Production of $K\alpha$ and $L\alpha$ X Rays by Protons of 1.0–3.7 MeV*

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Cross sections for the production of x rays by protons have been determined at 1.00, 2.25, and 3.00 MeV. The $K\alpha$ production was measured for Ti, V, Fe, Ni, Cu, Ge, Rb, Zr, Ag, Sn, and Sb. The $L\alpha$ production was measured for Ce, Sm, Dy, Tm, W, Au, Pb, Bi, Th, and U. In addition, excitation functions were determined in 100-keV steps from 1.00 to 3.70 MeV for $K\alpha$ production in Ni, Ge, and Ag and for $L\alpha$ production in Sm, W, and Th. The results compare well with the predictions of the plane-wave Born approximation and the binary-encounter approximation.

I. INTRODUCTION

Recently it has been observed[1] that only a few data exist concerning K-shell ionization induced by protons of energy greater than 4 MeV. While it is true that more data exist for protons < 4 MeV than for protons > 4 MeV, there is hardly a surfeit. Lewis *et al.*[2] reported data on five elements (Mo, Ag, Ta, Au, and Pb) for bombarding energies between 1.70 and 3.15 MeV. The classic paper of Merzbacher and Lewis[3] added data on Ti, Fe, Ni, and U, in addition to verifying some of the results of Lewis *et al.* Khan *et al.*[4] studied ten elements but were limited to proton energies below 1.7 MeV. Recent results have been reported[1,5] for Ca, Ti, Ni, and Ag with protons of energy greater than 2 MeV. In summary, detailed information is available for only a few elements with protons of energy between 1.7 and 4.0 MeV. Additional data would have practical application in elemental analysis by proton-induced x-ray fluorescence, a field presently enjoying great popularity.

On the theoretical side, recent work by Garcia[6,7] on the binary-encounter approximation (BEA) gives evidence that such a description is in good agreement with the more standard plane-wave Born approximation (PWBA) of Merzbacher and co-workers[3,8] and, more importantly, agrees with the available experimental data. The advantage of the Garcia description is its scaling law: If u is the binding energy of the ionized shell and E is the proton bombarding energy, then $u^2\sigma_j$ is a function of E/u, independent of the atomic number, where σ_j is the ionization cross section for the jth subshell.

The purpose of this paper is twofold: (i) to report detailed x-ray-production cross-section data over the proton energy range available to us (1.0–3.7 MeV), and (ii) to test the applicability of the BEA model.

II. EXPERIMENTAL TECHNIQUE

Twenty-one targets were obtained[9] either as self-supporting foils or evaporated onto 40-μg/cm^2 carbon backings. Some of the targets were prepared in elemental form and others as compounds. The composition of the targets and their areal densities are listed in Table I. The targets were mounted in a chamber[10] designed for elemental analysis and were bombarded by protons from the Los Alamos Scientific Laboratory (LASL) 3.75-MV Van de Graaff.

The targets were viewed by two detectors, each at 90° with respect to the beam direction and on opposite sides of the target, as indicated in Fig. 1. The x-ray detector was a Si (Li) detector[11] having a 0.025-mm-thick Be window and a resolution of 175 eV at 5.9 keV. In reaching the detector the x rays traversed a 0.013-mm Mylar window and a 4.1-cm air path. Absorbers were inserted into this path to reduce x-ray counting rates so that dead-time corrections were less than 1%. The effect of each attenuator was determined to an ac-

TABLE I. Target thicknesses and measured proton-induced $K\alpha$ or $L\alpha$ x-ray-production cross sections.

Target	Thickness (μg/cm^2)	Transition measured	Cross sections (b)[a] 1.00 MeV	2.25 MeV	3.00 MeV
Ti	185	$K\alpha$	33	163	258
V	84	$K\alpha$	···	126	···
Fe	94	$K\alpha$	10	65	110
Ni	110	$K\alpha$	6.3	47	80
Cu	88	$K\alpha$	4.8	37	66
Ge	111	$K\alpha$	2.0	19	35
$RbNO_3$	72	$K\alpha$	0.49	6.4	13
Zr	···	$K\alpha$	0.26	3.0	6.3
Ag	109	$K\alpha$	0.053	0.85	1.9
Sn	94	$K\alpha$	0.026	0.45	1.1
Sb	98	$K\alpha$	0.021	0.44	1.0
Ce_2O_3	129	$L\alpha$	23	97	150
Sm	83	$L\alpha$	16	80	126
Dy	163	$L\alpha$	12	60	94
Tm	100	$L\alpha$	7.9	45	75
WO_3	81	$L\alpha$	4.8	29	49
Au	124	$L\alpha$	2.7	18	32
Pb	97	$L\alpha$	1.8	13	24
Bi	100	$L\alpha$	1.6	12	22
ThF_4	168	$L\alpha$	0.72	6.4	12
UF_4	122	$L\alpha$	0.55	5.4	10

[a]Standard deviation of ± 15%.

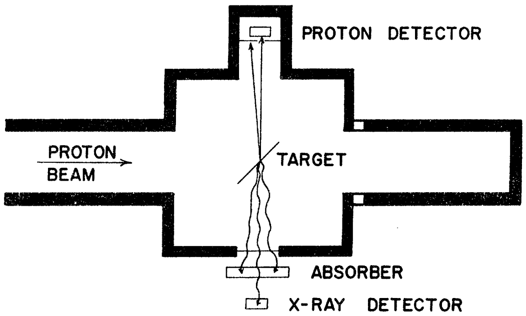

FIG. 1. Schematic representation of the apparatus. Details are given in Ref. 10.

curacy better than 2% for each radiation measured. The proton detector,[11] a surface-barrier detector of 300-μ depletion depth, viewed the targets through a 3.2-mm circular aperture placed 10.2 cm from the beam spot. Carbon slits defined the beam spot to be ~1×3 mm. This maintained constant solid angles to better than 2% for each detector.

In order to take account of target deterioration, errors in target thickness, and target nonuniformity, the data were normalized at each point to the number of elastically scattered protons at 90°. In this manner, day-to-day reproducibility was limited only by the statistical accuracy of the measurements.

Since it can be assumed[12,13] that for the elements and proton energies studied here the proton scattering cross section is approximately Rutherford, one can show that the x-ray-production cross section is given as

$$\sigma_x = 5.184 \frac{N_x}{N_p}\left(\frac{Z^2}{E^2}\right)\frac{\Omega_p}{\Omega_x \epsilon} \text{ mb},$$

where N_x is the number of x rays observed, N_p is the number of elastically scattered protons of incident energy E (MeV), Z is the atomic number of the target, Ω_p is the proton-detector solid angle in steradians, and $\Omega_x\epsilon$ is the total x-ray-detector efficiency. It has been assumed[3,5] that the x-ray production is isotropic. As an additional test of the assumption of Rutherford scattering, the elastic scattering cross section at 90° was determined at 3.00 MeV for each elemental target. The cross sections were found to be within 10% of Rutherford for each target.

The solid angle of the proton detector was determined from geometry. The x-ray detector efficiency was measured using the method described by Bissinger *et al.*[1]: An aluminum disk with a hole of 3.2-mm diam was centered over the detector. A ^{57}Co and an ^{88}Y source were placed in turn at the target position. Comparing the counting rates for ~14-keV photons with the disk in and out allowed the determination of the Si (Li) efficiency relative to that defined by the accurately measured disk. The intrinsic efficiency of the Si (Li) detector was assumed to be 100% at 14 keV. The relative efficiency as a function of x-ray energy was measured using ^{241}Am, ^{109}Cd, and ^{57}Co sources.[14] The overall efficiency was determined by correcting for the effects of the Mylar window, the air path, and the attenuation in the target itself. These corrections amounted to less than 1% for 14-keV photons and increased to 17% for the 5-keV $K\alpha$ x ray of Ti.

All data were taken to a statistical accuracy of ≤2%. The uncertainty introduced by normalization to the elastically scattered protons is estimated[12] to be less than 10%. Our own results confirmed this. While this error should be largely random, a systematic error may arise, since deviations from Rutherford usually manifest themselves as a relative decrease[12] in the cross section at 90°. The errors introduced by the determinations of the solid angles and the intrinsic Si (Li) efficiency are estimated to be approximately 10% total. We therefore assign an absolute standard deviation of 15% to our data, but relative measurements should be accurate to ≤10%.

III. RESULTS

The cross sections for $K\alpha$ x-ray production were determined for Ti, V, Fe, Ni, Cu, Ge, Rb, Zr, Ag, Sn, and Sb at 1.00, 2.25, and 3.00 MeV. The $L\alpha$ cross sections were measured for Ce, Sm, Dy, Tm, W, Au, Pb, Bi, Th, and U at the same energies. The results are listed in Table I and are plotted in Fig. 2. For all of these elements the $K\alpha$ or $L\alpha$ line was fully resolved from adjacent spectral lines. Excitation functions were measured in steps of 100 keV from 1.00 to 3.70 MeV for the $K\alpha$ transitions in Ni, Ge, and Ag and for the $L\alpha$ transitions in Sm, W, and Th. These data are shown in Table II and in Figs. 3 and 4.

Results for 3.00-MeV-proton bombardment of Ti have been reported[5] which are 30% larger than ours. Since the estimated accuracy of both measurements is 15% and since at 3 MeV the 90° elastic scattering cross section for Ti may be below that predicted by Rutherford (our cross sections would then have to be increased), the results are consistent. Our values for the Ag$K\alpha$ production can be compared with the AgK yields of Bissinger *et al.*[1] for 2.00 and 3.00 MeV by multiplying Bissinger's K x-ray-production cross sections by the $K\alpha/K_{\text{total}}$ ratio for silver. At both energies the agreement is within the 15% uncertainty of each measurement.

IV. COMPARISON WITH THEORY

Shown in Figs. 2–4 are the predictions made using the BEA theory of Garcia,[6,7] including the

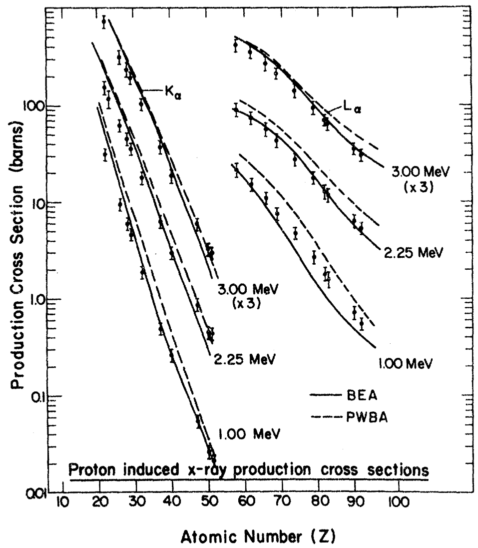

FIG. 2. Cross sections for $K\alpha$ and $L\alpha$ production by 1.00-, 2.25-, and 3.00-MeV protons plotted vs the atomic number Z. The data for 3.00 MeV have been multiplied by 3 to separate them from the other data. The BEA and PWBA curves were generated as described in the text.

effect of Coulomb deflection, and the corrected PWBA theory of Khandelwal *et al.*[8] Both of these theories predict K- and L-subshell ionization cross sections, so it is necessary to multiply them by the appropriate fluorescence yield (ω_K or ω_{L3}). Since our measurements are of $K\alpha$ and $L\alpha$ transitions, it is also necessary to correct for the yield of

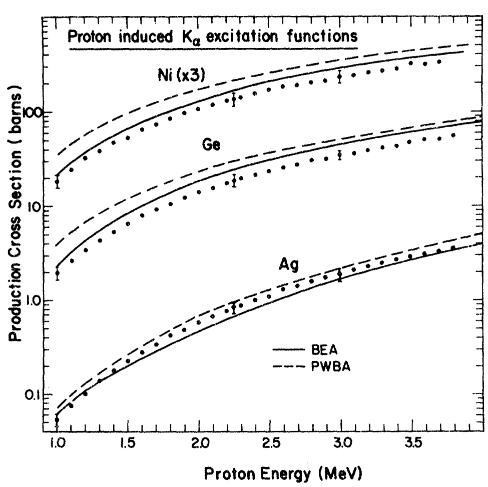

FIG. 3. $K\alpha$-production cross sections for Ni, Ge, and Ag plotted vs the proton bombarding energy. The Ni cross sections have been multiplied by 3 to separate them from the other data. The theoretical curves were generated as described in the text.

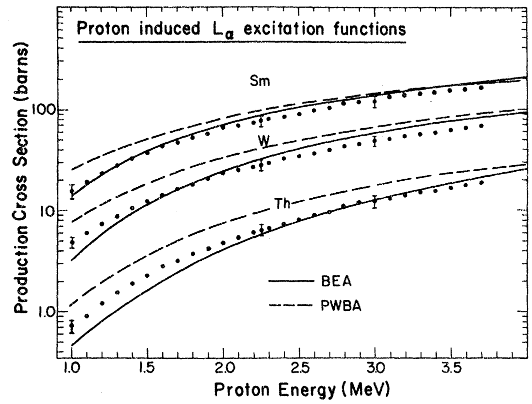

FIG. 4. $L\alpha$-production cross sections plotted vs the proton bombarding energy for Sm, W, and Th. The theoretical curves were generated according to the prescription in the text.

other K or L transitions. The values of ω_K used here were taken from Bambynek *et al.*[15] The ratios of $K\alpha$ transitions to all K transitions were

TABLE II. Excitation functions for proton-induced $K\alpha$ or $L\alpha$ x-ray production.

Proton energy (MeV)	X-ray-production cross sections (b)[a] $K\alpha$ Ni	Ge	Ag	$L\alpha$ Sm	W	Th
1.00	6.3	2.0	0.053	16	4.8	0.72
1.10	8.3	2.6	0.075	20	6.0	0.90
1.20	11	3.4	0.10	24	7.4	1.2
1.30	13	4.4	0.14	29	8.9	1.5
1.40	16	5.5	0.18	34	11	1.9
1.50	19	6.7	0.23	39	13	2.3
1.60	22	8.1	0.28	45	14	2.8
1.70	26	9.3	0.34	49	17	3.2
1.80	30	11	0.42	55	19	3.7
1.90	33	13	0.49	60	21	4.2
2.00	37	14	0.58	69	23	4.8
2.10	41	16	0.67	72	26	5.4
2.20	45	18	0.77	77	27	6.0
2.25	47	19	0.85	80	29	6.4
2.30	49	20	0.88	83	30	6.7
2.40	54	22	1.0	89	33	7.4
2.50	58	24	1.1	94	35	8.2
2.60	64	26	1.3	102	37	9.0
2.70	66	28	1.4	107	41	9.7
2.80	71	31	1.6	114	44	11
2.90	75	32	1.7	119	46	12
3.00	80	35	1.9	126	49	12
3.10	84	37	2.1	132	52	13
3.20	89	40	2.3	139	55	14
3.30	93	42	2.5	142	58	15
3.40	98	44	2.7	148	59	16
3.50	109	48	2.9	152	63	17
3.60	108	50	3.1	157	67	18
3.70	115	53	3.3	163	70	19
3.80	···	56	3.6	···	···	···

[a]Standard deviation of ± 15%.

derived from Nelson and Saunders.[16] The values for ω_{L3} were obtained by smoothing the values given by Bambynek *et al.*,[15] and the ratio of $L\alpha$ intensity to all $L3$ transitions was derived from Salem and Schultz.[17]

Inspection of Figs. 2–4 indicates that, over all, both theories reproduce the trends of the experimental data better than they predict the absolute cross section. Agreement appears somewhat better for the K shell, but this is expected for two reasons. First, the K-shell cross section of Garcia[7] has been used to estimate the L-shell cross section, even though the electron-velocity distributions of the two shells are not expected to be the same. Second, and probably more important, the values for ω_{L3} are less accurately known than the values for ω_K. The values of ω_K are known typically to ~ 5%, whereas the values of ω_{L3} are uncertain to 20 or 30%.

V. CONCLUSIONS

Both the PWBA and BEA give reasonable descriptions of the data, with superior fits provided by the BEA. It is worth mentioning that for most elements the limit of accuracy of the present experiment is determined by the estimate of our Si (Li)-detector efficiency and our assumption of Rutherford scattering. An apparatus with the proton monitor at a more forward angle, say 45°, would then improve accuracy since it is known (see, for example, Ref. 12) that deviations from Rutherford increase with increasing scattering angle.

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Series Solutions for Nonrelativistic Quantum-Mechanical Three-Body Problems

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A method for obtaining series solutions to nonrelativistic three-body problems is investigated; a calculation of the 1^1S ground state of the helium atom is performed as a test, and results are compared with previous work.

I. INTRODUCTION

A method for finding solutions to three-body problems in terms of a suitably chosen complete set of functions in each of the particle coordinates is investigated in this paper. The system chosen for study is the helium atom, because its low-lying eigenvalues have been studied in detail by Pekeris[1] and by Schwartz,[2] using variational approaches. The helium problem has an extensive literature,[3] of course, and numerous calculational methods have been advanced over the years. What is reported here is a straightforward calculation of some wave functions using a series expansion