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Scattering of Electromagnetic Radiation from Liquid He^3 †

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The Landau theory of Fermi liquids is used to calculate the frequency power spectrum of density fluctuations in liquid He^3. Light-scattering intensities from "zero" sound, first sound, and quasiparticle-hole pairs are computed in detail. Finite-temperature x-ray structure factors are compared with experimental values and agreement is obtained within the considerable scatter of the experimental points. Light-scattering experiments are feasible and would provide detailed confirmation of Landau theory.

Although the Landau theory of Fermi liquids is an old and respected theory,[1] some of its very specific predictions for light- and x-ray-scattering experiments have not been computed in detail. In a previous letter[2] the Landau prediction for the ground-state x-ray structure factor was computed. These calculations are here extended to the finite-temperature regime. More important, the recent advances in light-scattering technology[3,4] make it feasible to provide much-needed confirmation of the Landau theory of liquid He^3. The purpose of this paper is to provide detailed numerical predictions of the frequency power spectrum of scattered light.

The cross section for an unpolarized photon to scatter off a single helium atom is well known[5]:

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{2}\alpha^2(1+\cos^2\theta)\lambda\!\!\!^{-}{}^{-4} \quad \text{(light)}, \tag{1a}$$

$$\frac{d\sigma}{d\Omega}=\frac{1}{2}\left(\frac{e^2}{mc^2}\right)^2(1+\cos^2\theta)\,|F(Q)|^2 \quad \text{(x ray)}. \tag{1b}$$

In Eqs. (1), α is the atomic polarizability, $\lambda\!\!\!^{-}$ is the wavelength of the photon divided by 2π, $F(Q)$ is the electronic form factor, i.e., the Fourier transform of the electron number density, and

$$Q=(2/\lambda\!\!\!^{-})\sin\tfrac{1}{2}\theta, \tag{2}$$

is the momentum transferred to the photon. For liquid He^3 the effects of scattered-amplitude interference from different atoms can be taken into account via the fluctuation-dissipation theorem, yielding[5]

$$\frac{d^2h}{d\Omega\, d\omega}=\frac{d\sigma}{d\Omega}\,\frac{\hbar\omega}{\pi}\,(1-e^{-\hbar\omega/k_BT})^{-1}\,\frac{\mathrm{Im}\chi(Q,\omega)}{\omega}, \tag{3}$$

where $\omega=\omega_i-\omega_f$ is the frequency shift in the scattered photon. We have expressed the scattered-photon distribution so that h_{tot} (the extinction coefficient) represents the inverse-mean-free-path length of the photon in the liquid. The crucial function to be calculated is $\chi(Q,\omega)$, i.e., the linear response in the density $\delta\rho$ to an external periodic potential δU_{ext} proportional to $e^{i(\vec{Q}\cdot\vec{r}-\omega t)}$. Present resolution allows x-ray experiments to measure only

$$\frac{dh}{d\Omega}=\int d\omega\,\frac{d^2h}{d\Omega\,d\omega}=\rho\,\frac{d\sigma}{d\Omega}\,S(Q). \tag{4}$$

We have calculated $\chi(Q,\omega)$ by putting the external potential δU_{ext} into the Landau–Boltzmann equation and finding the resulting response $\delta\rho$ in the density of quasiparticles.[2] Two approximations were made: (i) The collision integral was replaced by a single collision time in a manner consistent with the conservation laws. (ii) Only two nonzero Landau parameters[6] F_0 and F_1 were assumed in the spin symmeteric part of the quasiparticle interaction function. The resulting $\chi(Q,\omega)$ can be expressed in terms of a "dimensionless" frequency

$$\nu=\omega/QV_F \tag{5}$$

and "dimensionless" collision time

$$\sigma=\tau_c QV_F, \tag{6}$$

where V_F is the Fermi velocity p_F/m^*. We take[7]

$$\tau_c=(1.46\times10^{-12}\ \text{sec}\ {}^\circ\text{K}^2)/T^2 \tag{7}$$

to assure that the first-sound mode is damped by the measured viscosity. The final result is in basic agreement with Abrikosov and Khalatnikov[8]; the Landau prediction is

$$\chi(Q,\omega)=\chi_L(\nu,\sigma), \tag{8a}$$

$$(3\pi^2\hbar^3/m^*p_F)\,\chi_L(\nu,\sigma)=(1+\tfrac{1}{3}F_1)\,[\nu_0^2-\varphi(\nu,\sigma)-\nu^2]^{-1}, \tag{8b}$$

where

$$\nu_0^2=\tfrac{1}{3}(1+F_0)\,(1+\tfrac{1}{3}F_1), \tag{9a}$$

$$\varphi=(1+\tfrac{1}{3}F_1)\,\nu[(1/3s)\,(1+1/W)-s], \tag{9b}$$

$$s=\nu+i/\sigma, \tag{9c}$$

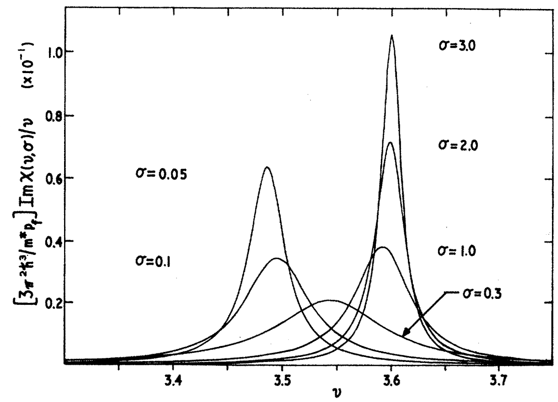

FIG. 1. Light-scattering intensity plotted against "dimensionless" frequency ν, for several "dimensionless" relaxation times σ. The choice of σ covers scattering in the first-sound ($\sigma = 0.05$, 0.1) transition ($\sigma = 0.3$) and zero-sound ($\sigma = 1.0$, 2.0, and 3.0) regimes.

$$W = \tfrac{1}{2} s \ln[(s+1)/(s-1)] - 1 \,. \tag{9d}$$

All of the predictions of the Landau theory for the finite-temperature frequency power spectrum of density oscillations are contained in Eqs. (8) and (9) in a very implicit form. For this reason we have made detailed numerical calculations of

$$\frac{\operatorname{Im}\chi_L(\nu, \sigma)}{\nu} = Q v_F \frac{\operatorname{Im}\chi(Q, \omega)}{\omega} \,. \tag{10}$$

Using the values shown in Figs. 1 and 2, together with Eqs. (3) and (10), allows one to compute the frequency power spectrum of scattered light in considerable detail for a range of temperatures and wavelengths.

In Fig. 1 we have plotted the "zero-" and first-sound peaks. The peak corresponds to an incident photon creating one phonon of sound. As the temperature is lowered, the peak first broadens and starts to shift from the first- to zero-sound regime. Further lowering of the temperature narrows the zero-sound peak. The shift takes place at temperatures of the order of 50 °mK and is accessible to experiment.

In Fig. 2 we have plotted the intensity due to a photon creating a single quasiparticle-hole pair. The intensity shows up as a broad central peak which *increases*[9] as the temperature is lowered. This is in contrast to a typical heat-diffusion peak, which would decrease with lowered temperature but which is entirely negligible in liquid He^3 at low temperatures. The central quasiparticle-hole-pair intensity is small in comparison with the sound-wave peak (about 0.1%), but it is clearly distinguishable from background effects and would be most valuable to observe, since it is a prediction unique to a Landau quasiparticle theory.

Using the sum rule

$$S(Q) = (\hbar/\pi\rho) \int_{-\infty}^{+\infty} (1 - e^{-\hbar\omega/k_B T})^{-1} \operatorname{Im}\chi(Q, \omega)\, d\omega \,, \tag{11}$$

one can predict finite-temperature x-ray structure factors from the Landau function $\chi_L(\nu, \sigma)$. By far the dominant contributor to the integral in Eq. (11) is the sound-wave peak.

In Fig. 3 we have plotted $S(Q)$ for a range of temperatures and find that finite-temperature effects are negligible even for the smallest Q values measured experimentally. Agreement between theory and experiment is qualitatively good considering the scatter in the experimental points presently available.[10,11] What is needed is more precise experimental points to reduce the scatter.

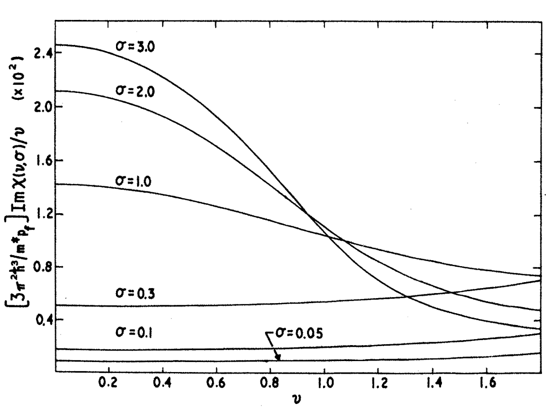

FIG. 2. Continuation of the plots in Fig. 1, for small values of ν, where for $\sigma \geq 1$ quasiparticle-hole pairs dominate the scattering.

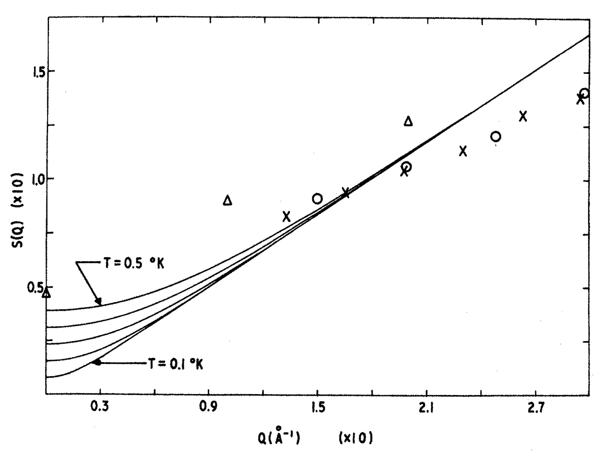

FIG. 3. Plot of $S_T(Q)$ for temperatures from 0.1 to 0.5 °K, at intervals of 0.1 superimposed over existing data at $T = 0.56$ (Ref. 11), 0.41, and 0.36 °K (Ref. 10) represented by triangles, circles, and crosses, respectively.

In conclusion we wish to make two points: (i) Light-scattering measurements of the zero- to first-sound transition in liquid He^3 would be valuable and are possible. (ii) More difficult but even more valuable would be the detection of quasiparticle-hole pairs by light scattering, since this would constitute an experimental confirmation of a particular prediction of the Landau theory.

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[6]Our definition is the same as that used by D. Pines and P. Nozieres, *The Theory of Quantum Liquids* (Benjamin, New York, 1966), Vol. I. We use the values $F_0 = 6.25$ and $F_1 = 10.8$.

[7]See, for recent viscosity value, M. P. Bertinat, D. S. Betts, D. F. Brewer, and G. J. Butterworth, Phys. Rev. Letters 28, 472 (1972). We find η via the relationship $\eta = \frac{1}{5}\pi\tau_\eta v_F^2\,(1+\frac{1}{3}F_1)$.

[8]A. A. Abrikosov and I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. 34, 198 (1958) [Sov. Phys. JETP 7, 135 (1958)]; 41, 544 (1961) [14, 389 (1961)]. The original paper treated the collision integral incorrectly; however, we are in agreement with the latter result (second paper).

[9]As the temperature is lowered from first- to "zero-" sound regime the central peak first increases and then decreases (below about 20 °mK) since an over-all T factor has begun to dominate.

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Optimized Jastrow Factor and Consistent Phonons in Quantum Crystals

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Optimization of the short-range correlation function in the Jastrow–Nosanow–Koehler form of trial ground-state wave function for quantum crystals, by functional minimization of the ground-state energy, is proved to result automatically in phonon frequencies which are fully consistent: frequencies defined from the one-phonon excitation energy and from the Gaussian portion of the ground-state wave function are identical.

Considerable progress has been achieved[1,2] in recent years in constructing wave functions and phonon dynamical matrices for quantum crystals, where the possibility of hard-core collisions must be dealt with explicitly. Nevertheless, certain disturbing difficulties have resisted treatment. The major problem[2] is that of ambiguity in the definition of the phonon frequencies; those appearing in the phonon zero-point portion of the ground state wave function do not agree with those appearing in the excitation energy of the one-phonon state. Another problem is the conceptual difficulty of specifying the separation of the ground-state wave function into short- and long-range correlation parts. Although the former is to be represented by a Jastrow function, and the latter by a phonon Gaussian function, in fact both are of the same functional form and cannot be distinguished analytically. Finally, the problem of how best to choose the short-range correlation function has given rise to a substantial literature.[1]

In this paper we demonstrate that all three problems are in fact closely related and can be resolved simultaneously. We show that when the short-range correlation function is determined by the variational requirement of functionally minimizing the ground-state energy, subject to a constraint which uniquely separates the pair distribution function into a short-range and a phonon part, then the previous ambiguity in the phonon dynamical matrix also disappears.

As a starting point we use the Koehler adaptation[2] of the method of correlated basis functions. First a trial ground-state wave function is exhibited for the solid,

$$|0\rangle = \Big(\prod_{i<j} f_{ij}(\vec{r}_{ij})\Big)\exp\Big(-\sum_{i<j}\frac{1}{4}\vec{u}_{ij}\cdot\overleftrightarrow{\Gamma}_{ij}\cdot\vec{u}_{ij}\Big), \quad (1)$$

where the Jastrow–Nosanow factors f_{ij} are specifically intended to account for short-range hard-core