); E. J. Bleser, L. M. Lederman, J. L. Rosen, J. E. Rothberg, and E. Zavattini, Phys. Rev. Lett. **8**, 288 (1962); E. Bertolini, A. Citron, G. Gialanella, S. Focardi, A. Mukhin, C. Rubbia, and F. Saporetti, in *Proceedings of the Internaitonal Conference on High-Energy Physics, Geneva,* 1962, edited by J. Prentki (CERN, Geneva, 1962), p. 421; J. E. Rothberg, E. W. Anderson, E. J. Bleser, L. M. Lederman, S. L. Meyer, J. L. Rosen, and I.-T. Wang, Phys. Rev. **132**, 2664 (1963).

[3] A. Alberigi Quaranta, A. Bertin, G. Matone, F. Palmonari, G. Torelli, P. Dalpiaz, A. Placci, and E. Zavattini, Phys. Rev. **177**, 2118 (1969).

[4] I.-T. Wang, E. W. Anderson, E. J. Bleser, L. M. Lederman, S. L. Meyer, J. L. Rosen, and J. E. Rothberg, Phys. Rev. B **139**, 1528 (1965).

[5] A. Placci, E. Zavattini, A. Bertin, and A. Vitale, Phys. Rev. Lett. **25**, 475 (1970).

[6] S. S. Gershtein, Zh. Eksp. Teor. Fiz. **43**, 706 (1962) [Sov. Phys.-JETP **16**, 501 (1963)].

[7] M. Schiff, Nuovo Cimento **22**, 66 (1961).

[8] V. P. Dzhelepov, P. F. Ermolov, E. A. Kushnirenko, V. I. Moskalev, and S. S. Gershtein, Zh. Eksp. Teor. Fiz. **42**, 439 (1962) [Sov. Phys.-JETP **15**, 306 (1962)].

[9] G. Conforto, C. Rubbia, E. Zavattini, and S. Focardi, Nuovo Cimento **33**, 1001 (1964).

[10] S. C. Basiladze, P. F. Ermolov, and K. O. Oganesian, Zh. Eksp. Teor. Fiz. **49**, 1042 (1965) [Sov. Phys.-JETP **22**, 725 (1966)].

[11] A. Alberigi Quaranta, A. Bertin, G. Matone, F. Palmonari, A. Placci, P. Dalpiaz, G. Torelli, and E. Zavattini, Nuovo Cimento B **47**, 96 (1967).

[12] A. Placci, E. Zavattini, A. Bertin, and A. Vitale, Nuovo Cimento A **64**, 1053 (1969).

[13] A. Placci, E. Zavattini, A. Bertin, and A. Vitale, Nuovo Cimento A **52**, 1274 (1967).

[14] A. Bertin, M. Bruno, A. Vitale, A. Placci, and E. Zavattini, Nuovo Cimento Lett. **4**, 449 (1972).

[15] V. P. Dzhelepov, P. F. Ermolov, and V. V. Fil'chenkov, Zh. Eksp. Teor. Fiz. **49**, 393 (1965) [Sov. Phys.-JETP **22**, 275 (1966)].

[16] A. Alberigi Quaranta, A. Bertin, G. Matone, F. Palmonari, A. Placci, P. Dalpiaz, G. Torelli, and E. Zavattini, Nuovo Cimento B **47**, 72 (1967).

[17] S. Cohen, D. L. Judd, and R. J. Riddell, Phys. Rev. **119**, 397 (1960).

[18] V. P. Dzhelepov, P. F. Ermolov, V. I. Moskalev, V. V. Fil'chenkov, and M. Friml, Zh. Eksp. Teor. Fiz. **47**, 1243 (1964) [Sov. Phys.-JETP **20**, 841 (1965)].

[19] L. Bulgarelli, S. Focardi, E. Zavattini, and M. P. Zerbetto, Report No. INFN/AE-66/4 (1966) (unpublished).

[20] E. J. Bleser, E. W. Anderson, L. M. Lederman, S. L. Meyer, J. L. Rosen, J. E. Rothberg, and I.-T. Wang, Phys. Rev. **132**, 2679 (1963).

[21] V. P. Dzhelepov, P. F. Ermolov, V. I. Moskalev, and V. V. Fil'chenkov, Zh. Eksp. Teor. Fiz. **50**, 1235 (1966) [Sov. Phys.-JETP **23**, 820 (1966)].

[22] J. C. Sens, Phys. Rev. **113**, 679 (1959).