bound two-body systems can give rise to a series of excited states in the corresponding three-body systems.[8]

If one assumes for the ^{3}He trimer the same wave function and interaction, its ground-state binding energy can likewise be calculated. It is found to be unbound by 0.127 °K, a result in agreement with Stenschke's.[9]

Of some interest is the form of the correlation function; $f(r_{ij})$ resembles closely those derived by Nosanow[10] and Massey and Woo[11] from their variational calculations for solid ^{3}He. This is shown in Fig. 3.

In summary, the following conclusions can be drawn: (i) The *ab initio* potential of Bertoncini and Wahl leads to a bound trimer of ^{4}He. Other realistic interactions should lead to the same result. (ii) The trimer of ^{3}He is probably unbound. If the ^{3}He-^{3}He potential is deeper than the ^{4}He-^{4}He potential, as suggested by Bennewitz *et al.*,[12] a bound trimer could result. (The author is at present studying this.) (iii) The correlation function derived in this paper is similar to those obtained in solid-^{3}He calculations. (iv) The behavior of the E-vs-ϵ curve from this work indicates inaccuracies in the Kruger calculations.

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High-Energy K-Shell Ionization by Heavy Projectiles*

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The ionization of K-shell electrons by heavy particles is examined in the energy region where the projectile is moving faster than the orbiting electron. An explanation suggested for the rise of the experimental results above the Born approximation is the mechanism of "charge exchange to the continuum." An estimate of the effect shows reasonable agreement with experiment.

At sufficiently high energies, one expects that the K-shell ionization cross section of an atom by a charged projectile would be well described by the first Born approximation. This approximation predicts a cross section proportional to Z_1^2 (Z_1 is the projectile charge number), so that the ratio $R = \sigma(Z_1)/Z_1^2\sigma(1)$ is expected to be unity where the projectile velocities are the same. In fact, experimental results for projectiles of protons and α particles show significant deviations from unity.[1] Attempts have been made[2] to explain these deviations in both the low- and high-velocity regions of the curve. We discuss only the high-energy end here.

Two mechanisms[2,3] have been proposed to explain the deviation of R from unity at higher energies based on initial-state polarization of the K-shell electron by the projectile. The first[2] corresponds to a second-order correction in the standard perturbation expansion in Z_1e^2/hv which, it is expected, extends over a region comparable to the size of the K-shell electric orbit. Since this initial-state polarization is the adiabatic response of the bound electron to the projectile, we suggest that this effect is more appropriate for the low-velocity region. The second initial-state polarization effect[3] extends over projectile impact parameters larger than the atomic radius and could contribute at moderately high projectile velocities where the target still has time to adjust.

In this paper we propose another mechanism to explain the experimental deviations at the higher energies. It has been called[4] "charge exchange to the continuum," or polarization of the final state. When an electron is ionized by the projectile, it can correlate strongly with the projectile in the final state, but still not be bound to it. Even at high velocities the polarization at the final state

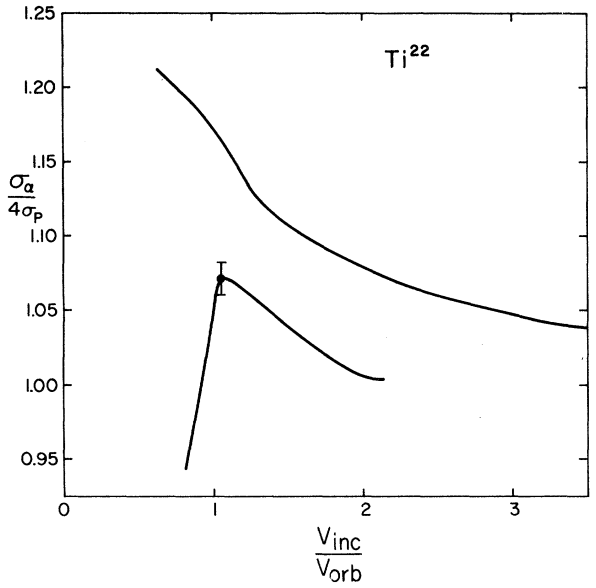

FIG. 1. R vs projectile velocity for α particles and protons incident on titanium. The lower curve represents the experimental data of Lewis, Watson, and Natowitz. The upper curve corresponds to Eq. (10). The ratio $V_{inc}/v_{orb}=V/Z_2$, where Z_2 is the effective nuclear charge seen by a K-shell electron. In the approximations of Born, Gryzinski, and Garcia, $R=1$.

can be large since the relative velocity of the electron and projectile can be small. Furthermore, this final-state polarization will be stronger the larger the projectile charge, which is in the right direction to explain the deviation of R from unity.

An accurate calculation of this effect is by no means a simple task. One has to calculate the matrix element for ionization to a state k and then integrate over all possible states. We consider here two kinds of final states: one in which the electron is correlated with its initial center (the atom), and another in which it is correlated with the projectile. In integrating over final electron momenta, care must be taken to assure that states are not counted twice. However, if the relative velocity of the projectile and atom is high enough, the interference between the two kinds of states becomes less important.

We expect the first kind of state to populate a volume in momentum space whose size is of the order $k\approx Z_2/a_0$ (Z_2 is the charge on the atomic nucleus). The second kind of state will probably populate a volume of momentum space centered about the moving projectile with radius $\sim Z_1/a_0$. That is, this kind of electron will be likely to have momenta $\sim mV+q$, where V is the projectile velocity, m is the electron mass, and $q\sim Z_1/a_0$.

If the projectile velocity is sufficiently high [$mV \gg (Z_1+Z_2)a_0^{-1}$], it is reasonable to expect that there is no significant interference between the two kinds of final states, and, in effect, the total ionization cross section is just the sum of the two kinds of ionizations, namely,

$$\sigma_{tot}=\sigma^{(1)}+\sigma^{(2)}=\sigma^{(1)}(1+\sigma^{(2)}/\sigma^{(1)}). \tag{1}$$

In order to calculate the second cross section we shall use the impact-parameter formalism to describe the motion of the projectile, assume that the spectator electrons do not change their states during the collision, and describe the transition by the Brinkman–Kramers matrix element.[5] In rydbergs this matrix element is

$$A_q(\vec{b})=i\int dt\, e^{i\Delta Et}\int d^3x\,\phi_q^{(-)*}(\vec{x}-\vec{R},Z_1)e^{-i\vec{x}\cdot\dot{\vec{R}}/2}\times(-2Z_1/|\vec{x}-\vec{R}|)\phi_{1s}(\vec{x},Z_2), \tag{2}$$

where ϕ_{1s} is the K-shell wave function and $\phi_q^{(-)}$ is the (incoming wave) Coulomb wave function of an electron with momentum q relative to the projectile Z_1 and with unit amplitude at infinity. The change of energy of the electron is

$$\Delta E=q^2+\tfrac{1}{4}V^2-W_{1s} \tag{3}$$

and the position of the projectile is given

$$\vec{R}=\vec{b}+\vec{V}t, \tag{4}$$

where $\vec{b}$ is the impact parameter of the collision. The cross section for this ionization is

$$\sigma_q=\int d^2b\,|A_q(\vec{b})|^2 \tag{5}$$

and the total ionization cross section is

$$\sigma_{tot}=\int\frac{d^3q}{(2\pi)^3}\,\sigma_q. \tag{6}$$

It may readily be seen that σ_{tot} is a function of Z_1 and Z_2 only through the variables Z_1/V and Z_2/V. Since we are interested in the region $V\geq Z_2$ and $Z_1\ll Z_2$, we can then exploit the inequality $Z_1/V\ll 1$ by expanding in this parameter. The lowest-order contribution to σ_{tot} is proportional to Z_1^2, and we note from Eq. (2) that this contribution is obtained by replacing $\phi_q^{(-)}$ by a plane wave. However, the use of a plane wave in Eq. (2) and the subsequent calculation of σ_{tot} is exactly the first Born approximation with the final state of the electron correlated to neither center. Thus we see that the term in σ_{tot} proportional to Z_1^2 is just the first Born result (the Brinkman–Kramers form) calculated in this roundabout way. The remaining part of σ_{tot} is the enhancement of the cross section due to the final-state interaction contained in $\phi_q^{(-)}$.

Instead of calculating this enhancement absolutely we shall take the ratio of the Z_1^3 term in Eq. (6) to the Z_1^2 which is in lowest order, just the ratio $\sigma^{(2)}/\sigma^{(1)}$ in Eq. (1). This calculation is greatly simplified by the use of the momentum-space representation[6] of ϕ_q. The result is

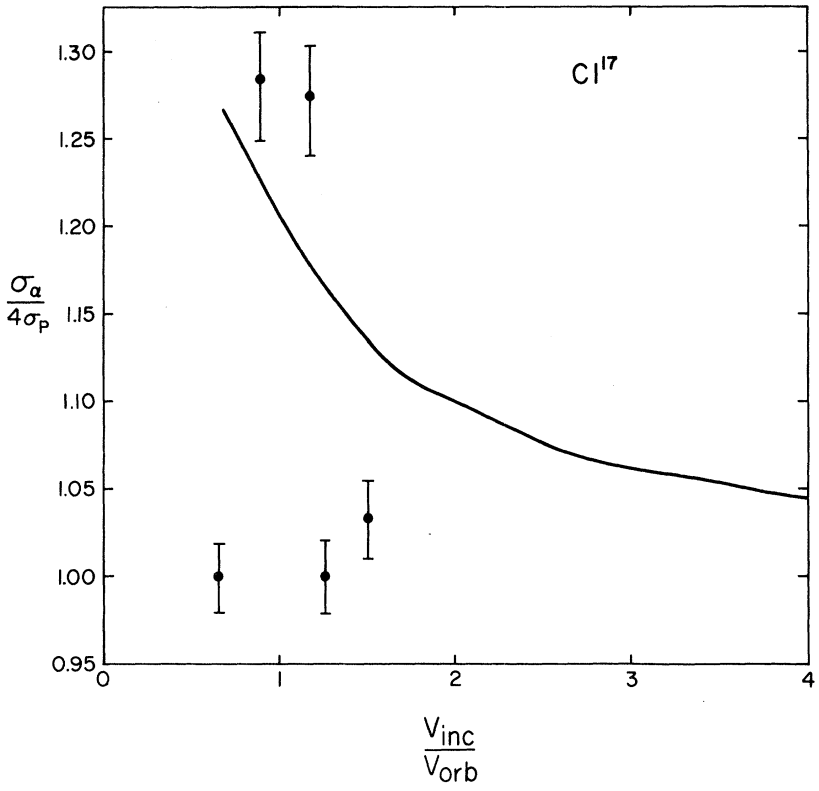

FIG. 2. R vs projectile velocity for α particles and protons incident on chlorine. The data are taken from Ref. 1, and the solid curve corresponds to Eq. (10).

$$\sigma^{(2)}/\sigma^{(1)} = (Z_1/Z_2) f(\lambda), \tag{7}$$

where

$$f(\lambda) = \pi\sqrt{\lambda}\int_0^\infty \frac{ds}{[(s-1+\lambda)^2+4\lambda]^5} \times\left(\int_0^\infty \frac{ds\sqrt{s}}{[(s-1+\lambda)^2+4\lambda]^5}\right)^{-1} \tag{8}$$

and

$$\lambda = 4Z_2^2/V^2 \simeq (W_{1s}/E_{1ab})(M/m). \tag{9}$$

Here E_{1ab} is the lab energy of the projectile and M its mass.

The ratio for α particles and protons[7]

$$R = \sigma(Z_1)/Z_1^2\sigma(1)\big|_{z_1=2} = \frac{1+(2/Z_2)f(\lambda)}{1+(1/Z_2)f(\lambda)} \tag{10}$$

is plotted with experimental data[1] for targets of Ti^{22} and Cl^{17}. The Z_2 dependence of Eq. (10) is such as to make the deviation of R from unity smaller the higher Z_2. This is qualitatively in the right direction but the deviation from unity in R is greater than experiment. This is understandable since the Brinkman–Kramers matrix element has been used to calculate the "exchange" cross section. This approximation is known to yield results which are often a factor of 5 or so too high for charge transfer in the energy range considered here. The defects of the matrix element can be corrected at the expense of extensive numerical calculations. In this paper, however, we are primarily interested in pointing out this effect due to charge exchange, and showing that it is the right order of magnitude.

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[7]The Born approximation, or any calculation which ignores charge exchange, may be used for $\sigma^{(1)}$.

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Recalculation of hfs Constants of Muonic Rotational Nuclei

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Magnetic hyperfine splittings of rotational levels in muonic deformed nuclei are calculated microscopically within the self-consistent cranking model, taking into account the finite extension of the nucleus. A previous oversight in the theory of the magnetic interaction between the muon and the nucleus is corrected. The splittings are found to be about 25% larger than the earlier estimates. These new values seem to remove the discrepancy between muonic and Mössbauer isomer shifts in the rare-earth region.

The change of nuclear charge radii $\delta\langle r_p^2\rangle$ owing to collective rotation has been measured by two different techniques: (a) Mössbauer effect,[1] and (b) muonic atoms.[2,3] In both cases one observes the isomer shift $\Delta E^{\text{is (expt)}}$ of a nuclear γ transition. Whereas the measured shift in a Mössbauer experiment is directly proportional to the change of the nuclear charge radii, the nuclear γ transition in a muonic atom is shifted due to the Coulomb interaction (isomer shift) and to a magnetic hyperfine interaction between the bound muon and the nucleus[4,5]:

$$\Delta E_\mu^{\text{expt}} = \Delta E_\mu^{\text{is (expt)}} + \Delta E_\mu^{\text{magn}} . \tag{1}$$

In order to derive the muonic isomer shift $\Delta E_\mu^{\text{is (expt)}}$ from the measured shift $\Delta E_\mu^{\text{expt}}$, one needs to know the magnetic contribution $\Delta E_\mu^{\text{magn}}$. These magnetic shifts have not yet been measured for deformed nuclei and there exists only one theoretical estimate[5] of this effect. Using this earlier estimate and comparing the values $\delta\langle r_p^2\rangle$ as derived from muonic isomer shifts with those derived from Mössbauer experiments, one obtains large discrepancies. In some cases even the sign is different. These differences have stimulated speculations about the charge distribution of excited 2^+ states[6] and the importance of polarization effects.

In this paper we report new microscopic calculations of the hyperfine splitting of 2^+ rotational levels in deformed nuclei where we take into account, in linear response, the residual interaction.[7] We find larger values of the magnetic hyperfine splitting than reported earlier[5]; this result removes the discrepancy of the isomer-shift measurements.

The magnetic-energy shift is determined by

$$\Delta E_\mu^{\text{magn}} = \tfrac{1}{2}[F(F+1) - I(I+1) - \tfrac{3}{4}]\, a_I . \tag{2}$$

This formula holds for the muon in its $1s$ state, nuclear spin I, and total spin F. When the finite extension of the nucleus is taken into account, the hyperfine splitting (hfs) constant a_I is given by[8,9]

$$a_I = \tfrac{8}{3} e\mu_N F_0 \langle \Psi_{I,M=I} \mid M_z(R) \mid \Psi_{I,M=I}\rangle / I \tag{3}$$

and

$$\vec{\mathrm{M}}(R) = \sum_{i=1}^{A} \{ g_l^{(i)} \vec{\mathrm{l}}_i [F_1(R) + F_2(R)] + g_s^{(i)} \vec{\mathrm{s}}_i F_1(R) + g_s^{(i)} \vec{\mathrm{a}}_i F_2(R) \} . \tag{4}$$

Here, $\vec{\mathrm{l}}$, $\vec{\mathrm{s}}$, g_l, and g_s are the orbital and spin angular momenta, respectively, and the corresponding g factors; $\vec{\mathrm{a}}$ stands for the tensor part, μ_N is the nuclear magneton, R the radial coordinate, and

$$F_0 = \int_0^\infty \frac{fg}{r^2}\, dr, \quad F_1(R) = \frac{1}{F_0}\int_R^\infty \frac{fg}{r^2}\, dr, \quad F_2(R) = \frac{1}{F_0 R^3}\int_0^R rfg\, dr , \tag{5}$$

with the muonic wave functions f and g. In the limit of a point nucleus, $\vec{\mathrm{M}}(R=0)$ is the usual magnetic-moment operator. The R dependence of $\vec{\mathrm{M}}$ takes into account the spatial distribution of the magnetic moment in the nucleus, as tested by the muon.

The nuclear matrix element in Eq. (3) has to be evaluated with the wave function Ψ_I of the rotating nucleus. As shown in Ref. 10, this can be done for well-deformed nuclei in the form