Quadrupole Moments of Dy^{163} and Dy^{161}

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The hyperfine structure of the spectrum of Dy I was studied by means of a Fabry–Perot etalon. The dipole- and quadrupole-coupling constants of the levels $4f^{10}(^5I_{7,8})6s\,6p\,(^3P_{2,1})$ with $J=9$ were deduced; these levels were found to be free from configuration mixing. From the quadrupole-coupling constant of the level $[4f^{10}(^5I_7)6s\,6p\,(^3P_2)]_9$ the contribution of the $6p$ electron alone was separated, from which it was deduced that $Q^{163}=+2.46(21)b$ and $Q^{161}=+2.33(20)b$, the Sternneimer correction being taken into account.

I. INTRODUCTION

Using a natural sample of Dy, the hfs of the spectrum of Dy I was studied previously by Kamei and the author,[1] but only the isotope shift of 164–162 could be resolved, whereas the study of the odd isotopes 163 and 161 was fruitless, owing to insufficient resolving power.

Striganov *et al.*[2] determined the relative shift of all the even isotopes by photographic method, and Dekker *et al.*[3] reported accurate measurement of the relative shifts for six lines, using recording interferometer. Ross[4] has measured the isotope shifts in 165 lines in visible region of the Dy I spectrum by photographic method. After the finding of the nuclear spins and approximate nuclear magnetic moments by Park,[5] Ebenhöh *et al.*[6] published the odd isotope hfs of the ground level $4f^{10}\,6s^2\,{}^5I_8$ of Dy I obtained by atomic-beam magnetic resonance experiment. Childs[7] extended the analysis for the two levels $4f^{10}\,6s^2\,{}^5I_{8,7}$ with better precision. In addition to the calculation of μ^{163} and μ^{161}, Childs and Ebenhöh *et al.* obtained Q^{163} and Q^{161} with and without Sternheimer correction, respectively. Denoting the quadrupole moment with and without Sternheimer correction by Q and Q' $[Q=(1+\Delta)Q']$, respectively, Childs assumed $\Delta=0.11\pm0.07$ for the $4f$ electron shell.

It would be highly desirable to check the value of Q by the contribution of a $6p$ electron alone. The present work is concerned with such an investigation.

II. EXPERIMENTAL PROCEDURE

The spectrum of Dy I was excited by a liquid-nitrogen-cooled hollow-cathode discharge, using a natural sample of Dy. The hfs was recorded by means of a pressure-scanned Fabry–Perot interferometer. The resolving power was greatly increased as compared with Ref. 1. In addition to this, photographic plates of Fabry–Perot patterns that were obtained by Ross (Winter Park) using separated isotopes of Dy 163 and Dy 161 and a liquid-nitrogen-cooled hollow-cathode discharge stood at the author's disposal.

III. RESULTS AND DISCUSSION

In the present study only the hfs of the lines having the ground level $4f^{10}\,6s^2\,{}^5I_8$ as the final level and levels $(4f^{10}\,6s6p)_9$ as the initial levels were studied. The reason for the choice of the initial levels is that the $(4f^{10}\,6s6p)_{10}$ level is not detected as yet, and some of the $(4f^{10}\,6s6p)_J$ levels with $J\leqq8$ are perturbed by configuration mixing. Since both Dy 163 and Dy 161 have the same nuclear spin $\frac{5}{2}$ and the J values involved are high, only six diagonal components could be measured in each odd isotope in each line.

Since the hfs of the ground level is known from Refs. 6 and 7, the hfs of the initial levels could be calculated from the observed hfs. The interval factor A and the quadrupole constant B for three levels are tabulated in Table I. The hfs of the 23736 level that represents $[4f^{10}(^5I_8)6s6p(^1P_1)]_9$ was also studied but was difficult to resolve.

Racah[8] showed that, in the case of the configuration $4f^n\,6s6p$ of rare earths heavier than Tb, the ions $4f^n$ and $6s6p$ can be represented by LS-coupled $L_{J_{\rm I}}$ and ${}^{3,1}P_{J_{\rm II}}$, respectively, and $J_{\rm I}$ and $J_{\rm II}$ form the resultant J via jj coupling. In the case of Dy I we make the assumption that the wave function of the $(23736)_9$ level is purely $[4f^{10}(^5I_8)6s6p(^1P_1)]_9$. Since Meggers *et al.*[9] found the intensities of $\lambda(4211)$ $[\nu(23736)]$, $\lambda(6259)$ $[\nu(15972)]$, $\lambda(5639)$ $[\nu(17727)]$, and $\lambda(4577)$ $[\nu(21838)]$ (the numbers in parentheses in units of Å and cm^{-1}, respectively) to be 1300, 34, 13, and 34, respectively, the uncertainty of the above-mentioned assumption can be estimated by the sum of the intensities of $\lambda(6259)$, $\lambda(5639)$, and $\lambda(4577)$ divided by that of $\lambda(4211)$, namely 6.2%.

We now apply the sum rule, equating the sum of the observed values of A^{163} of the three levels 15972, 17727, and 21839 cm^{-1} (see Table I) to the sum of the A's of the wave functions $[4f^{10}(^5I_8)\,6s6p(^3P_2)]_9$, $[4f^{10}(^5I_8)\,6s6p(^3P_1)]_9$, and

TABLE I. hfs of the levels $[4f^{10}(^5I_{J_1})6s6p(^3P_{J_2})]_9$.

			Dy 163		Dy 161	
Level[a]	J_1	J_2	A[b]	B[b]	A[b]	B[b]
15972	8	1	8.74(2)	−8.7(8)	−6.29(2)	−8.3(8)
17727	8	2	6.73(15)	67.7(40)	−4.84(15)	64.1(40)
21839	7	2	9.06(2)	73.5(13)	−6.52(2)	70.2(13)

[a]In units of cm^{-1} and according to Ref. 14.
[b]In units of 10^{-3} cm^{-1}.

$[4f^{10}(^5I_7)\,6s6p(^3P_2)]_9$; then we insert the value of a_f that is known from the work of Childs[7] and assume that $(dn^*/dn)/n^{*3}=0.372$[10] and $\zeta(6p)=1550$.[11] In this way we get $\mu^{163}=0.633(37)$nm. It is necessary to estimate the correction due to the excitation of the s electron in the core. Applying the procedure given in the literature,[12] one gets $\mu^{163}=0.614(37)$ nm. It is difficult to estimate the uncertainty of the core-polarization correction, so it is not included in the estimation of error of μ.

The error given here is not the standard deviation, but the greater part is an estimate of error (6.2%) caused by the neglect of mixing of $[4f^{10}(^5I_8)\,6s6p(^1P_1)]_9$ to $[4f^{10}(^5I)\,6s6p(^3P)]_9$. This neglect has always the effect of subtraction from the true value, so that we may rather express our result as $\mu^{163}=0.651(37)$ nm. In the same way we get $\mu^{161}=-0.465(25)$ nm. Ebenhöh *et al.* and Childs obtained $\mu^{163}=0.66(13)$ nm, $\mu^{161}=-0.47(9)$ nm and $\mu^{163}=0.65(6)$ nm, $\mu^{161}=-0.46(5)$ nm. Agreement with the previous authors is so good that we may assume that the three states $4f^{10}(^5I_{7,8})6s6p(^3P)$, $J=9$ are not perturbed by configuration mixing. In the same way we can prove that the level 21839 is represented by pure $[4f^{10}(^5I_7)6s6p(^3P_2)]_9$ to a good approximation, not being mixed with $[4f^{10}(^5I_8)6s6p(^3P_2)]_9$ or $[4f^{10}(^5I_8)6s6p(^3P_1)]_9$.

We now turn to calculation of the nuclear quadrupole moment from the constant B^{163} of the level 21839. In this case we have

$$Q=-\{B[4f^{10}(^5I_7)6s6p(^3P_2)_9]-B[4f^{10}(^5I_7)]\}\times 0.7455/[(-2/5)R'\langle r^{-3}\rangle_{6p}]\ , \quad (1)$$

where R' is Casimir's relativity correction factor for $\langle r^{-3}\rangle_p$. By putting these values into Eq. (1), (i) the observed value of B^{163} (Table I) into the first B, (ii) the value of B^{163} $(4f^{10}6s^2\,^5I_7)$ observed by Childs[7] into the second B, and also (iii) $R'=1.175$, and putting (i) $Z_p^*=66-6=60$, (ii) $H=1.09$, and (iii) ζ $(6p)=1550$ in the usual formula for $\langle r^{-3}\rangle_p$, we get $Q'(Dy^{163})=+2.73(5)$ b. We assume that the Sternheimer correction for the $6p$ electron[13] is $\Delta=-0.10\pm 0.04$, then we get $Q^{163}=+2.46$ (21) b. In the same way we get $Q^{161}=+2.33$ (20) b. Childs[7] obtained $Q^{163}=+2.51(30)$ b and $Q^{161}=+237(28)$ b from the contribution of the $4f^{10}$ shell. These values are in good agreement with the corresponding values obtained in the present work, so that the Sternheimer correction for the $4f$ electron seems to have been proved.

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