Drift Instabilities in Nonuniform Streaming Plasmas

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The stability of electromagnetic waves propagating transverse to the direction of streaming in a system comprised of identical nonuniform plasmas contrastreaming along a nonuniform magnetic field is investigated. It is found that the low-frequency waves $\omega^2 \ll \Omega_i^2$(ω and Ω_i being the wave frequency and the ion cyclotron frequency, respectively) can become unstable due to the excitation of either resonant ion instability or resonant electron instability. The latter instability can be excited only in the presence of streaming. The drift cyclotron instability gets excited for some bounded values of U_e, U_i, k, β, and μ (U_e being the electron streaming velocity, U_i the ion streaming velocity, k the characteristic wave number, β the ratio of kinetic pressure to magnetic pressure, and μ the inhomogeneity in the magnetic field). The effect of ion temperature on this mode is stabilizing and the typical growth rates associated with this unstable mode are of the order of Ω_i. The presence of even a weak inhomogeneity can render the otherwise marginally stable waves unstable, their growth rates being as high as 0.05 Ω_e (Ω_e being the electron cyclotron frequency). However, the effect of μ and ∇n on the transversal instability (for frequencies $\Omega_i^2 \ll \omega^2 \ll \Omega_e^2$) is to increase the growth rate of the unstable waves slightly.

I. INTRODUCTION

The study of instabilities in nonuniform plasmas is of great interest as such plasmas are encountered in controlled-fusion machines as well as in some astrophysical situations.[1,2] The presence of gradients not only can give rise to various kinds of drift modes but it can affect the modes of the uniform plasmas and make them unstable. The electrostatic drift modes have been discussed by several authors,[3–9] but little work has been done on electromagnetic modes. Krall and Rosenbluth[10] and Krall[11] have shown that for nonuniform plasmas in an inhomogeneous magnetic field, electromagnetic drift-wave instability can exist. Chamberlain[1] has explained the mechanism of auroral bombardment on the basis of this instability. He has also given the physical picture of this instability as arising due to the resonant interaction of drifting ions with the drift waves. Recently Wu[12] has extended the work of Krall and Rosenbluth[10] by including temperature gradients. He finds that in some cases the electron and ion temperature gradients can give rise to a new instability which is due to the resonant interaction between electrons and the drift wave.

Here we have investigated some electromagnetic instabilities which can arise in contrastreaming nonuniform plasmas. In particular we have studied (a) low-frequency drift instability, (b) drift cyclotron instability, and (c) the effect of gradients on transversal instabilities. For case (a) we find that the low-frequency waves, $\omega^2 \ll \Omega_i^2$, can become unstable due to the excitation of either resonant ion instability or resonant electron instability. The latter instability can be excited only in the presence of streaming and its growth rates are M/m times larger than those in the former case. The growth rates, corresponding to resonant ion instability, increase with the increase of β, μ, U_e, and U_i and with the decrease of T_i/T_e and k, where $\beta = 8\pi nKT/B_0^2$, $\mu = \nabla B/B_0$, and $T_{e(i)}$ are the electron (ion) temperatures. For resonant electron instability, however, the growth rates are enhanced due to an increase in T_i/T_e, μ, and U_e and due to a decrease in β and U_i.

For case (b) we find that the drift cyclotron instability can exist only for nonuniform streaming plasmas and for some bounded values of U_e, U_i, k, β, and μ. The ion temperature has a stabilizing effect on this instability.

For case (c) we find that for $\Omega_i^2 \ll \omega^2 \ll \Omega_e^2$ the effect of the gradients is toward increasing the growth rate slightly for the already unstable waves. However, even a very weak gradient can make the marginally stable waves unstable with the growth rate as high as 0.05 Ω_e.

In Sec. II we have derived the general dispersion relation and some special cases are analyzed in Sec. III. The results are summarized in Sec. IV.

II. DISPERSION RELATION

We consider a system comprising of two plasmas, which are nonuniform in the x direction, and are streaming along the direction of the external magnetic field which initially is uniform and is directed along the z axis. The drift currents owing to the plasma nonuniformities render the magnetic field nonuniform, and the equilibrium state is reached only when these currents are counterbalanced by the drift currents owing to the magnetic field gradients. The equilibrium state is completely described by the Vlasov–Maxwell equations, namely,

$$\vec{V}\cdot\frac{\partial f_{0j}^{s}}{\partial\vec{x}}+\frac{e_j}{m_j}\frac{1}{c}(\vec{V}\times\vec{B})\cdot\frac{\partial f_{0j}^{s}}{\partial\vec{V}}=0 \qquad (1)$$

and

$$\nabla\times\vec{B}=(4\pi/c)\textstyle\sum_s\sum_j e_j\int d\vec{V}\,\vec{V}f_{0j}^{s}\;, \qquad (2)$$

where f_{0j}^{s} and $\vec{B}(x)=B_0(1+\mu x)\hat{e}_z$ are the equilibrium distribution function and the magnetic field, respectively. Here j refers to the particle species and is to be summed over i and e for ions and electron, respectively; s specifies the plasma streams. A simple distribution function which satisfies Eqs. (1) and (2) can be written

$$f_{0j}^{s}=(\alpha_j/\pi)^{3/2}\,n_{0j}^{s}\exp[-\alpha_j^{s}V_\perp^2-\alpha_j^{s}(V_z-U_j^{s})^2]$$
$$\times[1-\epsilon_j(x+V_y/\Omega_j)]\;, \qquad (3)$$

where

$$\alpha_j^{s}=m_j/2kT_j^{s}=\tfrac{1}{2}V_{t_j}^{s\,-2}\quad\text{and}\quad\Omega_j=e_jB_0/m_jc$$

is the cyclotron frequency of the jth species. $\epsilon_j=-\nabla n_j/n_{0j}$ denotes the inhomogeneity in the density. However, the charge-neutrality condition requires that $\epsilon_i=\epsilon_e=\epsilon$.

The y component of Eq. (2) demands that $\mu/\epsilon=\frac{1}{2}\beta$. The x and z components are identically satisfied when the plasmas are identical and contrastreaming and thus yield no further information regarding the equilibrium state.

Equation (3) describes the equilibrium nonuniform streaming plasmas; we wish to examine the stability of such a system in the presence of a small perturbation for which the distribution function can be written as $f_j^{s}=f_{0j}^{s}+f_{1j}^{s}$. The perturbed distribution function f_{1j}^{s} can be obtained by integrating the linearized Vlasov equation over the unperturbed orbits,[10,11,13] and on making use of the local approximation we arrive at the following dispersion relation

$$|\overleftrightarrow{R}|=0\;, \qquad (4)$$

where

$$\overleftrightarrow{R}=(c^2k^2-\omega^2)\overleftrightarrow{I}-\vec{k}\vec{k}-\omega\sum_s\sum_{l,m}\frac{\omega_{p_j}^{s\,2}}{n_{0j}^{s}}\int d\vec{V}\,\vec{V}\vec{A}f_{0j}^{s}J_l\left(\frac{kV_\perp}{\Omega_j}\right)J_m\left(\frac{kV_\perp}{\Omega_j}\right)\exp[-i(l-m)(\tfrac{1}{2}\pi-\theta)]\;, \qquad (5)$$

where

$$\vec{A}=-V_\perp\alpha_j^{s}\left(1+\frac{U_j^{s}k_z}{\omega}-\frac{\epsilon k_y}{2\alpha_j^{s}\Omega_j\omega}\right)A_+\hat{e}_x+\left\{-iV_\perp\alpha_j^{s}\left(1+\frac{U_j^{s}k_z}{\omega}\right)A_--\left[\frac{\epsilon}{\Omega_j}\left(1+\frac{k_zV_z}{\omega}\right)+2\alpha_j^{s}V_d\left(1+\frac{U_j^{s}k_z}{\omega}\right)\right]A_0\right\}\hat{e}_y$$
$$+\left\{iV_\perp\alpha_j^{s}\frac{U_j^{s}k_y}{\omega}A_-+\left[-2(V_z-U_j^{s})\alpha_j^{s}+\left(\frac{\epsilon V_z}{\Omega_j}+2\alpha_j^{s}U_j^{s}V_d\right)\frac{k_y}{\omega}\right]A_0\right\}\hat{e}_z\;, \qquad (6)$$

with

$$A_\pm=e^{-i\theta}[\omega+k_zV_z+k_yV_d+(l+1)\Omega_j]^{-1}\pm e^{i\theta}[\omega+k_zV_z+k_yV_d+(l-1)\Omega_j]^{-1}\;, \qquad (7)$$

and

$$A_0=(\omega+k_zV_z+k_yV_d+l\Omega_j)^{-1},\qquad V_d=V_\perp^2\mu/2\Omega_j\,. \qquad (8)$$

For identical contrastreaming plasmas ($U_1=-U_2=U$) the ordinary mode, i.e., the mode with $k_z=0$ and $\vec{E}_1$ (perturbed electric field) parallel to $\hat{e}_z$ gets decoupled from the rest of the modes, and the dispersion relation for this mode according to Eq. (4) comes out to be

$$c^2k^2-\omega^2=\omega\sum_j\sum_{l=-\infty}^{\infty}2\alpha_j\omega_{p_j}^2\int_0^\infty dV_\perp V_\perp$$
$$\times\left[\left(\frac{2\alpha_jU_j^2\Omega_jl}{\omega}-1\right)\left(1-\frac{\epsilon l}{k}\right)+\frac{\epsilon k(1+2\alpha_jU_j^2)}{2\omega\Omega_j\alpha_j}+\frac{\alpha_jU_j^2k\mu V_\perp^2}{\omega\Omega_j}\right]\frac{J_l^2(kV_\perp/\Omega_j)e^{-\alpha_jV_\perp^2}}{\omega+l\Omega_j+k\mu V_\perp^2/2\Omega_j}\;. \qquad (9)$$

For $\omega^2\ll\Omega_i^2$ and $U_j=0$, Eq. (9) becomes similar to that of Krall and Rosenbluth[10] and Wu[12] and for $\epsilon=\mu=0$ it reduces to that of Buti and Lakhina.[14,15]

III. ANALYSIS OF DISPERSION RELATION

In this section we shall discuss the solution of Eq. (9) for some special cases.
Eq. (9) for some special cases.

A. Low-Frequency Drift Instability ($\omega^2 \ll \Omega_i^2$)

Let us consider the case for which $c^2k^2 \gg \omega^2$. On neglecting $\omega + k\mu V_\perp^2/2\Omega_j$ compared to $l\Omega_i (l \neq 0)$, the dispersion relation (9) reduces to

$$D(k,\omega) \equiv c^2k^2 - \sum_j 2\alpha_j U_j^2 \omega_{p_j}^2 [1 - I_0(\lambda_j)e^{-\lambda_j}] - \sum_j \omega_{p_j}^2 \int_0^\infty d\eta \left(-\omega + \frac{\epsilon k}{2\alpha_j\Omega_j}(1+2\alpha_j U_j^2) + \frac{k\mu U_j^2 \eta}{\Omega_j}\right) \frac{J_0^2((k/\Omega_j)(\eta/\alpha_j)^{1/2})e^{-\eta}}{(\omega + k\mu\eta/2\Omega_j\alpha_j)} = 0\,, \quad (10)$$

where $\lambda_j = k^2 V_{t_j}^2/\Omega_j^2$. Let us write $\omega = \omega_r - i\nu$, where $\nu > 0$ corresponds to instability. Let us further assume that $|\nu| \ll |\omega_r|$; then we can express the real and the imaginary parts of the function D as follows:

$$\mathrm{Re}D(k,\omega_r) = c^2k^2 - \sum_j 2\alpha_j U_j^2 \omega_{p_j}^2 [1 - I_0(\lambda_j)e^{-\lambda_j}] - \sum_j \omega_{p_j}^2 P\int_0^\infty d\eta \left(-\omega_r + \frac{\epsilon k}{2\alpha_j\Omega_j}(1+2\alpha_j U_j^2) + \frac{k\mu U_j^2\eta}{\Omega_j}\right) \frac{J_0^2((k/\Omega_j)(\eta/\alpha_j)^{1/2})e^{-\eta}}{\omega_r + k\mu\eta/2\Omega_j\alpha_j} \quad (11)$$

and

$$\mathrm{Im}D(k,\omega_r) = -\frac{2\alpha_j\Omega_j\pi}{k\mu}\sum_j \omega_{p_j}^2 \int_0^\infty d\eta\, e^{-\eta}\left(-\omega_r + \frac{\epsilon k}{2\alpha_j\Omega_j}(1+2\alpha_j U_j^2) + \frac{k\mu U_j^2\eta}{\Omega_j}\right) J_0^2\left(\frac{k}{\Omega_j}\left(\frac{\eta}{\Omega_j}\right)^{1/2}\right)\delta\left(\eta + \frac{2\alpha_j\Omega_j\omega_r}{k\mu}\right). \quad (12)$$

Evidently the $\mathrm{Im}D(k,\omega_r)$ would be nonzero only if

$$2\alpha_j\Omega_j\omega_r/k\mu < 0\,. \quad (13)$$

The real frequency ω_r can be determined from the relation $\mathrm{Re}D(k,\omega_r) = 0$, whereas the growth rate ν can be obtained from the relation

$$\nu = \frac{\mathrm{Im}D(k,\omega_r)}{(\partial/\partial\omega_r)[\mathrm{Re}D(k,\omega_r)]}\,. \quad (14)$$

Assuming $|\omega_r| \gg |k\mu\eta/2\alpha_j\Omega_j|$, the principal integral in Eq. (11) can be easily evaluated to yield

$$\omega_r = -\frac{[A_e - (m/M)A_i]}{\chi}\,, \quad (15)$$

where

$$A_e = \frac{\mu k}{2|\Omega_e|\alpha_e}\left(\frac{2I_0(\lambda_e)e^{-\lambda_e}}{\beta} + \frac{d}{d\lambda_e}[\lambda_e I_0(\lambda_e)e^{-\lambda_e}]\right) \times (1+2\alpha_e U_e^2)\,, \quad (16)$$

$$\chi = \frac{2\lambda_e(1+T_i/T_e)}{\beta} + I_0(\lambda_e)e^{-\lambda_e} - 2\alpha_e U_e^2[1 - I_0(\lambda_e)e^{-\lambda_e}] - 2\frac{T_e}{T_i}\alpha_e U_i^2[1 - I_0(\lambda_i)e^{-\lambda_i}] + \frac{m}{M}I_0(\lambda_i)e^{-\lambda_i}, \quad (17)$$

and A_i is obtained from A_e by replacing the subscript e by i.

Now $A_e > (m/M)A_i$ unless $\lambda_i < \lambda_e$ and $U_i^2 > U_e^2$, V_{te}^2, in which case for $\chi > 0$ only the ions can satisfy the inequality (13) and thus only they can contribute to $\mathrm{Im}D(k,\omega_r)$ and consequently to ν. In this sense the ions in this case are having resonant interaction with the wave.[9,12] On the other hand, for $\chi < 0$, it is obvious from inequality (13) that the instability will be excited by the resonant electrons.

The growth rates for resonant ion and for resonant electron instability we shall determine separately.

1. Resonant Ion Instability

As mentioned above, for $\chi > 0$ the low-frequency mode is driven unstable by the resonance interaction of the ion with the wave. The growth rates, for this mode, on using Eq. (14), are found to be

$$\nu = \frac{2\pi m}{M}\frac{\alpha_i\Omega_i}{k\mu}\frac{\omega_r^2 e^{-\eta_i}}{A_e - (m/M)A_i}\left(-\omega_r + \frac{k\mu}{\alpha_i\Omega_i\beta}\times(1+2\alpha_i U_i^2) + \frac{k\mu U_i^2}{\Omega_i}\eta_i\right) J_0^2(x_i)\,, \quad (18)$$

where

$$\eta_i = k|\omega_r|/(\mu\Omega_i\lambda_i)$$

and

$$x_i = [2k\,|\omega_r|/(\mu\Omega_i)]^{1/2}\,.$$

For $x_i \gg 1$ we can replace $J_0^2(x_i)$ by $1/(\pi x_i)$ in Eq. (18) and on doing so, we get

$$\nu = \frac{m}{M}\,\frac{x_i}{2}\,\frac{\Omega_i}{\chi\lambda_i}\,e^{-\eta_i}\left(\frac{|\omega_r|}{\Omega_i} + \frac{2\mu\lambda_i}{k\beta}\right) \times \left(1 + \frac{T_e}{T_i}\,\frac{M}{m}\,\frac{U_i^2}{V_{te}^2}\right)\,. \quad (19)$$

We have evaluated Eq. (19) numerically and some results for the growth rate ν are shown in Figs. 1–3.

From Fig. 1 we observe that the growth rate increases with the increase of β as well as that of ion streaming. For $\beta > 9$, the condition $\omega^2 \ll \Omega_i^2$ breaks down for all the curves shown in the figure and hence the growth rates corresponding to $\beta > 9$ are not considered.

From Fig. 2 we observe that ν increases with the increase of μ and decrease of T_i/T_e. When $(\mu V_{te}/\Omega_e)$ exceeds a certain value say, $(\mu V_{te}/\Omega_e)_{\max}$, the growth rates are not consistent with the assumption $\omega^2 \ll \Omega_i^2$ and hence are dropped out.

From Fig. 3 we observe that the range of unstable electron streaming velocities U_e increases with the increase of λ_e. The growth rate, however, increases with the decrease of λ_e and with the increase of U_e.

2. Resonant Electron Instability

This instability can occur when $\chi < 0$, which is satisfied if $\beta > \beta_c$ where β_c, the critical value of β, is given by

$$\beta_c = 2\lambda_e\left(1 + \frac{T_i}{T_e}\right)\bigg/\left(\frac{U_e^2}{V_{te}^2}\,[1 - I_0(\lambda_e)e^{-\lambda_e}] - I_0(\lambda_e)e^{-\lambda_e} + \frac{T_e}{T_i}\,\frac{U_i^2}{V_{te}^2}\,[1 - I_0(\lambda_i)e^{-\lambda_i}] - \frac{m}{M}\,I_0(\lambda_i)e^{-\lambda_i}\right)\,. \quad (20)$$

The quantity β_c is meaningful only when the denominator in the expression (20) is positive, i.e., if

$$U_e^2 > V_{te}^2\left(I_0(\lambda_e)e^{-\lambda_e} + \frac{m}{M}\,I_0(\lambda_i)e^{-\lambda_i} - \frac{U_i^2}{V_{te}^2}\,\frac{T_e}{T_i}\,[1 - I_0(\lambda_i)e^{-\lambda_i}]\right)\bigg/[1 - I_0(\lambda_e)e^{-\lambda_e}]\,. \quad (21)$$

Equation (21) puts restrictions on the minimum value of U_e^2 or U_i^2 or both. The growth rates in this case according to Eq. (14) are given by

$$\nu = \frac{2\pi\alpha_e|\Omega_e|}{k\mu}\,\frac{\omega_r^2 e^{-\eta_e}}{A_e - (m/M)A_i}\left(\omega_r + \frac{\mu k}{\beta\alpha_e|\Omega_e|}\right) \times \left(1 + \frac{U_e^2}{V_{te}^2}\right)J_0^2(x_e)\,, \quad (22)$$

where

$$\eta_e = k\omega_r/\mu\Omega_e\lambda_e$$

and

$$x_e = (2k\omega_r/\mu\Omega_e)^{1/2}\,.$$

Now because of the presence of Ω_e in the denominator in the expression for x_e, we may take $x_e \ll 1$, i.e., $J_0^2(x_e) \to 1$; then Eq. (22) is simplified to yield

$$\nu = \pi\,\frac{x_e^2}{2}\left|\frac{\Omega_e}{\chi}\right|\frac{1}{\lambda_e}\,e^{-\eta_e}\left(\frac{\omega_r}{|\Omega_e|} + \frac{2\mu\lambda_e}{k\beta}\right)\left(1 + \frac{U_e^2}{V_{te}^2}\right)\,. \quad (23)$$

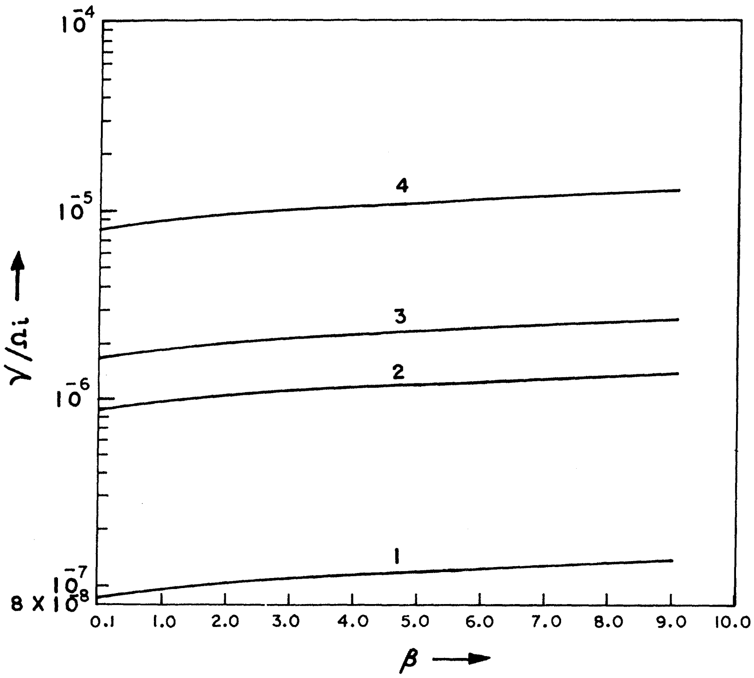

FIG. 1. Variation of growth rate ν/Ω_i vs β for $\lambda_e = 2$, $\mu V_{te}/\Omega_e = 10^{-3}$, $T_i/T_e = 10$, $U_e = 0$ and for $U_i^2/V_{te}^2 = 0.0$, 0.05, 0.1, and 0.5 for the curves 1, 2, 3, and 4, respectively.

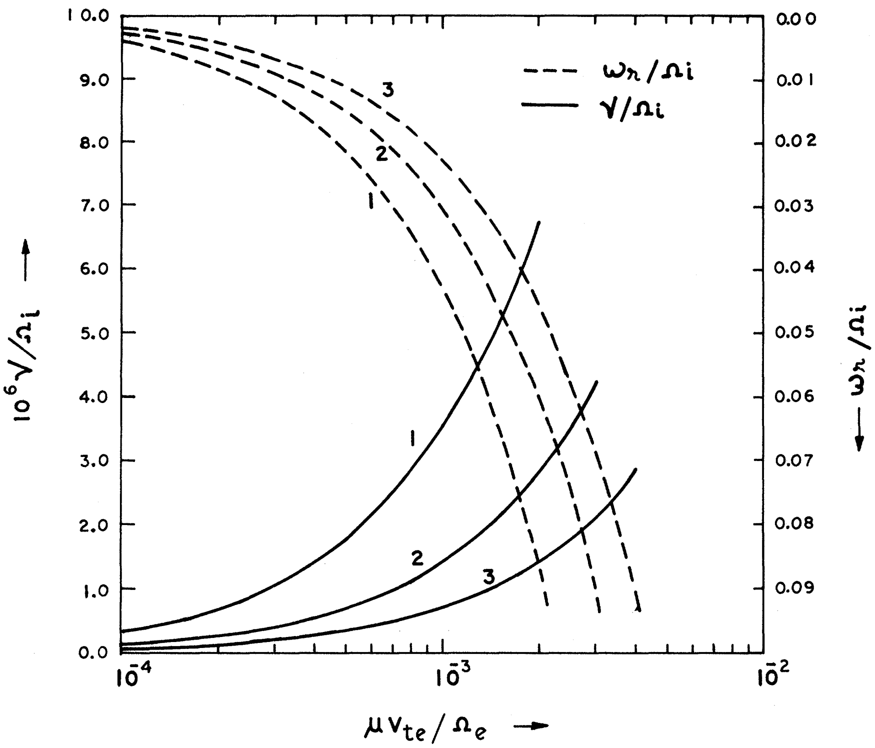

FIG. 2. Variation of ν/Ω_i vs $\mu V_{te}/\Omega_e$ for $\beta=1.0$, $\lambda_e=2$, $U_e=0.0$, $U_i^2/V_{te}^2=0.2$ and for $T_i/T_e=10$, 15, and 20 for the curves 1, 2, and 3, respectively.

For $\lambda_e \ll 1$, the condition $\chi < 0$ cannot be satisfied by the electron streaming alone and, moreover, in this case $\eta_e \gtrsim 1$, and so the growth rates will be quite small. For $\lambda_e \gtrsim 1$, electron streaming can drive this mode unstable by the resonant electron instability with or without the ion streaming. For

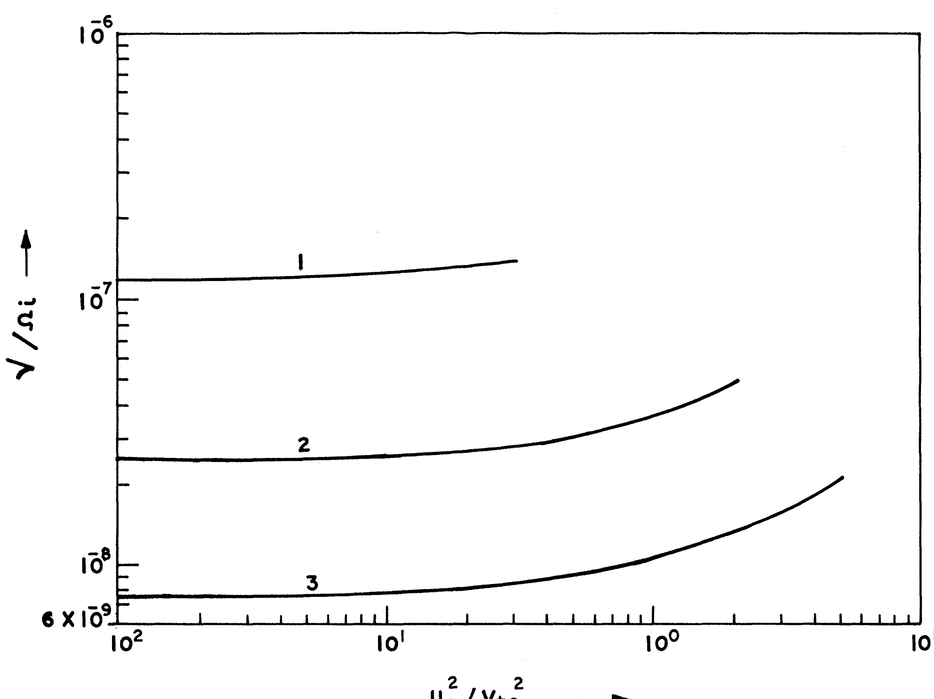

FIG. 3. Variation of ν/Ω_i vs U_e^2/V_{te}^2 for $\beta=5$, $\mu V_{te}/\Omega_e=10^{-3}$, $T_i/T_e=10$, $U_i=0$ and for $\lambda_e=2, 5$, and 10 for the curves 1, 2, and 3, respectively.

this case, $\eta_e \ll 1$, and hence we can replace $e^{-\eta_e}$ by unity; the growth rate is then given by

$$\nu = \frac{\pi x_e^2}{2\lambda_e}\left|\frac{\Omega_e}{\chi}\right|\left(\frac{\omega_r}{|\Omega_e|}+\frac{2\mu\lambda_e}{k\beta}\right)\left(1+\frac{U_e^2}{V_{te}^2}\right). \quad (24)$$

From the above expression we notice that ν will increase with the increase of U_e^2 and μ. As in both ω_r and x_e, β occurs in the denominator [cf. Eqs. (15)–(17)]; the growth rate will increase with the decrease of β. We also observe that the growth rates are M/m times larger than those for the case of resonant ion instability.

From Eq. (24) we further notice that once the instability is initiated, ν will decrease with the increase of U_i^2 and T_e/T_i. This is because of the fact that the ω_r decreases and $|\chi|$ (with $\chi<0$) increases with U_i^2 and with T_e/T_i.

It is worth pointing out that the conditions $\chi>0$ or $\chi<0$ are the necessary but not sufficient conditions for the resonant ions or electrons instability, respectively. For $|\chi|\approx 0$ neither the ions nor the electrons can have resonant interaction with the wave because in this case $\omega^2 \gtrsim \Omega_i^2$ [cf. Eqs. (15)–(17)] which is incompatible with our assumption $\omega^2 \ll \Omega_i^2$.

B. Drift Cyclotron Instability

In this section we consider the case

$$|\omega + l\Omega_i + kV_d| \ll (l+1)\Omega_i \ ,$$

where

$$V_d = k\mu V_\perp^2/2\Omega_i$$

is the drift velocity of the ions due to ∇B. Let us consider the case where l is such that

$$|k\mu V_\perp^2/2\Omega_e| \ll \omega \sim l\Omega_i \ll \Omega_e \ .$$

Then the contribution due to the electron terms in Eq. (9) can be written

$$\psi_e = 2\alpha_e U_e^2\,\omega_{p_e}^2[1-I_0(\lambda_e)e^{-\lambda_e}] - \frac{\omega_{p_e}^2}{\omega}\left[\left(\omega+\frac{\epsilon k}{2|\Omega_e|\alpha_e}(1+2\alpha_e U_e^2)\right)I_0(\lambda_e)e^{-\lambda_e} + \frac{\mu k}{2\alpha_e|\Omega_e|}\frac{d}{d\lambda_e}[\lambda_e I_0(\lambda_e)e^{-\lambda_e}](1+2\alpha_e U_e^2)\right] . \quad (25)$$

For the ion contribution we need retain only the lth harmonic term in the ion series and on further assuming $kV_\perp/\Omega_i \gg 1$, i.e.,

$$J_l^2(kV_\perp/\Omega_i) \to (1/\pi)\ \Omega_i/kV_\perp \ ,$$

the ion contribution ψ_i, can be written in the form

$$\psi_i = \frac{2\omega\omega_{p_i}^2\,\Omega_i^2\,\alpha_i^{3/2}}{\pi^{1/2}k^2\mu}\left\{\left[\left(\frac{2\alpha_i U_i^2 l\Omega_i}{\omega}-1\right)\times\left(1-\frac{\epsilon l}{k}\right)+\frac{\epsilon k}{2\omega\Omega_i\alpha_i}(1+2\alpha_i U_i^2)\right]\frac{Z(\xi)}{\xi} + \frac{k\mu}{\omega\Omega_i}U_i^2[1+\xi Z(\xi)]\right\} , \quad (26)$$

where

$$\xi^2 = -\frac{(\omega+l\Omega_i)2\Omega_i\alpha_i}{k\mu} = -\frac{(\omega+l\Omega_i)}{kV_{t_i}\mu R_i} \ ,$$

with $R_i = V_{t_i}/\Omega_i$ as the ion Larmor radius and

$$Z(\xi) = \pi^{-1/2}\int_{-\infty}^{\infty} dt\,\frac{e^{-t^2}}{t-\xi} \quad \text{with } \operatorname{Im}\xi>0 \ ,$$

is the plasma dispersion function.[16] If μ and k are such that $(\mu R_i)(kR_i) \ll 1$ then $|\xi^2| \gg 1$ and we can use the asymptotic expansion for $Z(\xi)$ in Eq. (26). On combining the resulting equation with Eq. (25) and taking $c^2k^2 \gg \omega^2$ the dispersion relation can be written in the form

$$p\omega^2 + q\omega + r = 0 \ , \quad (27)$$

where

$$p = \frac{2\lambda_e}{\beta}\left(1+\frac{T_i}{T_e}\right) + e^{-\lambda_e}I_0(\lambda_e) - \frac{U_e^2}{V_{te}^2}[1-I_0(\lambda_e)e^{-\lambda_e}] , \quad (28)$$

$$q = (Wp + Q - F) \ , \quad (29)$$

$$r = QW \ , \quad (30)$$

$$Q = \frac{k\mu}{2\alpha_e|\Omega_e|}(1+2\alpha_e U_e^2)\left(2\frac{I_0(\lambda_e)e^{-\lambda_e}}{\beta} + \frac{d}{d\lambda_e}[\lambda_e I_0(\lambda_e)e^{-\lambda_e}]\right) , \quad (31)$$

with

$$F = \frac{m}{M}\frac{(kR_i)^{-1}}{(2\pi)^{1/2}}\left[2\alpha_i U_i^2 l\Omega_i\left(1-\frac{2\mu l}{k\beta}\right) + \frac{\mu k}{2\alpha_i\Omega_i}\left(\frac{2(1+2\alpha_i U_i^2)}{\beta}+\alpha_i U_i^2\right)\right] , \quad (32)$$

$$W = l\Omega_i + k\mu/4\alpha_i\Omega_i \ . \quad (33)$$

Equation (27) for ω yields

$$\omega = -q/2p \pm (1/2p)(q^2-4rp)^{1/2} \ . \quad (34)$$

From Eq. (34) it is obvious that the instability can occur only if

$$q^2 - 4rp < 0 \ . \quad (35)$$

For $p \le 0$, the inequality (35) is violated and hence the necessary and sufficient conditions for the stability of this mode is simply $p\le 0$, i.e.,

$$\beta \ge \frac{2\lambda_e(1+T_i/T_e)}{(U_e^2/V_{t_e}^2)[1-I_0(\lambda_e)e^{-\lambda_e}] - I_0(\lambda_e)e^{-\lambda_e}} \ , \quad (36)$$

with

$$U_e^2 > V_{t_e}^2 I_0(\lambda_e)e^{-\lambda_e}/[1-I_0(\lambda_e)e^{-\lambda_e}] \ . \quad (37)$$

For $p>0$ and in particular for $U_e=0$ it can be shown from inequality (35) that the instability can occur only for some bounded values of ion streaming, i.e., $U^2_{\min} < U^2_i < U^2_{\max}$, where

$$U^2_{\min} = V^2_{t_e}\{[(Wp_0)^{1/2} - Q_0^{1/2}]^2 - S_0\}/G \ , \qquad (38)$$

$$U^2_{\max} = V^2_{t_e}\{[(Wp_0)^{1/2} + Q_0^{1/2}]^2 - S_0\}/G \ ; \qquad (39)$$

S_0 and G are, however, given by

$$S_0 = \frac{m}{M}\,\frac{(kR_i)^{-1}}{(2\pi)^{1/2}}\,\frac{\mu k}{\alpha_i\Omega_i}\,\frac{1}{\beta} \qquad (40)$$

and

$$G = \left(W + \frac{2\mu\Omega_i}{k\beta}\,(k^2R_i^2 - l^2)\right)\frac{T_e}{T_i}\,\frac{(kR_i)^{-1}}{(2\pi)^{1/2}} \ , \qquad (41)$$

p_0 and Q_0 being the corresponding values of p and Q for $U_e = 0$. We may point out that only those solutions for ω, as given by Eq. (34), would be valid which satisfy the basic requirement $\mathrm{Re}\,\omega \simeq -(l\Omega_i + k\mu/4\alpha_i\Omega_i)$, on which the analysis of this section is based. In this sense inequality (35) is the necessary but not sufficient condition for stability. In order that the solutions given by Eq. (34) should represent the valid solutions we must satisfy the requirement $q/2p \simeq W$; in which case the inequality (35) is automatically satisfied and hence $q/2p \simeq W$ represents the necessary and sufficient condition for instability. The growth rates, consistent with this, are simply given by

$$\nu = (WF/p)^{1/2} \ . \qquad (42)$$

In Figs. 4–6 we have shown some numerical results for the growth rates as calculated from Eq. (42). In Fig. 4, when β exceeds some particular value, say $\beta_{\max}$, the relation $q/2p \simeq W$ is not satisfied and so the growth rates corresponding to $\beta > \beta_{\max}$ are redundant and hence dropped out. The same thing happens for ion streaming which is shown in Fig. 5.

From Figs. 4 and 5 we observe that the instability exists for some bounded values of U_e^2. As for $\beta > \beta_{\max}$ and $U_i^2 > U^2_{\max}$ the condition $q/2p \simeq W$ breaks down, therefore, the instability can occur only for some bounded values of β and U_i^2. The effect of T_i/T_e is stabilizing.

In Fig. 6 we cannot go below $\lambda_e = 0.3$ as then the assumption $k^2R_i^2 \gg 1$ does not hold good. From this figure we observe that, in general, the instability will occur for some bounded values of λ_e and $\mu V_{t_e}/\Omega_e$ (i.e., k and μ), respectively, because the values of V_{t_e} and Ω_e have been fixed while assigning values to other parameters.

We may point out that for homogeneous plasmas, i.e., $\epsilon = \mu = 0$, this mode is stable as seen from Eq. (35). Moreover, when ϵ or μ are finite but the plasmas are stationary, i.e., $U_e = U_i = 0$, this mode is again stable because then the condition $q/2p \simeq W$ cannot be satisfied without violating the assumption $|k\mu V_\perp^2/2\Omega_e| \ll l\Omega_i$.

C. Transverse Instabilities

In this section we study the effect of ∇B and ∇n on the unstable electromagnetic modes of uniform contrastreaming plasmas. We consider the special case of

$$\Omega_i^2 \ll \omega^2 \ll \Omega_e^2 \quad \text{with} \quad |\omega| \gg |k\mu V_\perp^2/2\Omega_e| \ .$$

In this case the dispersion relation (9) reduces to the following simple form

$$\omega^2 + \xi - \Phi/\omega = 0 \ , \qquad (43)$$

where

$$\xi = \omega^2_{p_e}\left(\frac{U_e^2}{V^2_{t_e}}[1 - I_0(\lambda_e)e^{-\lambda_e}] - I_0(\lambda_e)e^{-\lambda_e} - \frac{2\lambda_e}{\beta}\right), \qquad (44)$$

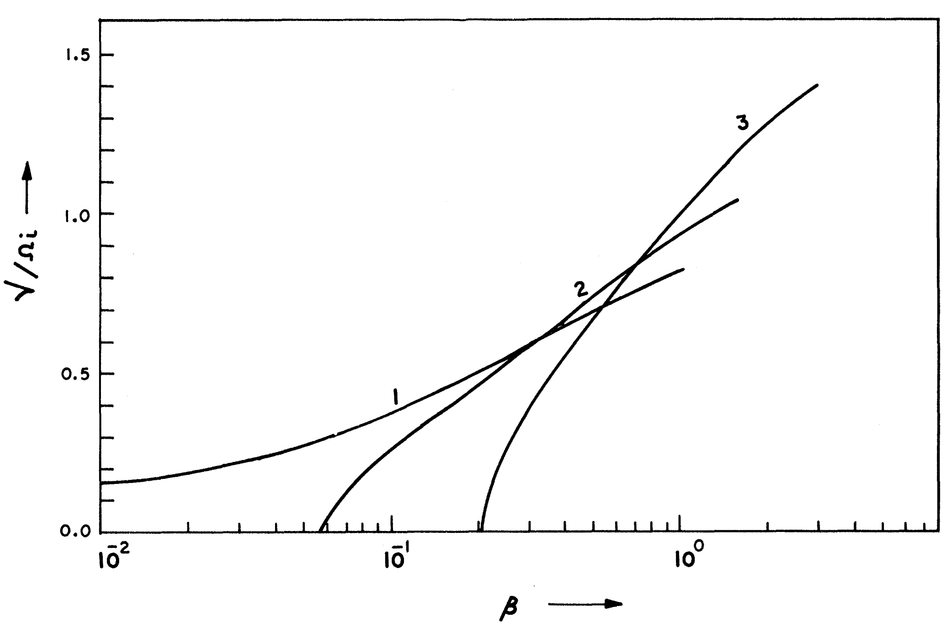

FIG. 4. Variation of growth rate ν/Ω_i vs β for $\lambda_e = 0.5$, $\mu V_{t_e}/\Omega_e = 0.7\times10^{-3}$, $U_i^2/V_{t_e}^2 = 0.1$, $T_i/T_e = 0.01$ and for $U_e^2/V_{t_e}^2 = 0.1$, 0.5, and 1.0 for the curves 1, 2, and 3, respectively (for $l = 1$).

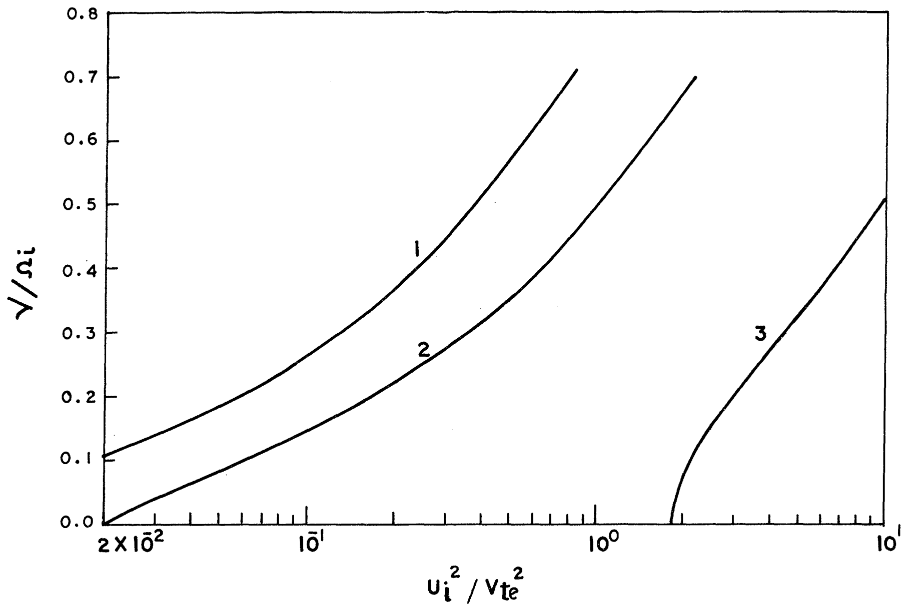

FIG. 5. Variation of ν/Ω_i vs $U_i^2/V_{t_e}^2$ for $\beta=1.0$, $\lambda_e=0.5$, $\mu V_{t_e}/\Omega_e=0.7\times10^{-3}$ $U_e=0.0$ and for $T_i/T_e=0.05$, 0.1, and 0.5 for the curves 1, 2, and 3, respectively (for $l=1$).

and

$$\Phi=\omega_{p_e}^2\,\frac{k\mu V_{t_e}^2}{\Omega_e}\left(\frac{2I_0(\lambda_e)e^{-\lambda_e}}{\beta}+\frac{d}{d\lambda_e}\left[\lambda_e I_0(\lambda_e)e^{-\lambda_e}\right]\right)\times\left(1+\frac{U_e^2}{V_{t_e}^2}\right)\quad. \tag{45}$$

We can solve Eq. (43) considering Φ/ω as the perturbation and applying the successive approximation. In the zeroth approximation we see that $\xi>0$ gives rise to a purely growing wave, i.e.,

$$\omega_i^0=-\xi^{1/2}\;. \tag{46}$$

Then the first-order approximation yields

$$\omega_i'=-(\xi^{1/2}+\Phi^2/8\xi^{5/2}) \tag{47}$$

and

$$\omega_r'=\tfrac{1}{2}(\Phi/\xi)(1-\Phi^2/8\xi^3)\;. \tag{48}$$

From Eq. (47) we observe that the gradients tend to increase the growth rates by a small factor of $\Phi^2/8\xi^{5/2}$, which one should, as a matter of fact, expect because the gradient drifts are very much weaker than the streaming velocity required for producing the transverse instability. In fact, it would be more appropriate and interesting to study the effect of gradients on the marginally stable waves, i.e., $\xi\to0$; for this, if once again we write $\omega=\omega_r-i\nu$ and solve Eq. (43), we obtain

$$\omega_r=\tfrac{3}{2}\Phi/(4\nu^2-\xi) \tag{49}$$

and

$$\nu^2=\xi+\tfrac{27}{4}\,\Phi^2/(4\nu^2-\xi)^2\;. \tag{50}$$

Now for $\xi\to0$, Eq. (50) for the growth rate simply

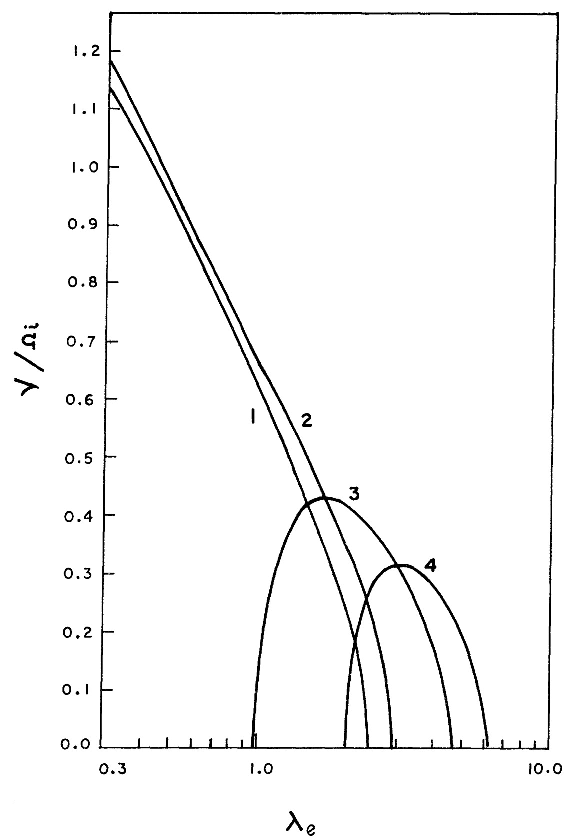

FIG. 6. Variation of ν/Ω_i vs λ_e for $\beta=1.0$, $U_i^2/V_{t_e}^2=0.1$, $T_i/T_e=0.01$, $U_e^2/V_{t_e}^2=0.5$, and $\mu V_{t_e}/\Omega_e=0.7\times10^{-3}$, 0.2×10^{-2} and 0.2×10^{-2} for the curves 1, 2, 3, and 4, respectively (for $l=1$).

gives

$$\nu = \tfrac{1}{2}\sqrt{3}\,(\Phi^*)^{1/3} \quad ; \qquad (51)$$

Φ^* in Eq. (51) is the value of Φ corresponding to $\xi = 0$. Figure (7) shows the growth rates as calculated from Eq. (51). From this figure we observe that a weak nonuniformity such as $\mu V_{t_e}/\Omega_e \sim 10^{-7}$ can render the otherwise marginally stable waves unstable, their growth rates being of the order of 3–5% of Ω_e. From Eqs. (45) and (51) it is obvious that ν will vary as $\mu^{1/3}$.

IV. CONCLUSIONS

In identical nonuniform plasmas contrastreaming along an external magnetic field, the electromagnetic waves propagating transverse to the direction of the magnetic field can become unstable. In particular the low-frequency waves $\omega^2 \ll \Omega_i^2$, can become unstable owing to the excitation of either resonant ion instability or resonant electron instability, the latter instability being much stronger than the former one. For the case of resonant ion instability the growth rates increase with the increase of β, μ, U_e, and U_i and with the decrease of T_i/T_e and k. For large values of β, μ, U_e, and U_i, however, the assumption $\omega^2 \ll \Omega_i^2$ breaks down and so the corresponding growth rates are inconsistent. On the other hand the growth rates for the resonant electron instability show increase with the increase of μ, T_i/T_e, and U_e and with the decrease of β and U_i. These results may prove useful in understanding some astrophysical processes[1,2,12] such as hydromagnetic discontinuities in the solar wind, etc.

Apart from the low-frequency instability the existence of the unstable drift cyclotron mode is also predicted. This instability occurs in nonuniform streaming plasmas and for some bounded values of U_e, U_i, k, β, and μ (or ϵ). The ion temperature has a stabilizing effect and the growth rates are of the order of Ω_i.

The effect of nonuniformities on the transverse instability, for frequencies $\Omega_i^2 \ll \omega^2 \ll \Omega_e^2$ is to increase the growth rate of the unstable waves slightly. However, even a weak nonuniformity such as $\mu V_{t_e}/\Omega_e \sim 10^{-7}$ can render the otherwise marginally stable waves unstable, their growth rate being 3–5% of Ω_e. These results may prove useful in some plasma confinement experiment.

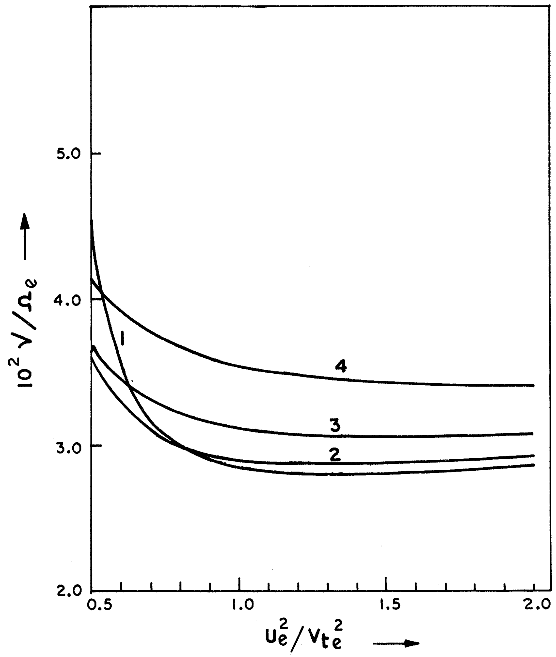

FIG. 7. Variation of growth rate ν/Ω_e vs $U_e^2/V_{t_e}^2$ for $\mu V_{t_e}/\Omega_e = 10^{-7}$, $C^2/V_{t_e}^2 = 200$ and for $\lambda_e = 2, 3, 5$, and 10 for the curves 1, 2, 3, and 4, respectively.

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