the Navier-Stokes equations are linearized.

[9]For $m=0$ the integral diverges, because the exponential does not then restrict the volume of phase space which contributes. A natural cutoff is then given by the requirement that if $\omega_k t \gg 1$, the corresponding k will no longer contribute to the correlation function at time t. A final cutoff is given by the necessity for $ka \lesssim 1$, where a is an average intermolecular spacing.

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Comments on "Critique of Electromagnetic Turbulent-Plasma Scattering Theories" by D. S. Bugnolo*

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In a recent paper Bugnolo suggested that first-order smoothing theory describing wave propagation in media with random fluctuations of the refractive index should not be used for wave numbers larger than the correlation length. This suggestion is shown to be incorrect.

Bugnolo's[1] intention is to show that the first-order smoothing theory[2-4] for wave propagation in media with random fluctuations of the refractive index cannot be applied in the high-frequency case, $k_0 l_0 \gg 1$, where k_0 is the wave number of the wave and l_0 is the correlation length of the density fluctuations $n_e(\vec{r})$. [It was assumed that $\langle n_e(\vec{r}_1) n_e(\vec{r}_2)\rangle = \langle n_e^2\rangle e^{-|\vec{r}_2-\vec{r}_1|/l_0}$.] The conclusion was based on seemingly contradictory results of two theories on wave energy transport: (i) first-order smoothing theory (FOST),[2-4] and (ii) a transport theory using the photon picture, as developed by Bugnolo[1,5] (B). A thorough comparison was only made for those terms ("coherent terms") which describe energy loss due to scattering of radiation out of the wave, and a discrepancy was found there between FOST and B for $k_0 l_0 \gg 1$. I should like to point out that this discrepancy does not in fact exist and was owing only to an erroneous evaluation of the relevant equations by Bugnolo. Within certain limits both theories give the same scattering losses. Bugnolo's conclusion that FOST should not be used for $k_0 l_0 \gg 1$ is therefore not supported by his work, and instead the opposite conclusion is suggested.

The dispersion equation including scattering losses, as given by, for example, Eq. (23) from Ref. 3, reads in Bugnolo's notation

$$k_{\mathrm{eff}} = (\sqrt{2}\; l_0)^{-1}\{k_0^2 l_0^2 - (1 - ik_0 l_0)^2 \pm [(1-2ik_0 l_0)^2 + 4(4\pi)^2 r_e^2 l_0^4 \langle n_e^2\rangle]^{1/2}\}^{1/2}. \quad (1)$$

For $k_0 l_0 \gg 1$ and small fluctuations

$$\lambda_0^2 r_e^2 l_0^2 \langle n_e^2\rangle \ll 1, \quad (2)$$

($\lambda_0 = 2\pi/k_0$, $r_e = e^2/mc^2$), one obtains the two approximate solutions;

$$k'_{\mathrm{eff}} \approx k_0 (1 + i/k_0 l_0), \quad (3)$$

$$k''_{\mathrm{eff}} \approx k_0 [1 + (2\pi^2 r_e^2/k_0^4)\langle n_e^2\rangle(1 + 2ik_0 l_0)]. \quad (4)$$

Equation (3) is the solution given by Bugnolo[1] (apart from a factor of 2 which is missing). The relevant k_{eff} is, however, given by Eq. (4) since, owing to relation (2) k''_{eff} corresponds to a wave with less damping than the one with k'_{eff}. The damping coefficient due to the imaginary part k_2 of k''_{eff} is

$$2k_2 = 2r_e^2 \langle n_e^2\rangle\, l_0 \lambda_0^2, \quad (5)$$

and this agrees exactly with the value derived from Bugnolo's theory for $k_0 l_0 \gg 1$, Eq. (20) in Ref. 1 (apart from the misprint λ_0^{-2} instead of λ_0^2 in Ref. 1).

The first-order smoothing theory has been applied to the case $k_0 l_0 \gg 1$ from the very beginning of the theory,[2,6] and its justification for this case has been investigated by several authors.[7-9] For $k_0 l_0 \gg 1$ their results seem to agree.[9] The range of applicability of FOST as quoted by, for example, Ref. 9, Eq. (32b) in fact exactly agrees with condition (2) of this paper.

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