Second-harmonic generation of Bogoliubov excitations in a two-component Bose-Einstein condensate

Guoxiang Huang,[1,2] Xin-qi Li,[1,3] and Jacob Szeftel[2]

[1]*Department of Physics and Key Laboratory for Optical and Magnetic Resonance Spectroscopy, East China Normal University, Shanghai 200062, China*

[2]*Laboratorie de Physique Théorique de la Matière Condensée, Case 7020, Université Paris VII—Denis Diderot, 2 Place Jussieu, F-75251 Paris Cedex 05, France*

[3]*National Laboratory for Superlattices and Microstructures, Institute of Semiconductors, Chinese Academy of Sciences, P. O. Box 912, Beijing 100083, China*

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A second-harmonic generation (SHG) is predicted for the Bogoliubov excitations in a two-component Bose-Einstein condensate. It is shown that, because the linear dispersion curve of the excitations displays two branches, the phase-matching condition for the SHG can be fulfilled if the wave vectors and frequencies of fundamental and second-harmonic waves are selected suitably from different branches. The nonlinearly coupled envelope equations for the SHG are derived by using a method of multiple scales. The explicit solutions of these envelope equations are provided and the conversion efficiency of the SHG is also discussed.

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Since the successful experimental realization of Bose-Einstein condensation in weakly interacting atomic gases, much progress has been made on the study of linear collective excitations (or called Bogoliubov quasiparticles) in Bose-Einstein condensates (BECs) [1]. The nonlinear collective excitations in BECs have also attracted much attention [2–10]. The investigation on the nonlinear collective excitations up to now can be classified into two types. One of them is the low-energy excitations with the size the same as that of condensate. The eigenfrequencies of such excitations are discrete, i.e., they are standing wave modes. The nonlinear frequency shift and mode coupling have been explored both theoretically and experimentally [4–10]. A very interesting work in this aspect is the experimental observation by Hechenblaikner *et al.* on the harmonic generation from a low-lying mode to a high-lying mode of condensed rubidium gas in a harmonic trap [7]. The other type of excitations explored are those with the size much smaller than that of condensate. In this case the excitations have higher energy and their eigenfrequencies are continuous (or quasi-continuous), characterizing the intrinsic bulk property of the condensate [11]. Such collective excitations can propagate a fairly long time before reaching the boundary of condensate. The most typical nonlinear excitations of such kind explored in BECs are solitary excitations, including dark [2] and bright [3] solitons. Recently, Ozeri *et al.* [12] investigated the mixing of three propagating wave modes with energy down-conversion in a homogeneous, single-component BEC with a repulsive interatomic interaction. However, as far as we know so far no work has been reported on a second-harmonic generation (SHG) for *propagating* collective excitations in BECs. It is this topic that will be addressed here.

Note that for a single-component BEC the SHG is not possible because its excitation spectrum, which takes the form $\omega(q)=q(c^2+q^2/4)^{1/2}$ [1] with q the wave number of the excitation and c the sound speed of the system, cannot satisfy the phase-matching condition for the SHG. Thus we consider a two-component BEC, whose excitation spectrum displays two branches and hence provides us with the possibility to fulfill the SHG phase-matching condition. Since the SHG is a process of energy up-conversion, at zero temperature such process can be well described by two coupled Gross-Pitaevskii (GP) equations. Using a method of multiple scales we derive the nonlinearly coupled envelope equations describing the SHG and give their explicit solutions. We show that an experimental realization of such SHG may give information about the interaction between different components of the condensate.

We consider a two-component BEC, which is a binary mixture of condensate. Such mixture may consist of different particles such as ^{87}Rb and ^{23}Na, or different isotopes such as ^{87}Rb and ^{85}Rb, or different hyperfine spin states of the same species. Denoting $\Psi_j(\mathbf{r},t)$ as the order parameter of species j with particle number $N_j=\int d\mathbf{r}|\Psi_j|^2$ $(j=1,2)$, Ψ_j satisfy the GP equations

$$i\hbar\frac{\partial\Psi_1}{\partial t}=[-(\hbar^2/2m_1)\nabla^2+V_1(\mathbf{r})+g_{11}|\Psi_1|^2+g_{12}|\Psi_2|^2]\Psi_1, \tag{1}$$

$$i\hbar\frac{\partial\Psi_2}{\partial t}=[-(\hbar^2/2m_2)\nabla^2+V_2(\mathbf{r})+g_{21}|\Psi_1|^2+g_{22}|\Psi_2|^2]\Psi_2, \tag{2}$$

where m_j and $V_j(\mathbf{r})$ are respectively the atomic mass and external trapping potential for the species j, $g_{jl}=2\pi\hbar^2a_{jl}/m_{jl}$ is interaction parameter with a_{jl} $(j,l=1,2)$ being the s-wave scattering length between the species j and the species l ($a_{jl}>0$ for repulsive interaction) and $m_{jl}=m_jm_l/(m_j+m_l)$ being the reduced mass. We consider a quasi-one-dimensional (1D) trap with a negligible axial confinement. Such a system can be realized experimentally by using, e.g., a toroidal trap of radius R and cross area πr^2 with the condition $r\ll R$ satisfied. Thus the trapping potentials take the form $V_j(\mathbf{r})=(m_j/2)\omega_{j\perp}^2(y^2+z^2)]$, where $\omega_{j\perp}$ are the

trap frequencies of the species j in transverse directions. For simplicity we assume $m_1=m_2=m$ and $\omega_{1\perp}=\omega_{2\perp}=\omega_\perp$. Such assumption is only for getting simplified expressions and clarifying essential physics. A more general case can be considered with a similar result given below.

Expressing the order parameters in terms of their modulus and phases, i.e., $\Psi_j=\sqrt{n_j}\exp(i\phi_j)$, we obtain a set of coupled nonlinear equations for n_j and ϕ_j $(j=1,2)$. By letting the condensate densities, time, axial spatial coordinate and transverse spatial coordinates are measured respectively in the units $n_0=N_1/(La_\perp^2)$, $t_0=\omega_\perp^{-1}$, L (condensate length in the axial direction), and $a_\perp=[\hbar/(m\omega_\perp)]^{1/2}$ (harmonic oscillator length in the transverse directions), these equations become dimensionless. We assume that the transverse confinement is strong enough so that the conditions $a_\perp \ll l_0$ [where $l_0=(4\pi n_0 g_{11})^{-1/2}$ is healing length], and $n_0 g_{11} \ll \hbar\omega_\perp$ can be satisfied. Under these conditions the system can be taken as quasi-1D, which implies that at sufficiently low temperature the transverse motion of the particles is essentially "frozen" and governed by the ground-state wave functions of corresponding transverse harmonic oscillators [13,14]. Thus one can take $\sqrt{n_j}=A_j(x,t)\psi_0(y)\psi_0(z)$ and $\phi_j=\phi_j(x,t)(j=1,2)$, where $\psi_0(y)\psi_0(z)=\pi^{-1/2}\exp[-(y^2+z^2)/2]$ is the normalized ground state wave function of a 2D harmonic oscillator. We obtain

$$\partial A_j/\partial t+\varepsilon[(\partial A_j/\partial x)(\partial\phi_j/\partial x)+\tfrac{1}{2}A_j(\partial^2\phi_j/\partial x^2)]=0, \tag{3}$$

$$(\partial\phi_j/\partial t+1)A_j+\varepsilon[-\tfrac{1}{2}\partial^2/\partial x^2+\tfrac{1}{2}(\partial\phi_j/\partial x)^2+(1/2\pi)(G_{jj}A_j^2+G_{j3-j}A_{3-j}^2)]A_j=0, \tag{4}$$

with $G_{ij}=g_{ij}/g_{11}$. The parameter $\varepsilon=n_0 g_{11}/(\hbar\omega_\perp)$, i.e., the ratio between the atomic interaction and the strength of the transverse confinement, is a natural expansion parameter for solving Eqs. (3) and (4) approximately.

The stationary (ground) state solution of the above equations is given by $A_1=A_{1GS}=1/\sqrt{L}$, $A_2=A_{2GS}=\sqrt{N_2/(N_1L)}$, $\phi_{1GS}=-(1+\mu_1^{(1)})t$, and $\phi_{2GS}=-(1+\mu_2^{(1)})t$ with $\mu_1^{(1)}=(G_{11}+G_{12}N_2/N_1)/(2\pi L)$, and $\mu_2^{(1)}=(G_{21}+G_{22}N_2/N_1)/(2\pi L)$. The linear dispersion relation of an excitation from the ground state is given by

$$\omega^2(q)/q^2=\tfrac{1}{2}(\tilde{G}_{11}+\tilde{G}_{22})+\tfrac{1}{4}q^2\pm\tfrac{1}{2}[(\tilde{G}_{11}-\tilde{G}_{22})^2+4\tilde{G}_{12}\tilde{G}_{21}]^{1/2}, \tag{5}$$

where $\tilde{G}_{11}=G_{11}/(2\pi L)$, $\tilde{G}_{12}=G_{12}N_2/(2\pi LN_1)$, $\tilde{G}_{21}=G_{21}/(2\pi L)$, and $\tilde{G}_{22}=G_{22}N_2/(2\pi LN_1)$. q and ω are wave vector and frequency of the excitation, respectively. From Eq. (5) we see that the dispersion curve of the collective modes has two branches, i.e., the upper branch $\omega_+(q)$ and the lower branch $\omega_-(q)$; both of them are acoustic.

A necessary condition for the SHG is that the phase-matching condition, i.e.,

$$q_2=2q_1,\quad \omega_2=2\omega_1, \tag{6}$$

must be fulfilled, where $q_1(q_2)$ and $\omega_1(\omega_2)$ are the wave vector and frequency of the fundamental (second–harmonic) wave, respectively. By choosing $\omega_1=\omega_+(q_1)$ and $\omega_2=\omega_-(q_2)=\omega_-(2q_1)$, the condition (6) is equivalent to $\omega_-(2q_1)=2\omega_+(q_1)$, which results in the solution

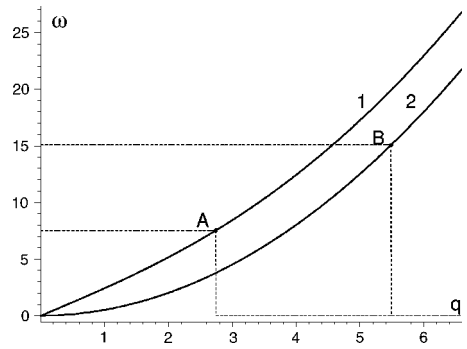

FIG. 1. The dispersion curve of a two-component BEC consisting of two different hyperfine spin states in a trap with the particle numbers $N_1=N_2=2\times10^6$. Curves 1 and 2 represent the upper branch, $\omega_+(q)$, and the lower branch, $\omega_-(q)$, respectively. The interaction parameters of the system are $G_{11}=1.0$, $G_{12}=G_{21}=0.9926$ and $G_{22}=1.0027$. For the fundamental wave and the second-harmonic wave, the phase-matched wave vectors and frequencies for the SHG are $(q_1=2.743,\omega_1=7.536)$ and $(q_2=5.486,\omega_2=15.072)$, which have been illustrated by the points $A=(q_1,\omega_1)$ and $B=(q_2,\omega_2)$, respectively.

$$q_1=(2/\sqrt{3})[(\tilde{G}_{11}-\tilde{G}_{22})^2+4\tilde{G}_{12}\tilde{G}_{21}]^{1/4}. \tag{7}$$

Thus for the two-component condensate the SHG phase-matching condition can be satisfied based on the multivalue property of the linear dispersion relation. Shown in Fig. 1 is the dispersion curve of the collective modes of the two-component BEC consisting of different hyperfine spin states in the trap with the particle numbers $N_1=N_2=2\times10^6$. For this system one has $G_{11}=1.0$, $G_{12}=0.9926$ and $G_{22}=1.0027$ [15]. The modes satisfying the phase-matching condition (6) have been clearly shown as the point $A=(q_1,\omega_1)$ (the fundamental wave) and the point $B=(q_2,\omega_2)$ (the second-harmonic wave). From Eq. (7) we obtain $q_1=2.743$ and hence $\omega_1=7.536$, $q_2=5.486$ and $\omega_2=15.072$.

We know that an optical SHG occurs in active media with no inversion symmetry. For trapped atoms this symmetry is not broken and hence the SHG in the BEC is possible only when the ground state (condensate) is not depleted by the excitations. This imposes a constraint that the amplitude of the excitations cannot be too large. Here we develop a weak nonlinear theory for the SHG in the BEC by making the asymptotic expansion $A_j-A_{jGS}=A_{jGS}(\varepsilon F_j^{(1)}+\varepsilon^2F_j^{(2)}+\cdots)$ and $\phi_j-\phi_{jGS}=\varepsilon\phi_j^{(1)}+\varepsilon^2\phi_j^{(2)}+\cdots$, where $F_j^{(l)}$ and $\phi_j^{(l)}$ are the functions of the fast variables x, τ and the slow variables $X=\varepsilon x$, $T=\varepsilon\tau$. The expansion parameter $\varepsilon[=n_0g_{11}/(\hbar\omega_\perp)]$ can be small as long as the typical value of the chemical potential is less than the level spacing of the harmonic oscillator. Using such expansion Eqs. (3) and (4) are transferred into a set of equations for $F_j^{(l)}$ and $\phi_j^{(l)}$ $(j=1,2;l=1,2,3,\ldots)$. In the first-order $(l=1)$ we get the solution in linear approximation. For the SHG we take

$$F_1^{(1)}=U_1\exp(i\theta_1)+U_2\exp(i\theta_2)+\text{c.c.}, \tag{8}$$

$$F_2^{(1)} = (\tilde{G}_{12}q_1^2)^{-1}L_1(\omega_1,q_1)U_1\exp(i\theta_1) + (\tilde{G}_{12}q_2^2)^{-1}L_1(\omega_2,q_2)U_2\exp(i\theta_2) + \text{c.c.}, \quad (9)$$

$$\phi_1^{(1)} = -i(2\omega_1/q_1^2)U_1\exp(i\theta_1) - i(2\omega_2/q_2^2)U_2\exp(i\theta_2) + \text{c.c.}, \quad (10)$$

$$\phi_2^{(1)} = -i2\gamma_m\omega_1(\tilde{G}_{12}q_1^4)^{-1}L_1(\omega_1,q_1)U_1\exp(i\theta_1) - i2\gamma_m\omega_2(\tilde{G}_{12}q_2^4)^{-1}L_1(\omega_2,q_2)U_2\exp(i\theta_2) + \text{c.c.}, \quad (11)$$

where $L_1(\omega,q)=\omega^2-(\tilde{G}_{11}+q^2/4)q^2$, U_1 and U_2 are respectively the envelope functions of the fundamental wave (with the phase $\theta_1=q_1x-\omega_1\tau$) and the second-harmonic wave (with the phase $\theta_2=q_2x-\omega_2\tau$). q_1, q_2, ω_1 and ω_2 are chosen according to the SHG phase-matching condition (6), i.e., $\omega_1=\omega_+(q_1)$ and $\omega_2=\omega_-(q_2)$ with $q_2=2q_1$.

At the second order ($l=2$), solvability conditions give the closed equations for U_1 and U_2. After making the transformation $U_j=\varepsilon u_j$ and noting that $X=\varepsilon x$ and $T=\varepsilon\tau$, we get

$$\partial u_1/\partial\tau + v_{g1}(\partial u_1/\partial x) + i\Gamma_1 u_1^* u_2\exp(-i\Delta q x) = 0, \quad (12)$$

$$\partial u_2/\partial\tau + v_{g2}(\partial u_2/\partial x) + i\Gamma_2 u_1^2\exp(-i\Delta q x) = 0, \quad (13)$$

where $v_{gj}=(d\omega_+/dq)_{q=q_j}$ is the group velocity of jth wave, $\Delta q=q_2-2q_1$ is a possible phase mismatch. The expressions of v_{gj} ($j=1,2$) is given by

$$v_{gj} = \frac{L_1(\omega_j,q_j)(\omega_j^2+q_j^4/4)+L_2(\omega_j,q_j)(\omega_j^2+q_j^4/4)}{q_j\omega_j[L_1(\omega_j,q_j)+L_2(\omega_j,q_j)]}, \quad (14)$$

where $L_2(\omega,q)=\omega^2-(\tilde{G}_{22}+q^2/4)q^2$. The nonlinear coefficients appearing in Eqs. (12) and (13) read

$$\Gamma_1 = \Delta_1/\{2\omega_1[L_1(\omega_1,q_1)+L_2(\omega_1,q_1)]\}, \quad (15)$$

$$\Gamma_2 = \Delta_2/\{2\omega_2[L_1(\omega_2,q_2)+L_2(\omega_2,q_2)]\}, \quad (16)$$

with

$$\Delta_1 = L_2(\omega_1,q_1)[3(\omega_1^2+\tilde{G}_{11}q_1^2)+L_1(\omega_1,q_1)+\tfrac{1}{4}L_1(\omega_2,q_2) + [L_1(\omega_1,q_1)L_1(\omega_2,q_2)]/(4\tilde{G}_{12}q_1^2)] + \tilde{G}_{12}\tilde{G}_{21}q_1^4 + \{[3(\omega_1^2+\tilde{G}_{22}q_1^2)]/(4\tilde{G}_{12}q_1^2)\}L_1(\omega_1,q_1)L_1(\omega_2,q_2) + \tilde{G}_{21}q_1^2[L_1(\omega_1,q_1)+\tfrac{1}{4}L_1(\omega_2,q_2)], \quad (17)$$

$$\Delta_2 = (24/\tilde{G}_{12})L_1^2(\omega_1,q_1)(\omega_1^2/q_1^2+\tilde{G}_{22}) + 8\tilde{G}_{12}\tilde{G}_{21}q_1^4 + 16\tilde{G}_{21}q_1^2L_1(\omega_1,q_1) + L_2(\omega_2,q_2)[6\omega_1^2+6\tilde{G}_{11}q_1^2 + 4L_1(\omega_1,q_1)] + [2L_1^2(\omega_1,q_1)](\tilde{G}_{12}q_1^2)]. \quad (18)$$

We now consider the solutions of Eqs. (12) and (13) corresponding to the SHG. For a stationary case ($\partial/\partial\tau=0$) and for $\Delta q=0$, Eqs. (12) and (13) admit the solution [16]

$$u_1 = (-\Gamma_1 W/v_{g1})^{1/2}\text{sech}[\Gamma_1/v_{g1}(-\Gamma_2 W/v_{g2})^{1/2}x]e^{i\varphi_0}, \quad (19)$$

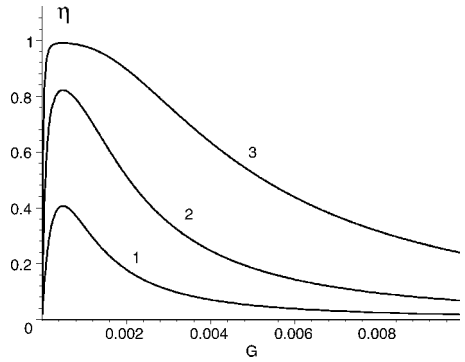

FIG. 2. The conversion efficiency as a function of interspecies interaction parameter $G(=G_{12}=G_{21})$ and propagating distance x in a stationary SHG for $G_{11}=1.0$, $G_{22}=1.0027$. Curves 1, 2, and 3 correspond to the input power $W=10.0$ with the propagating distance x taking the values 5.0, 10.0, and 20.0, respectively.

$$u_2 = (-\Gamma_2 W/v_{g2})^{1/2}\tanh[\Gamma_1/v_{g1}(-\Gamma_2 W/v_{g2})^{1/2}x]e^{i(2\varphi_0+\pi/2)}, \quad (20)$$

where $W(x)=-(v_{g1}/\Gamma_1)|u_1|^2-(v_{g2}/\Gamma_2)|u_2|^2=W(0)$ is a constant (denoting the input power of the excitation) and φ_0 is an arbitrary constant. At $x=0$, the fundamental wave takes the total power W of the system and thus power of the second-harmonic wave is zero. As x increases the power of the fundamental wave is converted gradually into the second-harmonic wave. At distance x, the conversion efficiency from the fundamental wave into the second-harmonic wave is given by

$$\eta = W_2(x)/W_1(0) = \tanh^2[\Gamma_1/v_{g1}(-\Gamma_2 W/v_{g2})^{1/2}x], \quad (21)$$

where $W_j(x)=-(v_{gj}/\Gamma_j)|u_j|^2$ is the power of jth wave. Thus the conversion efficiency of the SHG is determined by v_{gj}, Γ_j ($j=1,2$), W and the propagating distance x. From Eqs. (14)–(18) we see that, since v_{gj} and Γ_j depends on the interatomic interaction parameters g_{ij}, a larger conversion efficiency can be obtained by controlling g_{ij}.

Note that the solution (19) and (20) is valid only for $(\Gamma_1/v_{g1})(\Gamma_2/v_{g2})>0$. Because both v_{g1} and v_{g2} are positive, we require that Γ_1 and Γ_2 have the same sign. It can be shown that both Γ_1 and Γ_2 are negative for the interaction parameters $G_{11}=1.0$, $G_{22}=1.0027$ when G_{12} takes the value in the interval between zero and 1. Thus the solution (19) and (20) is physically realizable.

Shown in Fig. 2 is the conversion efficiency η as a function of $G_{12}(=G_{21})$ and the propagating distance (or sample length) x when $G_{11}=1.0$ and $G_{22}=1.0027$. The input power is taken as $W=10.0$. The curves 1, 2, and 3 correspond to the propagating distance x taking the values 5.0, 10.0, and 20.0, respectively. From Fig. 2 we see that to obtain a significant conversion efficiency of the SHG, in addition to a larger propagating distance and a larger input power, one must choose an appropriate interspecies interaction strength G_{12}. This provides us also a possibility for determining G_{12} by measuring the SHG conversion efficiency η.

Note that for a condensate of finite length L, to make the localized solution (19) and (20) be valid the size of the first

harmonic localization **(**i.e., $(v_{g1}/|\Gamma_1|)[v_{g2}/(|\Gamma_2|W)]^{1/2}$**)** should be less than L, which results in the requirement $L \geqslant (v_{g1}/|\Gamma_1|)[v_{g2}/(|\Gamma_2|W)]^{1/2}$. For $W=10.0$ one has $L \geqslant 8.0$ (in unit of the healing length l_0, which is about 1 μm). To get a larger conversion efficiency η in a real experiment, one should use a long enough condensate.

For very short-pulse excitations the walkoff effect due to different group velocities for the fundamental and the second-harmonic wave must be taken into account [16]. By the transformation $u_1=[v_{g1}v_{g2}/(\Gamma_1\Gamma_2)]^{1/2}w_1\exp(i\varphi)$ and $u_2=(v_{g1}/\Gamma_1)w_2\exp[i(2\varphi+\pi/2)]$ (w_1, w_2 and φ are real functions of x and τ) and assumption $\Delta q=0$, Eqs. (12) and (13) become

$$\partial w_1/\partial x + (1/v_{g1})(\partial w_1/\partial \tau) = w_1 w_2, \tag{22}$$

$$\partial w_2/\partial x + (1/v_{g2})(\partial w_2/\partial \tau) = -w_1^2. \tag{23}$$

Consider a traveling-wave solution, i.e., take w_j $(j=1,2)$ are the functions of x and $\zeta=\tau-x/v_{g1}$, Eqs. (22) and (23) are transferred as $\partial w_1/\partial x = w_1 w_2$, $\partial w_2/\partial x + \nu \partial w_2/\partial \zeta = -w_1^2$, where $\nu=1/v_{g2}-1/v_{g1}$ is a parameter denoting the walkoff (or group-velocity dispersion) effect. If at $x=0$ the fundamental wave and the second-harmonic wave take the form $w_1(0,\tau)=A_0/(1+\tau^2/\tau_0^2)$ and $w_2(0,\tau)=0$, where τ_0 is the initial pulse width and A_0 is a constant representing the initial amplitude of the fundamental wave, one can get the following solution [16]:

$$w_1(\zeta,x) = \frac{A_0}{(1+\tilde{\zeta}^2)^{1/2}[1+(\tilde{\zeta}-\tilde{x})^2]^{1/2}} \frac{1}{\cosh\,\xi + \tilde{\zeta}/f_0 \sinh\,\xi}, \tag{24}$$

$$w_2(\zeta,x) = \frac{\tau_{cr}}{\tau} \frac{A_0}{1+(\tilde{\zeta}-\tilde{x})^2} \frac{\tilde{x}\cosh\,\xi + [f_0 - \tilde{\zeta}(\tilde{\zeta}-\tilde{x})/f_0]\sinh\,\xi}{\cosh\,\xi + \tilde{\zeta}/f_0}, \tag{25}$$

where $\tilde{\zeta}=\zeta/\tau_0$, $\tilde{x}=x/L_\nu$, $f_0=(\tau_0^2/\tau_{cr}^2-1)^{1/2}$, $\tau_{cr}=\nu L_{NL}$, $\xi = f_0[\tan^{-1}\tilde{\zeta}-\tan^{-1}(\tilde{\zeta}-\tilde{x})]$ with $L_\nu=\tau_0/\nu$ (walk-off or dispersion length) and $L_{NL}=A_0^{-1}$ (nonlinear length). If the walk-off length is much larger than the nonlinear length, i.e., $L_\nu \gg L_{NL}$, the walk-off effect can be neglected. In this case the solution (24) and (25) is simplified into $w_1(\zeta,x)=A_0/(1+\tilde{\zeta}^2)\mathrm{sech}[A_0 x/(1+\tilde{\zeta})]$ and $w_2(\zeta,x)=A_0/(1+\tilde{\zeta}^2)\tanh[A_0 x/(1+\tilde{\zeta})]$. This situation corresponds to a quasistationary SHG and only in this case the conversion efficiency is significant.

In conclusion, we have made a theoretical prediction of SHG for propagating nonlinear collective excitations in a quasi-1D, two-component BEC. We have shown that the phase-matching condition of the SHG can be satisfied if the wave vectors and frequencies of fundamental and second-harmonic waves are selected suitably from different branches of the linear dispersion curve. We have derived the nonlinear envelope equations for the SHG by using a method of multiple scales and presented the SHG solutions. The conversion efficiency from the fundamental wave to the second-harmonic wave has also been discussed. The results presented here can also be generalized to the quasi-1D BEC with a slowly varying axial confining potential. In this case the envelope equations (12) and (13) are still valid but their coefficients will depend on the slow-variable X. Although solving the SHG problem in such trap is a further topic, one can expect that the main character of the solution will not be changed. To experimentally test the prediction given above, one can use the method developed in Ref. [17] to generate high frequency and weak nonlinear excitations of small size by suddenly modifying the trapping potential using optical dipole force of a focused laser beam. To get a larger conversion efficiency one can adjust the parameters of the trapping potential. It is better to use a condensate long enough (as in Ref. [17]) so that two wave modes can have a significant energy transfer. Another way is to change the interspecies interaction parameter G_{12} by using Feshbach resonance technique [1]. Inversely, the measurement of the conversion efficiency η in the SHG may provide a possibility to determine the interspecies interaction parameter G_{12} for a two-component BEC.

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