and Two Electron Atoms (Academic, New York, 1957), p. 45. The normalization given there is not that of a unit-amplitude wave at infinity. Their result has to be multiplied by the wave function at the origin in configuration space.

[7]The Born approximation, or any calculation which ignores charge exchange, may be used for $\sigma^{(1)}$.

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Recalculation of hfs Constants of Muonic Rotational Nuclei

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Magnetic hyperfine splittings of rotational levels in muonic deformed nuclei are calculated microscopically within the self-consistent cranking model, taking into account the finite extension of the nucleus. A previous oversight in the theory of the magnetic interaction between the muon and the nucleus is corrected. The splittings are found to be about 25% larger than the earlier estimates. These new values seem to remove the discrepancy between muonic and Mössbauer isomer shifts in the rare-earth region.

The change of nuclear charge radii $\delta\langle r_p^2\rangle$ owing to collective rotation has been measured by two different techniques: (a) Mössbauer effect,[1] and (b) muonic atoms.[2,3] In both cases one observes the isomer shift $\Delta E^{\text{is (expt)}}$ of a nuclear γ transition. Whereas the measured shift in a Mössbauer experiment is directly proportional to the change of the nuclear charge radii, the nuclear γ transition in a muonic atom is shifted due to the Coulomb interaction (isomer shift) and to a magnetic hyperfine interaction between the bound muon and the nucleus[4,5]:

$$\Delta E_\mu^{\text{expt}} = \Delta E_\mu^{\text{is (expt)}} + \Delta E_\mu^{\text{magn}} . \tag{1}$$

In order to derive the muonic isomer shift $\Delta E_\mu^{\text{is (expt)}}$ from the measured shift $\Delta E_\mu^{\text{expt}}$, one needs to know the magnetic contribution $\Delta E_\mu^{\text{magn}}$. These magnetic shifts have not yet been measured for deformed nuclei and there exists only one theoretical estimate[5] of this effect. Using this earlier estimate and comparing the values $\delta\langle r_p^2\rangle$ as derived from muonic isomer shifts with those derived from Mössbauer experiments, one obtains large discrepancies. In some cases even the sign is different. These differences have stimulated speculations about the charge distribution of excited 2^+ states[6] and the importance of polarization effects.

In this paper we report new microscopic calculations of the hyperfine splitting of 2^+ rotational levels in deformed nuclei where we take into account, in linear response, the residual interaction.[7] We find larger values of the magnetic hyperfine splitting than reported earlier[5]; this result removes the discrepancy of the isomer-shift measurements.

The magnetic-energy shift is determined by

$$\Delta E_\mu^{\text{magn}} = \tfrac{1}{2}[F(F+1) - I(I+1) - \tfrac{3}{4}]\, a_I . \tag{2}$$

This formula holds for the muon in its 1s state, nuclear spin I, and total spin F. When the finite extension of the nucleus is taken into account, the hyperfine splitting (hfs) constant a_I is given by[8,9]

$$a_I = \tfrac{8}{3} e\mu_N F_0 \langle \Psi_{I,M=I} |\, M_z(R)\, | \Psi_{I,M=I}\rangle / I \tag{3}$$

and

$$\vec{M}(R) = \sum_{i=1}^{A} \{ g_l^{(i)} \vec{l}_i [F_1(R) + F_2(R)] + g_s^{(i)} \vec{s}_i F_1(R) + g_s^{(i)} \vec{a}_i F_2(R) \} . \tag{4}$$

Here, $\vec{l}$, $\vec{s}$, g_l, and g_s are the orbital and spin angular momenta, respectively, and the corresponding g factors; $\vec{a}$ stands for the tensor part, μ_N is the nuclear magneton, R the radial coordinate, and

$$F_0 = \int_0^\infty \frac{fg}{r^2}\, dr, \quad F_1(R) = \frac{1}{F_0}\int_R^\infty \frac{fg}{r^2}\, dr, \quad F_2(R) = \frac{1}{F_0 R^3}\int_0^R rfg\, dr , \tag{5}$$

with the muonic wave functions f and g. In the limit of a point nucleus, $\vec{M}(R=0)$ is the usual magnetic-moment operator. The R dependence of $\vec{M}$ takes into account the spatial distribution of the magnetic moment in the nucleus, as tested by the muon.

The nuclear matrix element in Eq. (3) has to be evaluated with the wave function Ψ_I of the rotating nucleus. As shown in Ref. 10, this can be done for well-deformed nuclei in the form

TABLE I. Theoretical and experimental g_R factors and hfs constants for point nuclei and nuclei of finite extension. Columns 2 and 4 are correlated by Eq. (9).

Nucleus	$g_R^{\text{theor a}}$	$g_R^{\text{expt b}}$	Point nucleus[c] a_I (eV)	Finite nucleus[c] a_I (eV)
Nd^{150}	0.394	0.322 ± 0.009	429	320
Sm^{152}	0.396	0.339 ± 0.012	444	331
Sm^{154}	0.367	0.389 ± 0.019	412	310
Gd^{154}	0.406	0.427 ± 0.014	455	344
Gd^{156}	0.368	0.393 ± 0.007	413	316
Gd^{158}	0.367	0.327 ± 0.018	412	314
Gd^{160}	0.351	0.323 ± 0.015	394	301
Dy^{160}	0.375	0.364 ± 0.011	420	328
Dy^{162}	0.354	0.343 ± 0.014	397	311
Dy^{164}	0.324	0.336 ± 0.014	363	289
Er^{164}	0.328	0.353 ± 0.010	400	301
Er^{166}	0.307	0.312 ± 0.006	375	282
Er^{168}	0.322	0.333 ± 0.008	393	295
W^{180}	0.238	···	314	234
W^{182}	0.263	0.266 ± 0.09	347	253
W^{184}	0.306	0.295 ± 0.010	404	287
W^{186}	0.355	0.322 ± 0.013	469	327

[a]Reference 7. [b]Reference 12. [c]This theory.

$$\langle \Psi_{I,M=I} \,|\, M_z \,|\, \Psi_{I,M=I} \rangle = \frac{I}{\sqrt{I(I+1)}} \operatorname{Tr}\{M_x \rho^{\omega}\} , \qquad (6)$$

with the nuclear density matrix ρ^{ω} calculated from the self-consistent cranking model and ω determined by $\operatorname{Tr}\{J_x\rho^{\omega}\} = \sqrt{I(I+1)}$. With the first-order expansion $\rho^{\omega} = \rho^{(0)} + \omega\rho^{(1)}$ and keeping in mind $\sqrt{I(I+1)} = \omega J$, one obtains

$$a_I = \tfrac{8}{3}\, e\mu_N F_0 (\operatorname{Tr}\{M_x \rho^{(1)}\} / J) . \qquad (7)$$

J is the usual cranking moment of inertia. In the limit of a point nucleus

$$(\operatorname{Tr}\{M_x(R=0)\rho^{(1)}\}/J) = g_R \qquad (8)$$

yields the nuclear g_R factor and one has

$$a_I = \tfrac{8}{3}\, eF_0\, g_R\, \mu_N . \qquad (9)$$

The hfs constants are calculated from Eq. (7) simultaneously with the g_R factors [Eq. (8)], as discussed in Ref. 7. The results are shown in the last column of Table I and are found to be larger than the earlier estimates of Ref. 5 by about 25%. These earlier values had been calculated with a formula derived by Ehrlich *et al.*[11] In column 4 of Table I the hfs constants for a point nucleus [Eq. (9)] are shown. Comparing columns 4 and 5, one finds a reduction of about 25% due to the finite extension of the nucleus. In the calculations of Ref. 5 this reduction was nearly 40%. For comparison, we also give the experimental g_R factors[12] and the theoretical results of Ref. 7.

In order to understand the difference between our results and those of Ref. 5, one has to examine Eq. (4). In the nonrelativistic limit one gets

$$F_1(R) = -\frac{\rho(R)}{4m} , \quad F_2(R) = \frac{1}{4m} \int_0^R \frac{r^3}{R^3} \left(\frac{d\rho(r)}{dr} \right) dr . \qquad (10)$$

Here $\rho(R)$ and m are the density distribution and the mass of the muon, respectively. In the formula derived in Ref. 11, only the F_1 contribution was taken into account to calculate a_I. The difference between the present results and the previous calculation is mainly due to the F_2 contribution. In Fig. 1 we plot $F_1(R)$ and $F_2(R)$ given by Eq. (5). From this figure it appears obvious that F_2 gives appreciable contributions for states near to the Fermi surface. In Fig. 2 the scheme of the level splitting due to the magnetic hyperfine interaction is sketched. Under the assumption that the nuclear $E2$ transition takes place from the lower doublet partner (fast $M1$ interdoublet transition) to the ground state, the energy shift $\Delta E_{\mu}^{\text{magn}}$ is

$$\Delta E_{\mu}^{\text{magn}} = -\tfrac{3}{2} a_I , \qquad (11)$$

as given in column 3 of Table II. With these magnetic corrections one gets the experimental muonic isomer shifts $\Delta E_{\mu}^{\text{is (expt)}}$ from Eq. (1) which are shown in column 4 of Table II. In column 2 the measured shifts of Refs. 2 and 3 are given. Comparing our results of $E_{\mu}^{\text{is (expt)}}$ with those of Refs. 2

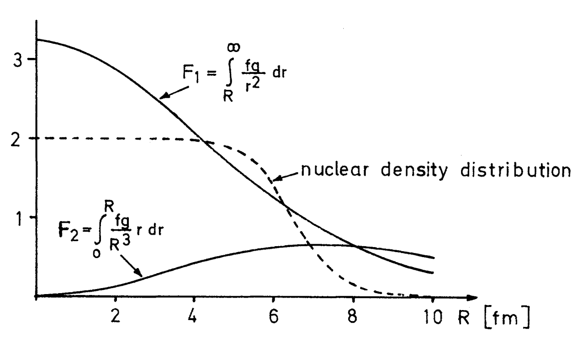

FIG. 1. Nuclear density distribution and R dependence of F_1 and F_2.

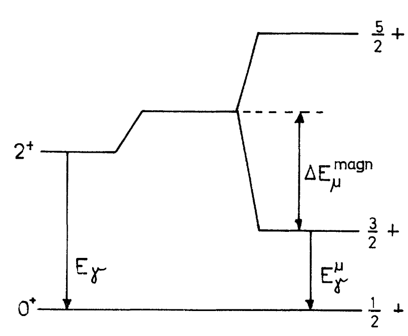

FIG. 2. Magnetic hyperfine splitting of a 2^+ nuclear level due to the presence of a $1s$ muon.

TABLE II. Comparison of muonic isomer shifts and Mössbauer $\delta\langle r_p^2\rangle$ as derived from Mössbauer data. The experimental muonic isomer shifts in column 4 are the differences of the experimental shifts given in column 2 and the calculated magnetic shifts shown in column 3. Column 5 shows the earlier results of Refs. 2 and 3, where the magnetic corrections of Ref. 5 are used. A detailed discussion of this table is given in the text.

Nucleus	$\Delta E_\mu^{\text{expt}}$ (eV)	$\Delta E_\mu^{\text{magn c}}$ (eV)	$\Delta E_\mu^{\text{is(expt)}}$ (eV)	$\Delta E_\mu^{\text{is(expt)}}$ Refs. 2 and 3 (eV)	$\delta\langle r^2\rangle_{\text{Möss}}^{\text{expt d}}$ (10^{-3} FM)
Sm^{152}	560 ± 60 [a] 500 ± 40 [b]	-496	1056 ± 60 996 ± 40	920 ± 70 [a] 770 ± 40 [b]	12 ± 4
Gd^{154}	670 ± 150 [a]	-515	1185 ± 150	980 ± 150 [a]	16 ± 3
Gd^{156}	-375 ± 80 [b]	-474	99 ± 80	0 ± 80 [b]	2 [a]
Gd^{158}	-595 ± 50 [b]	-471	-124 ± 50	-230 ± 50 [b]	0.4 ± 0.3
Gd^{160}	-434 ± 170 [b]	-451	16 ± 170	-40 ± 170 [b]	0.3 ± 0.6
W^{182}	-320 ± 100 [a] -290 ± 90 [b]	-380	60 ± 100 90 ± 90	-30 ± 100 [a] -40 ± 90 [b]	-0.44
W^{184}	-340 ± 100 [a] -350 ± 50 [b]	-430	90 ± 100 80 ± 50	-25 ± 100 [a] -75 ± 50 [b]	$+0.36$
W^{186}	-350 ± 100 [a] -400 ± 40 [b]	-490	140 ± 100 90 ± 40	-10 ± 100 [a] -120 ± 40 [b]	$+0.33$

[a] Reference 2.
[b] Reference 3.
[c] This theory.
[d] Reference 1.

and 3, in column 5, which were derived with the magnetic corrections of Ref. 5, one notices large differences. In many cases the sign of the isomer shift is changed due to the newly calculated corrections. A quantitative comparison of both kinds of isomer shifts is possible only by calculating the change of the charge distribution due to the collective rotation. This has been done in Ref. 13. Qualitatively, however, one would expect that the Mössbauer isomer shifts and the muonic isomer shifts are correlated since they are just proportional to different moments of the same charge distribution. One would in particular expect that both shifts have the same sign. Comparing the experimental Mössbauer isomer shifts[1] with the experimental[2,3] muonic isomer shifts, one notices in several cases a change of sign, especially in the tungsten isotopes, where the signs of the Mössbauer data are well known. The comparison of the Mössbauer data with the recalculated muonic isomer shifts shows, however, that the discrepancies vanish within the experimental and theoretical uncertainties, with the exception of Gd^{158}.

Finally, we wish to point out that these new values of the magnetic hyperfine splitting in muonic atoms not only give rise to large corrections of the present muonic-isomer-shift data but may also stimulate new direct measurements of the hyperfine splittings. Results of recent measurements in Os isotopes[14] are in agreement with our predictions.

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